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Some algorithms for equilibrium problems on Hadamard manifolds

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Abstract

In this paper, we suggest and analyze an iterative method for solving the equilibrium problems on Hadamard manifolds using the auxiliary principle technique. We also consider the convergence analysis of the proposed method under suitable conditions. Some special cases are considered. Results and ideas of this paper may stimulate further research in this fascinating and interesting field.

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1 Introduction

Equilibrium problems theory provides us with a unified, natural, novel and general framework to study a wide class of problems, which arise in finance, economics, network analysis, transportation and optimization. This theory has applications across all disciplines of pure and applied sciences. Equilibrium problems include variational inequalities and related problems as special cases; see [1–31]. Recently, much attention has been given to study the variational inequalities, equilibrium and related optimization problems on the Riemannian manifold and Hadamard manifold. This framework is useful for the development of various fields. Several ideas and techniques from the Euclidean space have been extended and generalized to this nonlinear framework. Hadamard manifolds are examples of hyperbolic spaces and geodesics; see [1, 3–5, 19, 20, 26–28] and the references therein. Nemeth [8], Tang *et al.* [28], Noor *et al.* [19, 20] and Colao *et al.* [3] have considered the variational inequalities and equilibrium problems on Hadamard manifolds. They have studied the existence of solutions of equilibrium problems under some suitable conditions. To the best of our knowledge, no one has considered the auxiliary principle technique for solving the equilibrium problems on Hadamard manifolds. In this paper, we use the auxiliary principle technique to suggest and analyze an implicit method for solving the equilibrium problems on a Hadamard manifold. As special cases, our result includes the recent results of Tang *et al.* [28] for variational inequalities on a Hadamard manifold. This shows that the results obtained in this paper continue to hold for variational inequalities on a Hadamard manifold, which are due to Noor and Noor [20], Tang *et al.* [28], and Nemeth [8]. We hope that the technique and idea of this paper may stimulate further research in this area.

2 Preliminaries

We now recall some fundamental and basic concepts needed for reading of this paper. These results and concepts can be found in the books on Riemannian geometry [1, 3, 4, 26, 29].

Let M be a simply connected m-dimensional manifold. Given $x \in M$, the tangent space of M at x is denoted by T_xM and the tangent bundle of M by $TM = \bigcup_{x\in M} T_xM$, which is naturally a manifold. A vector field A on M is a mapping of M into TM which associates to each point $x \in M$, a vector $A(x) \in T_xM$. We always assume that M can be endowed with a Riemannian metric to become a Riemannian manifold. We denote by $\langle\cdot,\cdot\rangle$ the scalar product on T_xM with the associated norm $\|\cdot\|_x$, where the subscript x will be omitted. Given a piecewise smooth curve $\gamma : [a,b] \longrightarrow M$ joining x to y (that is, $\gamma(a) = x$ and $\gamma(b) = y$) by using the metric, we can define the length of γ as $L(\gamma) = \int_a^b \|\gamma'(t)\|\,dt$. Then for any $x, y \in M$, the Riemannian distance $d(x,y)$, which includes the original topology on M, is defined by minimizing this length over the set of all such curves joining x to y.

Let Δ be the Levi-Civita connection with $(M,\langle\cdot,\cdot\rangle)$. Let γ be a smooth curve in M. A vector field A is said to be parallel along γ if $\Delta_{\gamma'}A = 0$. If γ' itself is parallel along γ, we say that γ is a geodesic and in this case $\|\gamma'\|$ is a constant. When $\|\gamma'\| = 1$, γ is said to be normalized. A geodesic joining x to y in M is said to be minimal if its length equals $d(x,y)$.

A Riemannian manifold is complete if for any $x \in M$, all geodesics emanating from x are defined for all $t \in R$. By the Hopf-Rinow theorem, we know that if M is complete, then any pair of points in M can be joined by a minimal geodesic. Moreover, (M,d) is a complete metric space, and bounded closed subsets are compact.

Let M be complete. Then the exponential map $\exp_x : T_xM \longrightarrow M$ at x is defined by $\exp_x v = \gamma_v(1,x)$ for each $v \in T_xM$, where $\gamma(\cdot) = \gamma_v(\cdot,x)$ is the geodesic starting at x with velocity v (*i.e.*, $\gamma(0) = x$ and $\gamma'(0) = v$). Then $\exp_x tv = \gamma_v(t,x)$ for each real number t.

A complete simply connected Riemannian manifold of nonpositive sectional curvature is called a *Hadamard manifold*. Throughout the remainder of this paper, we always assume that M is an m-dimensional Hadamard manifold.

We also recall the following well-known results, which are essential for our work.

Lemma 2.1 ([26]) *Let $x \in M$. Then $\exp_x : T_xM \longrightarrow M$ is a diffeomorphism, and for any two points $x, y \in M$, there exists a unique normalized geodesic joining x to y, $\gamma_{x,y}$, which is minimal.*

So, from now on, when referring to the geodesic joining two points, we mean the unique minimal normalized one. Lemma 2.1 says that M is diffeomorphic to the Euclidean space R^m. Thus, M has the same topology and differential structure as R^m. It is also known that Hadamard manifolds and Euclidean spaces have similar geometrical properties. Recall that a geodesic triangle $\triangle(x_1,x_2,x_3)$ of a Riemannian manifold is a set consisting of three points x_1, x_2, x_3 and three minimal geodesics joining these points.

Lemma 2.2 (Comparison theorem for triangles [3, 4, 26]) *Let $\triangle(x_1,x_2,x_3)$ be a geodesic triangle. Denote, for each $i = 1,2,3 \pmod 3$, by $\gamma_i : [0,l_i] \longrightarrow M$ the geodesic joining x_i to x_{i+1}, and $\alpha_i; = L(\gamma_i'(0), -\gamma_l'(i-1)(li-1))$, the angle between the vectors $\gamma_i'(0)$ and $-\gamma_{i-1}'(l_{i-1})$,* *and* $l_i; = L(\gamma_i)$. *Then*

$$\alpha_1 + \alpha_2 + \alpha_3 \leq \pi, \tag{2.1}$$

$$l_l^2 + l_{i+1}^2 - 2L_i; l_{i+1} \cos\alpha_{i+1} \leq l_{i-1}^2. \tag{2.2}$$

In terms of the distance and the exponential map, the inequality (2.2) *can be rewritten as*

$$d^2(x_i, x_{i+1}) + d^2(x_{i+1}, x_{i+2}) - 2\langle \exp_{x_{i+1}}^{-1} x_i, \exp_{x_{i+1}}^{-1} x_{i+2} \rangle \leq d^2(x_{i-1}, x_i), \tag{2.3}$$

since

$$\langle \exp_{x_{i+1}}^{-1} x_i, \exp_{x_{i+1}}^{-1} x_{i+2} \rangle = d(x_i, x_{i+1}) d(x_{i+1}, x_{i+2}) \cos\alpha_{i+1}.$$

Lemma 2.3 ([26]) *Let* $\triangle(x, y, z)$ *be a geodesic triangle in a Hadamard manifold* M. *Then there exist* $x', y', z' \in R^2$ *such that*

$$d(x, y) = \|x' - y'\|, \qquad d(y, z) = \|y' - z'\|, \qquad d(z, x) = \|z' - x'\|.$$

The triangle $\triangle(x', y', z')$ *is called the comparison triangle of the geodesic triangle* $\triangle(x, y, z)$, *which is unique up to isometry of* M.

From the law