Scientific Research and Essays Vol. 7(32), pp. 2857-2867, 16 August, 2012
Available online at http://www.academicjournals.org/SRE
DOI: 10.5897/SRE11.765
ISSN 1992-2248 ©2012 Academic Journals

Full Length Research Paper

Covering stack of layers consisting of a single layer: Local near-surface buckling instability

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Accepted 16 July, 2012

The local near-surface buckling of a material system consisting of a half-space, which is covered by the single layer and half-space materials is elastic within the framework of a three-dimensional linearized theory of stability (TLTS). The equations of TLTS are obtained from the three-dimensional geometrically nonlinear equations of the theory of viscoelasticity by using the boundary form perturbations technique. By employing the Laplace and Fourier transform, a method for solving the problem is developed. Numerical results on the critical compressive forces and the critical times are presented.

Key words: Buckling instability, curved-layer, critical time, local near-surface buckling, stability, viscoelastic layer.

INTRODUCTION

The results of the theoretical study of the growth of the initial imperfections one of which can be taken from the insignificant curving of the reinforcing layers or fibers in the structure of the composite materials can be used for estimation of the loading carried out by capacity of these materials. In recent years, it was established that the carbon-nanotubes or –nanofibers have a curving in the structure of the polymer-nanocomposites (Xiao et al., 2007). This statement also increases the significance of the study which shows that the mechanical behavior of the carbon-nanotubes influences the curving of the reinforcing elements in the structure of the composite materials. According to Guz (1990), Akbarov and Guz (2000), Akbarov and Guz (2004) and others, the curving of the reinforcing elements may be due to the design features (as in a woven composites), or to technological processes resulting from the action of various factors (as in a polymer-nanocomposites). Moreover, the aforementioned curving can be taken (Akbarov and Guz, 2000; Akbarov et al., 1997; Akbarov and Kosker, 2004) as a geometrical model for the structure of the composite materials for the investigation of the various type of fracture (internal or near-surface stability loss) problems for the unidirectional composites under compression along the reinforcing elements. Owing to such modeling, employing ‘‘boundary form perturbation’’ technique in the papers (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009), the three-dimensional linearized theory of stability (TLTS) (Biot, 1965; Guz, 1999) was developed for the internal and near-surface stability loss problems for viscoelastic composite materials by employing the initial imperfection criterion (Hoff, 1958). In this case, the development of these imperfections with the time flow is investigated within the scope of the piecewise homogeneous body model by the use of the three-dimensional geometrically nonlinear field equations of the theory of the viscoelasticity. Using the series representation of the sought values in small parameter characterizing the degree of the initial insignificant imperfections of the reinforcing elements, the solution of the nonlinear boundary value problems is reduced to the solution of the series linear boundary-value problems. By direct verification, it is proven that the linear equations and relations which are attained in these linear boundary

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value problems coincide with the corresponding ones of the TLTS. Just aforementioned statements allows the authors of the papers (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009) to take into account the initial imperfection in the relations of the TLTS and employ the TLTS to investigate the stability loss problems of the time-dependent materials within the framework of the initial imperfection criterion. Moreover, in the paper (Akbarov et al., 1997), it was proved that for the investigation of the stability loss problems and the determination of the values of the critical forces or critical time results obtained within the framework of only the zeroth and first approximations are enough.

Now, we consider some details of the results obtained in the papers (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009) and start with the paper (Akbarov et al., 1997) in which it was assumed that the mode of the initial imperfection of the reinforcing layers is the co-phase periodical plane curving (the plane-strain state was considered). In this case, by employing the aforementioned approach, the values of the critical forces for elastic composites and the values of the critical time for the viscoelastic composites were determined and it was established that in the particular cases, the values of the critical forces coincide with the corresponding results shown in Babich et al. (2001) which were attained by employing the Euler approach. In the paper (Akbarov and Kosker, 2004), the approach (Akbarov et al., 1997) was developed for the unidirectional fibrous viscoelastic composite materials. The near-surface stability loss problems for layered half-plane and half-space are studied in the paper of Akbarov and Tekercioğlu (2006) and Akbarov and Tekercioğlu (2007), respectively.

As applied to various structural elements, the conditions of existence of internal as well as surface instability can be represented as min$|p_{cr.}| < |p_{cr.}^{st}|$, $\ell_{cr.} << L$, where min$|p_{cr.}|$ and $|p_{cr.}^{st}|$ are the critical loads corresponding to the internal or surface instability and buckling of the whole structural element in question, $\ell_{cr.}$ is the half-wavelength of the mode of internal or surface instability and L is the characteristic (minimum) dimension of the structural element. Thus, the phenomenon of internal or surface instability exists in the case where the dependence $|p_{cr.}| = |p_{cr.}(\chi)|$ has a well-defined minimum under $\chi \neq 0$ and the critical value of external load is determined as min$|p_{cr.}|$ (where $\chi = \pi h / \ell$ for layered or $\chi = \pi R / \ell$ for fibrous materials; h is thickness of the reinforcing layer, R is a radius of fibre cross-section, ℓ is the half-wavelength of the initial imperfection mode). The value of the wave-generation parameter χ corresponding to min$|p_{cr.}|$ is taken as critical value of that and denoted as $\chi_{cr.} = \pi h / \ell_{cr.}$.

Note that for the viscoelastic composites, the foregoing procedures are made for the time t=0 and $t = \infty$ (where t is a time) separately and $p_{cr.0}$ (for $t = 0$) and $p_{cr.\infty}$ (for $t = \infty$) are defined. For these cases where $p_{cr.\infty} < p < p_{cr.0}$ the values of the critical time (denoted by $t_{cr.}$) are defined.

In the paper (Akbarov et al., 1999), the composite consisting of the alternating layers of two materials is also considered and as initial imperfection in the structure of the material, the local curving of the filler layers is taken. According to Akbarov et al. (1997), Akbarov and Kosker (2004), Akbarov and Tekercioğlu (2006) and Akbarov and Tekercioğlu (2007), it was assumed that this composite is compressed at infinity along the layers and the development of the aforementioned initial local imperfection with compressive force for elastic composite as well as with time for viscoelastic composite is investigated. As a result of these investigations which were made for the plane strain state, it was established that the critical values of the compressive force for the elastic composite as well as the values of the critical time for the viscoelastic composite does not depend on the initial local imperfection mode and on the values of the wave-generation type (as χ) parameter. By direct verification, it is proved that the critical values of the compressive force coincide with the corresponding values of the theoretical strength limit in compression (TSLC) which are determined within the framework of the continual approach (Guz, 1990, 1999). Consequently, the values obtained for the critical time were related to the TSLC of the corresponding viscoelastic composite material. Thus, in the paper (Akbarov et al., 1999), the approach proposed in Akbarov et al. (1997) was developed for the determination of the TSLC of the unidirectional elastic and viscoelastic composite materials within the framework of the piecewise homogeneous body model.

However, up to now, there were no studies on the near-surface local stability loss problems based on the development of the initial insignificant near-surface local curving (imperfection) of the reinforcing layer with external compressive force for the viscoelastic composite materials within the framework of the approach (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009). In the present paper, the first attempt is made in this field and the approach (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and

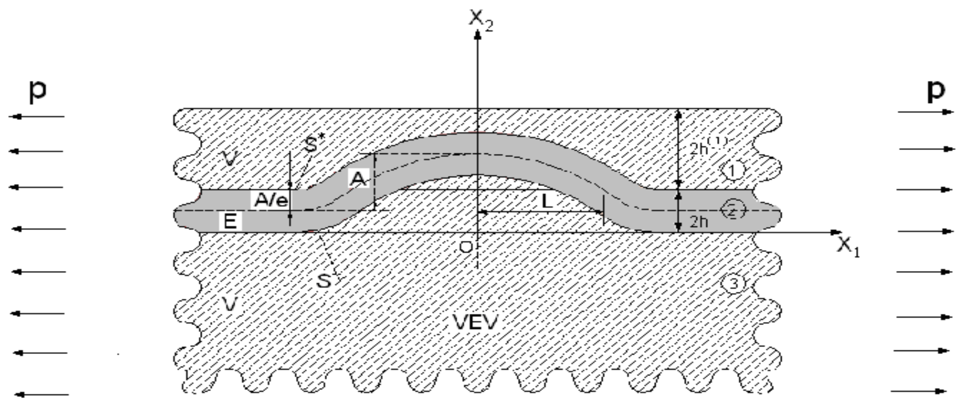

Figure 1. The geometry of the structure of the material considered.

Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009) is developed and applied for the investigation on the near-surface local stability loss of the system consisting of the elastic (viscoelastic) substrate, viscoelastic (elastic) bond layer and elastic (viscoelastic) covering layer. According to Akbarov et al. (1997), Akbarov and Kosker (2004), Akbarov and Tekercioğlu (2006), Akbarov and Tekercioğlu (2007), Akbarov et al. (1999), Aliyev (2007) and Argatov (2009), the equations and relations of the TLTS are attained from the geometrically nonlinear exact equations of the theory of the viscoelasticity by employing the boundary-form perturbation technique.

Problem formulation

Considering a semi-infinite half-space joined with the stack consisting of the finite number of the filler (reinforcing) and binder layers, we are assuming here that the filler layers in this system have an insignificant initial imperfection in the curving form. To ease the evaluation, one may suppose that the stack consists of three layers (Figure 1).

Values corresponding to the binder layers and half-space will be denoted by upper indices (2); values related with the filler layers by upper indices (1). Furthermore, the values related with each selected layer and half-space are represented by the additional index, indicating its sequence in the considered body. If we associate the corresponding Lagrangian coordinates $O_m^{(k)} x_{1m}^{(k)} x_{2m}^{(k)} x_{3m}^{(k)}$ (k = 1,2,3) which in their natural state coincide with Cartesian coordinates, then we obtain the new coordinate system from $Ox_1x_2x_3$ by parallel transfer along the Ox_2-axis, with the middle surface of each layer of the filler and binder. Figure 1 shows the cross-section of the considered system at $x_{1m}^{(k)} = const(x_{3m}^{(k)} = const$.

In the evaluation, the thickness of every filler layer will be assumed constant. It will also be proposed that the binder, filler and half-space materials are homogeneous, anisotropic and non-aging (hereditary) linearly viscoelastic. Now, we investigate the stress deformation state in the stated body under compression at “infinity” by uniformly distributed normal forces of intensity $p_1(p_3)$ in the direction of the $Ox_1(Ox_3)$-axis. For each layer and for half-space, we write the equilibrium equations, constitutive and geometrical relations as follows:

$$\frac{\partial}{\partial x_{jm}^{(k)}}\left[\sigma_{jn}^{(k)m}\left(\delta_i^n + \frac{\partial u_i^{(k)m}}{\partial x_{nm}^{(k)}}\right)\right] = 0,$$

$$\sigma_{ij}^{(k)m} = C_{ijrso}^{(k)m}\varepsilon_{rs}^{(k)m}(t) + \int_0^t C_{ijrs}^{(k)m}(t-\tau)\varepsilon_{rs}^{(k)m}(\tau)d\tau, \quad (1)$$

$$2\varepsilon_{ij}^{(k)m} = \frac{\partial u_i^{(k)m}}{\partial x_{jm}^{(k)}} + \frac{\partial u_j^{(k)m}}{\partial x_{im}^{(k)}} + \frac{\partial u_n^{(k)m}}{\partial x_{im}^{(k)}}\frac{\partial u_n^{(k)m}}{\partial x_{jm}^{(k)}}$$

$$i; j; n; r; s = 1,2,3,$$

$$k; m = 1,2$$

We assume that between the components of the considered system, there is a complete cohesion.

$$\left[\sigma_{jn}^{(1)1}\left(\delta_i^n + \frac{\partial u_i^{(1)1}}{\partial x_{n1}^{(1)}}\right)\right]\Bigg|_{S_1^-} n_j^{1-} = \left[\sigma_{jn}^{(2)1}\left(\delta_i^n + \frac{\partial u_i^{(2)1}}{\partial x_{n1}^{(2)}}\right)\right]\Bigg|_{S_1^-} n_j^{1-},$$

$$u_i^{(1)1}\Big|_{S_1^-} = u_i^{(2)1}\Big|_{S_1^-}, \quad (2)$$

$$\left[\sigma_{jn}^{(2)1}\left(\delta_i^n + \frac{\partial u_i^{(2)1}}{\partial x_{n1}^{(2)}}\right)\right]\Bigg|_{S_1^-} n_j^{2+} = \left[\sigma_{jn}^{(1)2}\left(\delta_i^n + \frac{\partial u_i^{(1)2}}{\partial x_{n2}^{(1)}}\right)\right]\Bigg|_{S_1^-} n_j^{2-},$$

$$u_i^{(2)1}\Big|_{S_2^+} = u_i^{(1)2}\Big|_{S_2^+},$$

$$\left[\sigma_{jn}^{(1)2}\left(\delta_i^n + \frac{\partial u_i^{(1)2}}{\partial x_{n2}^{(1)}}\right)\right]\Bigg|_{S_2^-} n_j^{2-} = \left[\sigma_{jn}^{(2)2}\left(\delta_i^n + \frac{\partial u_i^{(2)2}}{\partial x_{n2}^{(2)}}\right)\right]\Bigg|_{S_2^-} n_j^{2-},$$

$$u_i^{(1)2}\Big|_{S_2^-} = u_i^{(2)2}\Big|_{S_2^-},$$

In Equation 2, the values indicated by the upper index (2) 2 regard the half-space and these values satisfy also the decay conditions:

$$\sigma_{11}^{(2)2} \to p_1, \quad \sigma_{33}^{(2)2} \to p_3, \quad \sigma_{ij}^{(2)2} \to 0, \tag{3}$$

for $ij \neq 11;33$ as $x_{22}^{(1)} \to -\infty.$

Furthermore, on the upper surface of the $1^{(1)}$ layer the conditions are met.

$$[\sigma_{jn}^{(1)1}(\delta_j^n + \frac{\partial u_i^{(1)1}}{\partial x_n^{(1)}})]\Bigg|_{S_1^+} n_j^{1+} = 0 \tag{4}$$

In Equations 2 and 4, the components of the unit normal vector to the surfaces $S_m^{\pm}$ are denoted $n_j^{m\pm}$.

The other notation used in Equations 1, 2, 3 and 4 is well-known by the specialist. The initial insignificant imperfection of the filler layers is expressed by the equations of the middle surface of those as:

$$x_{2m}^{(1)} = F_m(x_{1m}^{(1)}, x_{3m}^{(1)}) = \varepsilon f_m(x_{1m}^{(1)}, x_{3m}^{(1)}), \tag{5}$$

Where ε is a dimensionless small parameter ($0 \le \varepsilon \le 1$).

In addition, we suppose that the functions $F_m(x_{1m}^{(1)}, x_{3m}^{(1)})$ and their first-order derivatives are continuous and satisfy the following conditions:

$$\left(\frac{\partial F_m}{\partial x_{1m}^{(1)}}\right)^2 + \left(\frac{\partial F_m}{\partial x_{3m}^{(1)}}\right)^2 \langle\langle 1. \tag{6}$$

SOLUTION METHODS

The general concepts of the solution procedure, which is used in the present investigation, can be considered according to Akbarov et al. (1997).

If we take into account the condition of constant thickness 2 h of the elastic layer and Equation 6, we can derive the equations for the surfaces $S^{\pm}$ (Figure 1), which can be presented as:

$$x_i^{(k)\pm} = x_i^{(k)\pm}(t_1, h, \varepsilon, f(t_1, t_3)) \quad (k\ ;\ i=1,2,3),$$

where t_1, t_3 is a parameter $(t_1, t_3 \in (-\infty, +\infty))$ and h is the half-thickness of the elastic-reinforcing layer. Using these equations, expressions for the components $n_j^{\pm}$ are also obtained. From the first condition in Equation 9, expressions for $x_i^{(k)\pm}$ and $n_i^{(k)\pm}$ are expanded to the power series in the small parameter ε. From previously discussed, the quantities characterizing the stress-strain state of arbitrary components of the systems considered are indicated as series in the parameter ε as follows:

$$\{\sigma_{ij}^{(k)}; \varepsilon_{ij}^{(k)}; u_i^{(k)}\} = \sum_{q=0}^{\infty} \varepsilon^q \{\sigma_{ij}^{(k),q}; \varepsilon_{ij}^{(k),q}; u_i^{(k),q}\} \tag{7}$$

If we substitute Equation 7 into Equations 1 to 4 and compare identical powers of ε, we reach the corresponding closed system of equations for contact and boundary conditions. From the linearity of the mechanical relations, these relations can be fulfilled for each approximation in Equation 7 separately. Furthermore, the fourth condition in Equation 2 and the conditions attained from boundary conditions in Equations 2 to 4 are also satisfied for each approximation. The remaining relations and conditions attained from Equations 1, 2 and 4 for each qth approximation contain the values of all approximations made. In this case, for the quantities of zeroth approximation, Equation 1, contact and boundary conditions in Equations 2 and 4 are fulfilled at $x_1 = t_1; x_2 = \pm h$ (instead of surfaces $S^{\pm}$). For the quantities of the first and subsequent approximations, we attain linear equations and relations by direct verification, coinciding with the corresponding ones of the TLTS (Guz, 1999). The equations of TLTS are also obtained from the corresponding nonlinear equations by employing the "boundary-form perturbation'' technique. However, in Guz (1999), the equations of TLTS were attained from the nonlinear equations, employing a linearization procedure. Exactly this distinction of the approach of Akbarov and Guz (2000) from the approach of Guz (1999) allows one to take into account the initial imperfection in the relations of TLTS and to employ the TLTS to investigate the stability loss problems of a time-dependent material within the framework of the initial imperfection criterion. Moreover, in it was proved that (Akbarov et al., 1997; Akbarov and Kosker, 2004; Akbarov and Tekercioğlu, 2006; Akbarov and Tekercioğlu, 2007; Akbarov et al., 1999; Aliyev, 2007; Argatov, 2009), for investigating instability problems and for determining the critical forces or critical times; we need only the results obtained within the framework of the zeroth and first approximations.

Now, we attempt to determine the quantities of zeroth and first approximations. To do this, we assume that the materials of the half-plane and the layers are moderately rigid and quantities regarding the zeroth approximation can be found from the corresponding linear equations. In addition to that, we presume that $\left|\partial u_i^{(k),0} / \partial x_j^{(k)}\right|$ <<1 and these quantities can be omitted in the equations of the first approximation.

Now, we applied the Laplace transform $\bar{\varphi}(s) = \int_0^{\infty} \varphi(t) \exp(-st) dt$ with $s > 0$ to all equations and relations corresponding to the zeroth approximation. From the structural arrangement of the problem for this approximation and the principle of correspondence, the Laplace transforms of quantities of this approximation are determined as follows:

$$\bar{\sigma}_{11}^{(k)m,0} = \frac{\bar{E}^{*(k)m}(\bar{p}_1 + \bar{\nu}^{*(k)m}\bar{p}_3)}{\bar{E}^{*(2)2}(1-(\bar{\nu}^{*(k)m})^2)} - \frac{\bar{E}^{*(k)m}(\bar{p}_3 + \bar{\nu}^{*(k)m}\bar{p}_1)}{\bar{E}^{*(2)2}(1-(\bar{\nu}^{*(k)m})^2)};$$

$$\bar{\sigma}_{11}^{(k)m,0} = \frac{\bar{E}^{*(k)m}(\bar{p}_1 + \bar{\nu}^{*(k)m}\bar{p}_3)}{\bar{E}^{*(2)2}(1-(\bar{\nu}^{*(k)m})^2)} - \frac{\bar{E}^{*(k)m}(\bar{p}_3 + \bar{\nu}^{*(k)m}\bar{p}_1)}{\bar{E}^{*(2)2}(1-(\bar{\nu}^{*(k)m})^2)} \tag{8}$$

$$\bar{\sigma}_{ij}^{(k)m,0} = 0 \qquad \text{for ij } \neq 11{,}33$$

Where $\bar{E}^{*(k)m}$ and $\bar{\nu}^{*(k)m}$ are Laplace transforms of the operators

$$E^{*(k)m} = E_0^{(k)m} + \int_0^t E^{(k)m}(t-\tau)d\tau,$$

$$\nu^{*(k)m} = \nu_0^{(k)m} + \int_0^t \nu^{(k)m}(t-\tau)d\tau. \tag{9}$$

Here $E_0^{(k)m}$ and $\nu_0^{(k)m}$ are the instantaneous values of modulus of elasticity and Poisson's ratio of the (*k*) *m*th material. We determine the original of the sought values by the use of the Schapery's method (Schapery, 1966).

Here, we determine the values of the first approximation for which the following equations and relations are obtained. The governing field equations:

$$\frac{\partial \sigma_{ji}^{(k)m,1}}{\partial x_{jm}^{(k)}} + \sigma_{11}^{(k)m,0}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \sigma_{33}^{(k)m,0}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \tag{10}$$

The mechanical and geometrical relations:

$$\sigma_{ji}^{(k)m,1} = \lambda^{*(k)m}\theta^{(k)m,1}\delta_i^j + 2\nu^{*(k)m}\varepsilon_{ji}^{(k)m,1}, \theta^{(k)m,1} = \varepsilon_{11}^{(k)m,1} + \varepsilon_{22}^{(k)m,1} + \varepsilon_{33}^{(k)m,1},$$

$$\varepsilon_{ji}^{(k)m,1} = \frac{1}{2}\left(\frac{\partial u_i^{(k)m,1}}{\partial x_{jm}^{(k)}} + \frac{\partial u_i^{(k)m,1}}{\partial x_{jm}^{(k)}}\right), \nu^{*(k)m} = \frac{1}{2}\left(\frac{E^{*(k)m}\nu^{*(k)m}}{(1+\nu^{*(k)m})(1-2\nu^{*(k)m})}\right), \tag{11}$$

$$\mu^{*(k)m} = \frac{E^{*(k)m}}{2(1+\nu^{*(k)m})}$$

The boundary conditions:

$$\sigma_{21}^{(1),1,1}\Big|_{x_2^{(1)}=-h_1^{(1)}} - \sigma_{11}^{(1),1,0}\Big|\frac{df}{dx_1^{(1)}} = 0,$$

$$\sigma_{22}^{(1),1,1}\Big|_{x_2^{(1)}=+h_1^{(1)}} = 0, \tag{12}$$

$$\sigma_{23}^{(1),1,1}\Big|_{x_2^{(1)}=+h_1^{(1)}} - \sigma_{33}^{(1),1,0}\Big|\frac{df}{dx_3^{(1)}} = 0,$$

The complete cohesion conditions:

$$\sigma_{2i}^{(1),1,1}\Big|_{x_2^{(1)}=-h_1^{(1)}} - \sigma_{2i}^{(2),1,1}\Big|_{x_2^{(1)}=+h_1^{(2)}} = (\sigma_{11}^{(1),1,0} - \sigma_{11}^{(2),1,0})\frac{df_1}{dx_{11}^{(1)}}\delta_i^1 + (\sigma_{33}^{(1),1,0} - \sigma_{33}^{(2),1,0})\frac{df_1}{dx_{31}^{(1)}}\delta_i^3$$

$$u_i^{(1)1,1}\Big|_{x_2^{(1)}=-h_1^{(1)}} - u_i^{(2)1,1}\Big|_{x_2^{(2)}=+h_1^{(2)}} = 0, \quad I = 1, 2, 3 \tag{13}$$

An advanced development of the method of solution is based upon the selection of the initial imperfection mode, that is, on the selection of the function in Equation 5. In the present investigation, according to the undulation stability loss mode dealt with in the regarding works carried out for the time-independent materials and tabulated in Equation 15, we presume that:

$$f = f_1(x_1).f_3(x_3); \qquad f = e^{-(\frac{x_1}{L_1})^2 - \gamma^2(\frac{x_3}{L_1})^2} \tag{14}$$

Where $\gamma = \frac{l_1}{l_3}$.

In the cause of $\gamma = \frac{l_1}{l_3} = 0 \quad (l_3 \to \infty)$ the results were attained (Aliyev, 2007) in the two-dimensional problems.

We propose that $L\langle\langle l_1$ and the dimensionless small parameter *ε* is identified by *ε=L/ℓ*$_1$. Eventually, the expressions for, and in Equations 11 and 12, according to Equations 5 and 13, are defined as:

$$\frac{df}{dx_1} = f_1^{'}(x_1).f_3(x_3); \quad \frac{df}{dx_3} = f_1(x_1).f_3^{'}(x_3) \tag{15}$$

Thus, for the considered case, the determination of the values of the first approximation is degraded to the solution of the integro-differential Equations 10 and 11 with boundary conditions in Equation 12) and contact conditions in Equation 13. Note that the coefficients $\sigma_{11}^{(k)m,0}$ and $\sigma_{33}^{(k)m,0}$ in Equation 10 are time-dependent, that is, $\sigma_{11}^{(k)m,0} = \sigma_{11}^{(k)m,0}(t)$ and $\sigma_{33}^{(k)m,0} = \sigma_{33}^{(k)m,0}(t)$. Just this statement violates the application of the principle of correspondence in the solution to the problems in Equations 10, 11, 12, 13, 14 and 15. We propound the following method for the solution to the problem.

If one applies Laplace transform to the Equations 10, 11, 12 and 13, the following equations are obtained from Equation 10:

$$\frac{\partial \bar{\sigma}_{ji}^{(k)m,0}}{\partial x_{jm}^{(k)}} + \int_0^\infty \sigma_{11}^{(k)m,0}(t)\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2}\exp(-st)dt + \tag{16}$$

$$\int_0^\infty \sigma_{33}^{(k)m,0}(t)\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2}\exp(-st)dt = 0$$

i;j = 1,2,3.

The other relations in Equations 11, 12 and 13 hold also for the Laplace transforms of the sought values with the changes:

$$\left\{\sigma_{ij}^{(k)m,0}, \varepsilon_{ij}^{(k)m,0}, u_i^{(k)m,0}, \lambda^{*(k)m}, \mu^{*(k)m}\right\} \Rightarrow \left\{\bar{\sigma}_{ij}^{(k)m,0}, \bar{\varepsilon}_{ij}^{(k)m,0}, \bar{u}_i^{(k)m,0}, \bar{\lambda}^{*(k)m}, \bar{\mu}^{*(k)m}\right\} \quad (17)$$

Now we consider the integral terms in Equation 16:

$$\int_0^\infty \sigma_{11}^{(k)m,0}(t)\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2}\exp(-st)dt$$

and

$$\int_0^\infty \sigma_{33}^{(k)m,0}(t)\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2}\exp(-st)dt \quad (18)$$

For the mechanical reasons, the values of $\sigma_{11}^{(k)m,0}(t)$ and $\sigma_{33}^{(k)m,0}(t)$ must be limited by their values obtained at $t = 0$ and ∞. In connection with this, one may write

$$\int_0^\infty \sigma_{jj}^{(k)m,0}(t)\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2}\exp(-st)dt \cong \sigma_{jj}^{(k)m,0}(t_*)\int_0^\infty \frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2}\exp(-st)dt =$$

$$\sigma_{jj}^{(k)m,0}(t_*)\frac{\partial^2 \bar{u}_i^{(k)m,0}}{\partial (x_{3m}^{(k)})^2} \qquad j=1,3;\ i=1,2,3. \quad (19)$$

To finding a certain value of t_* under which Equation 19 is satisfied exactly is generally impossible or very difficult. In the mean time, the exact values of the critical parameters of the investigated stability problems must be limited with those attained un de $t_* = 0$ and ∞. We represent the critical time obtained at $t_* = 0$ $(t_* = \infty)$ through $t_{cr.0}$ $(t_{cr.\infty})$.

From previously discussed, the determination of the values of t_{cr}, is reduced to the determinations of the values $t_{cr.0}$ and $t_{cr.\infty}$. In this case, the Equation 16 can be rewritten as follows:

$$\frac{\partial \bar{\sigma}_{ji}^{(k)m,1}}{\partial x_{jm}^{(k)}} + \sigma_{11}^{(k)m,0}\frac{\partial^2 \bar{u}_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \sigma_{33}^{(k)m,0}\frac{\partial^2 \bar{u}_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \quad (20)$$

Now, we try to find the solution to Equations 20, 11 and 17 which fulfills the boundary and contact conditions of Equations 12 and 13. Substituting Equation 11 into in Equation 20, we reach

$$\nabla^2 \bar{u}_i^{(k)m,1} + \left(1+\frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right)\frac{\partial}{\partial x_{im}^{(k)}}\bar{\theta}^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = 0. \quad (21)$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial (x_{1m}^{(k)})^2} + \frac{\partial^2}{\partial (x_{2m}^{(k)})^2} + \frac{\partial^2}{\partial (x_{3m}^{(k)})^2}. \quad (22)$$

From Equation 22, it can be seen that

$$\left(2+\frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right)\nabla^2\bar{\theta}^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2}{\partial (x_{1m}^{(k)})^2}\bar{\theta}^{(k)m,1} + \frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2}{\partial (x_{3m}^{(k)})^2}\bar{\theta}^{(k)m,1} = 0. \quad (23)$$

$$\nabla^2 \bar{u}_i^{(k)m,1} + \frac{\sigma_{11}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{1m}^{(k)})^2} + \frac{\sigma_{33}^{(k)m,0}(t_*)}{\bar{\mu}^{(k)m}}\frac{\partial^2 u_i^{(k)m,1}}{\partial (x_{3m}^{(k)})^2} = -\left(1+\frac{\bar{\lambda}^{(k)m}}{\bar{\mu}^{(k)m}}\right)\frac{\partial}{\partial x_{im}^{(k)}}\bar{\theta}^{(k)m,1}. \quad (24)$$

If double Fourier transformation is applied, then we obtain the following ordinary differential equations:

$$\frac{d^2\bar{\theta}_{13}^{(k)m,1}}{d^2 x_{2m}^{(k)}} - \left(S_1^2 + s_3^2 + \frac{\sigma_{11}^{(k)m,0}}{2\mu^{*(k)m}+\lambda^{*(k)m}}S_1^2 + \frac{\sigma_{33}^{(k)m,0}}{2\mu^{*(k)m}+\lambda^{*(k)m}}S_3^2\right)\theta_{13}^{(k)m} = 0 \quad (25)$$

$$\frac{d^2\bar{u}_i^{(k)m,1}}{d^2 x_{2m}^{(k)}} - \left(S_1^2 + s_3^2 + \frac{\sigma_{11}^{(k)m,0}}{\mu^{*(k)m}}S_1^2 + \frac{\sigma_{33}^{(k)m,0}}{\mu^{*(k)m}}S_3^2\right)\bar{u}_i^{(k)m} = -\left(1+\frac{\lambda^{*(k)m}}{\mu^{*(k)m}}\right)\left(\frac{d\bar{\theta}_{13}^{(k)m}}{dx_{2m}^{(k)m}}\delta_i^2 + \bar{\theta}_{13}^{(k)m,1}\delta_i^1 + \bar{\theta}_{13}^{(k)m,1}\delta_i^3\right) \quad (26)$$

Solutions of this equations can be given as follows:

$$\bar{\theta}_{13}^{(k)m,1} = \bar{A}_1^{(k)m,1}e^{r_1^{(k)m}x_{2m}^{(k)m,1}} + \bar{A}_2^{(k)m,1}e^{-r_1^{(k)m}x_{2m}^{(k)m,1}} \quad (27)$$

$$\bar{u}_1^{(k)m,1} = \bar{C}_{11}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + C_{12}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{11}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + CA_{12}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}}$$

$$\bar{u}_2^{(k)m,1} = \bar{C}_{21}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + C_{22}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{21}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + CA_{22}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}}$$

$$\bar{u}_3^{(k)m,1} = \bar{C}_{31}^{(k)m} e^{k_1^{(k)m} x_{2m}^{(k)}} + C_{32}^{(k)m} e^{-k_1^{(k)m} x_{2m}^{(k)}} + \bar{A}_{31}^{(k)m} e^{r_1^{(k)m} x_{2m}^{(k)}} + CA_{32}^{(k)m} e^{-r_1^{(k)m} x_{2m}^{(k)}}$$

The equations for the unknown constants entering the expressions of these functions are obtained from the boundary and contact conditions of Equations 12 and 13. In this way, we therefore, determine completely the Laplace transform of the values the first approximation. These functions are found by employing Schapery's method (Schapery, 1966) and the critical time is evaluated from the criterion:

$$\left|u_2^{(1)1,1}(0)\right| \to \infty, \text{ as } t \to t_{cr} . \tag{28}$$

At this point, we restrict ourselves to consideration of the method of solution. This method with the corresponding changing can be applied for investigation of the other analogous problems. Moreover, the approach can also be carried out for the time-independent materials. In this case by introducing the notation $p_3 = np_1$ the criterion of Equation 28 must be relocated with the following:

$$\left|u_2^{(1)1,1}(0)\right| \to \infty, \text{ as } p \to p_{1cr} \tag{29}$$

According to the problem statement and the expression of Equation 14, we employ the exponential double Fourier transformation $g_{13}(s_1, s_3) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x_1, x_2) e^{-i(s_1 x_1 + s_3 x_3)} dx_1 dx_3$ to the Equations 16, 17, boundary and contact conditions of Equations 12 and 13 with respect to the variable $x_1^{(k)}$.

NUMERICAL RESULTS AND DISCUSSION

We assume that the viscoelasticity of the material is described by the operators:

$$E^{*(k)} = E_0^{(k)}[1 - \omega_0 R_\alpha^*(-\omega_0 - \omega_\infty)],$$

$$\nu^{*(k)} = \nu_0^{(k)}[1 + \frac{1 - 2\nu_0^{(k)}}{\nu_0^{(k)}} \omega_0 R_\alpha^*(-\omega_0 - \omega_\infty)], \tag{30}$$

$$\lambda^{*(k)} = \lambda_0^{(k)}[1 + \frac{1 - 2\nu_0^{(2)}}{2\nu_0^{(2)}(1 + \nu_0^{(2)})} \omega_0 R_\alpha^*(-\frac{3}{2(1 + \nu_0^{(2)})} \omega_0 - \omega_\infty)], \tag{31}$$

$$\mu^{*(k)} = \mu_0^{(k)}[1 - \frac{3\omega_0}{2\nu_0^{(2)}(1 + \nu_0^{(2)})} R_\alpha^*(-\frac{3}{2(1 + \nu_0^{(2)})} \omega_0 - \omega_\infty)].$$

Where $E_0^{(k)}$ and $\nu_0^{(k)}$ are the instantaneous values of Young's modulus and the Poisson ratio, respectively; $\lambda_0^{(k)}$ and $\mu_0^{(k)}$ are the instantaneous values of Lame' constants; α, ω_0 and ω_∞ are rheological parameters; R_α^* is the Rabotnov fractional-exponentional operator (Rabotnov, 1977), which is defined as:

$$R_\alpha(\beta)\Psi(t) = \int_0^t R_\alpha(\beta, t - \tau)\Psi(\tau) d\tau,$$

$$R_\alpha(\beta) = t^\alpha \sum_{n=0}^{\infty} \frac{\beta^n t^{n(1+\alpha)}}{\Gamma[(n+1)(1+\alpha)]}, -1 < \alpha \le 0, \tag{32}$$

Where $\Gamma(x)$ is the gamma function.

We introduce the dimensionless rheological parameter $\omega = \omega_\infty / \omega_o$ and dimensionless time $t' = \omega_0^{1/(1+\alpha)} t$. First, we consider the case where the materials of the covering layer and half-plane are also pure elastic and analyze the distribution of the normal stress $\sigma_{nn}(= \varepsilon\sigma_{22}^{(1),1})$ acting on the interface surface between covering and bond layers. We assume that $\varepsilon = 0.015$, $E^{(2)}/E^{(1)} = E^{(2)}/E^{(3)} = 50$, $\nu_0^{(3)} = \nu^{(2)} = \nu_0^{(1)} = 0.3$, $\gamma = 1.0$. The influence of the geometrical nonlinearity on the mentioned distribution will be characterized through the parameter $e = p/E^{(2)} \times 10^3$. Thus, within the scope of the foregoing assumptions, we analyze the numerical results and begin this analysis with those regarding to the dependence between $\sigma_{nn}/\sigma_{11}^{(1),0}$ (at $x_1/L = 0.0$) and $h^{(1)}/L$. The graphs of this dependence constructed for various values of e under $h^{(2)}/L = 0.3$, $m = 0.0$ are shown in Figure 2.

According to the well-known mechanical consideration, the values of σ_{nn} must approach to its asymptotic (limit) values with $h^{(1)}/L$ and these limit values are the values of σ_{nn} attained for the case where layer 2 (Figure 1) is contained in the infinite body from the material of the half-plane 3 (Figure 1). This prediction is proved by the graphs shown in Figure 2 and the noted limit values under $|e| = 1.0$ almost coincide with those obtained

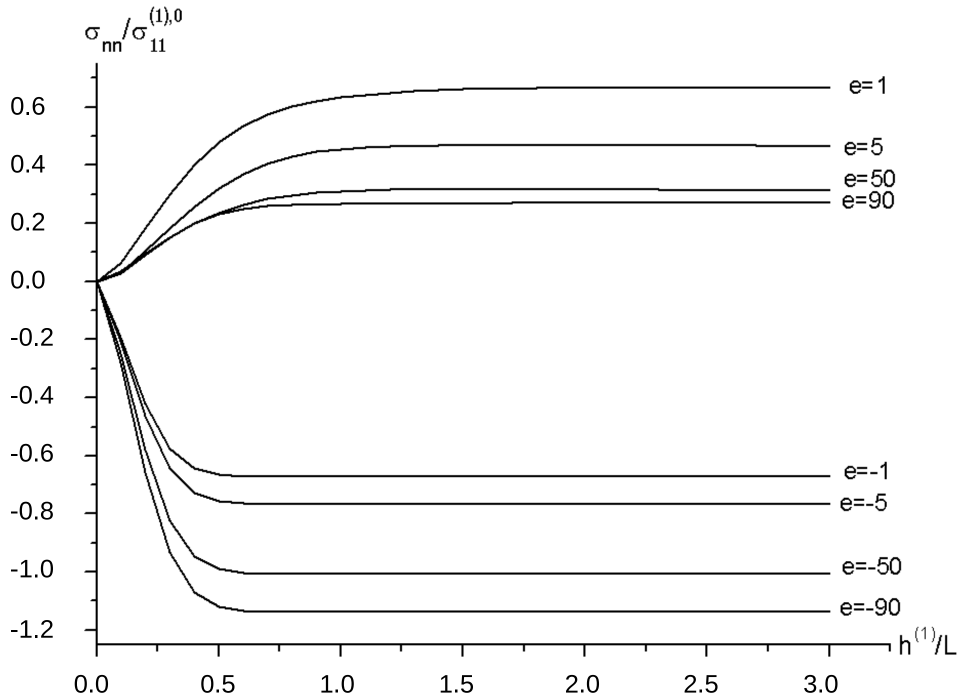

Figure 2. The relations between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and $h^{(1)}/L$ for values of e under $h^{(2)}/L$ = 0.3,m =0.0.

within the scope of the linear theory of elasticity and analyzed in the monograph (Akbarov and Guz, 2000). At the same time, the graphs shown in Figure 2 show that as a result of the accounting of the geometrical nonlinearity, the absolute values of σ_{nn} decrease (increase) with $|e|$ under tension (compression) of the considered material. In the qualitative sense, these results agree with the corresponding ones shown in the monograph (Akbarov and Guz, 2000) and in the paper of Akbarov and Kosker (2005). Consequently, the results illustrate the trustiness and validity of the employed algorithm and PC programs.

Consider the graphs shown in Figure 3 which show the dependence between $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ (at $x_1/L = 0.0$) and $h^{(2)}/L$ constructed for various values of e under $h^{(1)}/L = 0.45$. According to a lot of investigations detailed in the monograph (Akbarov and Guz, 2000), such type dependencies have a non-monotonic character. It follows from the graphs shown in Figure 3 that this character of the mentioned dependence occurs also for the considered case. Moreover, the graphs show that the value of $h^{(2)}/L$ under which $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ becomes maximum, increasing with e.

Let us analyze the influence of the parameter m which enters the expression of the function Equation 12, characterizing the local curving form of the reinforcing layer on the distribution of the $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ with respect to x_1/L. The graphs of this distribution attained in the case where $|e| = 5.0$, $h^{(1)}/L = 0.45$, $h^{(2)}/L = 0.3$ are shown in Figure 4. These results show that the values of $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ increase with m. Consequently, the oscillation character of the local curving form causes to increase values of the self-balanced normal stress.

Now, we assume that the materials of the covering layer and of the half-plane are the viscoelastic ones with operators of Equations 17 and 18. We investigate the influence of the rheological parameters ω and α on the values of $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ and assume that $t'_* = 0.0$ in Equations 13 and 14. Figure 5 shows the dependence between $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ (at $x_1/L = 0.0$) and dimensionless time t' for various values of ω (α) under $h^{(2)}/L = 0.3$, $h^{(1)}/L = 0.45$, $m = 0.0$, $\alpha = -0.5$ ($\omega = 3.0$). It follows from these results that the values of $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ increase with time and approach to the corresponding limit values of $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ attained in the case where the materials

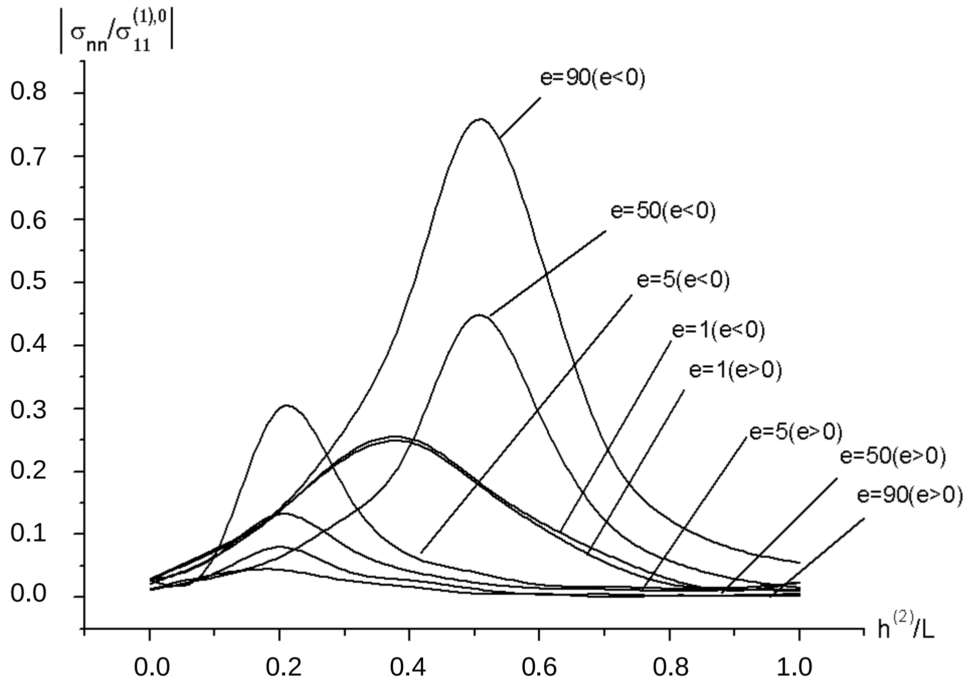

Figure 3. The relations between $\left|\sigma_{nn}/\sigma_{11}^{(1),0}\right|$ and $h^{(2)}/L$ for various e under $h^{(1)}/L = 0.45$.

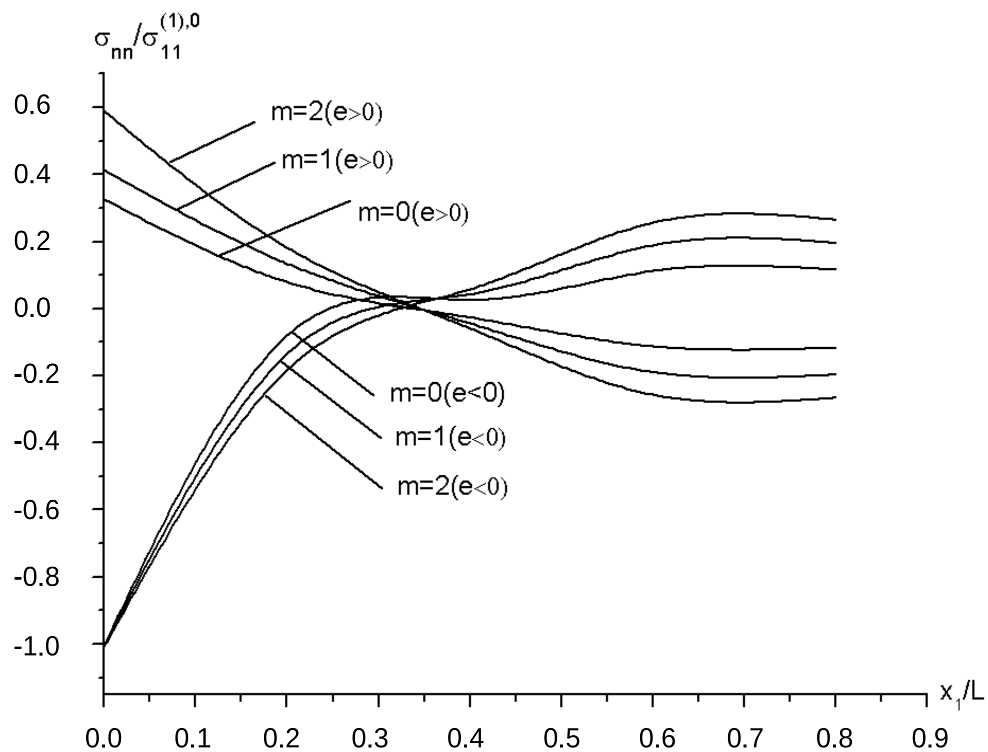

Figure 4. The relations between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and x_1/L for values m under $h^{(2)}/L = 0.3$; $h^{(1)}/L = 0.45$; $|e| = 5.0$.

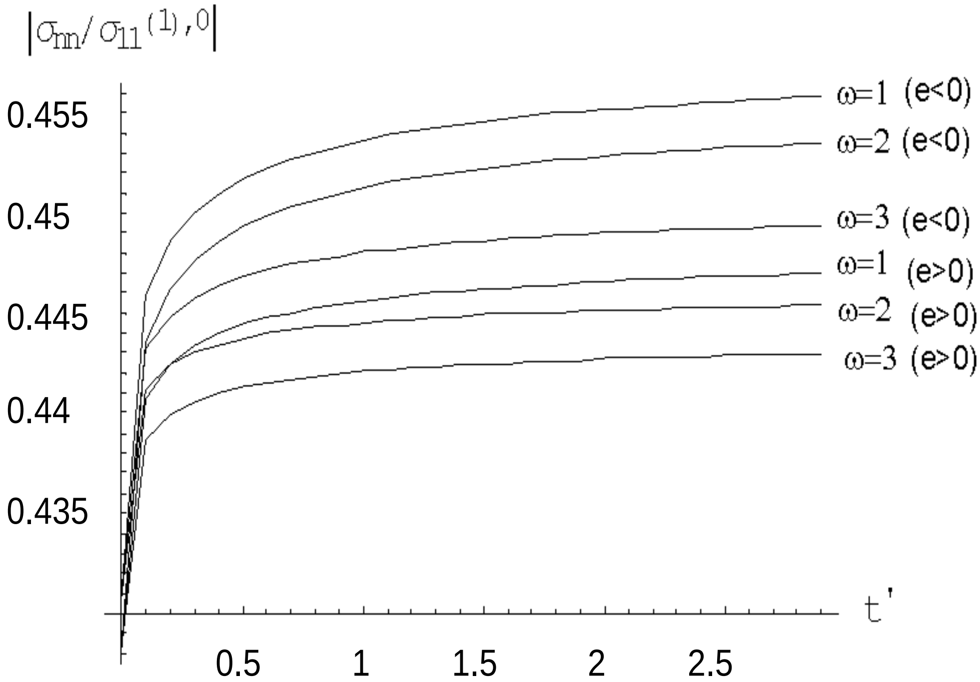

Figure 5. The relation between $\sigma_{nn}/\sigma_{11}^{(1),0}$ and t' for varions ω under $|e|=5.0;$ $h^{(2)}/L$ = 0.3; $h^{(1)}/L$ = 0.45; $\alpha=-0.5.$

of the covering layer and half-plane are the elastic ones with elasticity constants $E_\infty^{(1)}=E_0^{(1)}(1-1/(1+\omega))$, $\nu_\infty^{(1)}=\nu_0^{(1)}(1+\ (1+2\nu_0^{(1)})/(\nu_0^{(1)}(1+\omega))$. At the same time, the results show that the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ decrease with ω. This statement is explained with decreasing of the values of $E_\infty^{(1)}$ with ω. But the influence of the rheological parameter α on the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ has more complicate character. For the certain values before (after) t', the values of $|\sigma_{nn}/\sigma_{11}^{(1),0}|$ increase (decrease) with $|\alpha|$. Such type results were also obtained in the papers of Akbarov et al. (1997) and Akbarov and Kosker (2004).

With the foregoing investigations, we restrict our-selves to consideration of the self-balanced normal stress distribution. Note that the near-surface adhesion strengths of this material depend mainly on the values of this stress. Now, we consider the formulation of the failure criterion for the considered material. For this purpose, we introduce the following notation: $\Pi_1^{\pm}$ are the ultimate strengths, + and – representing, respectively tension and compression along the Ox_1 axis, Π_2^+ is the ultimate strength in tension in the direction of the Ox_2 axis. Since the values of Π_2^+ are determined mainly by the adhesion strength or by the matrix material we have:

$$\Pi_2^+/\Pi_1^+ << 1.0 \tag{33}$$

According to the experimental investigations given in the monograph (Tarnopolsky and Rose, 1960), for the class-fiber reinforced plastics the relation $\Pi_2^+/\Pi_1^+=0.055-0.10$ occurs and this relation satisfies the inequality of the Equation 19. Consequently, the near-surface failure of the considered material occurs when the relations

$$\sigma_{nn}=\Pi_2^+,\quad \sigma_{11}^{(1)0}<\Pi_1^+\ , \tag{34}$$

are hold. Foregoing numerical results show that in many considered cases, the inequality $\sigma_{nn}/\sigma_{11}^{(1)0}>\Pi_2^+/\Pi_1^+$ satisfies. From this inequality, it follows that for the investigated problem, the criterion of (34) is acceptable. Using the presented numerical results, the change range of the values of the problem parameters under which the criterion of (33), (34) can be satisfied and can be easily determined. Consequently, the presented near-surface failure model and criterion is very real.

Conclusions

Within the framework of a piecewise homogeneous body model with the use of the exact equations of the geometrical nonlinear theory for the viscoelastic body the near-surface self-balanced normal stress distribution in a body consisting of a viscoelastic half-plane, an elastic locally curved bond layer, and a viscoelastic covering layer has been investigated. A method for solving the problem considered by employing the Laplace and Fourier transformations was developed.

Numerical results on the self-balanced normal stress caused by the local curving (imperfection) of the elastic bond layer under stretching, as well as under compressing of the body mentioned along the free face plane were presented and analyzed. The viscoelasticity of the materials was described by the Rabotnov fractional-exponential operators (Rabotnov, 1977).

From the analyses performed, the following conclusions can be drawn:

1) The absolute values of the self-balanced normal stress decrease with approaching of the local curving elastic layer to the free surface which bounds the body considered;
2) The absolute values of the self-balanced normal stress increase (decrease) with absolute values of the external compressing (stretching) forces. This statement is explained with the accounting of the geometrical nonlinear effect;
3) The dependence between the self-balanced normal stress and ratio $h^{(2)}/L$ has non-monotone character;
4) The absolute values of the self-balanced normal stress grow with time and approach to its limit values which correspond the case where the materials of the covering layer and half-plane are pure elastic with material constants $E_{\infty}^{(1)} = E_0^{(1)}\left(1-1/(1+\omega)\right)$, $\nu_{\infty}^{(1)} = \nu_0^{(1)}\left(1+\left(1+2\nu_0^{(1)}\right)/\left(\nu_0^{(1)}(1+\omega)\right)\right)$;
5) An increase in the values of the rheological parameter ω causes to decreases in the self-balanced normal stress;
6) The macroscopic failure criterion of Equations 19 and 20 is presented and according to the obtained numerical results, it is established that this criterion is very real for the estimation of the near-surface failure of the considered type near-surface damage in the structure of the layered composite materials;
7) Taking the restrictions detailed in the papers (Guz et al., 2007; Zhuk and Guz, 2007) into account, the approach used in solving the problem considered can also be applied to studying the near-surface failure problems for nano composite materials in tension.

In the case of $\gamma = 0$, the results attained in Guz (1999) show that the three-dimensional system problem reduces to the two-dimensional one.

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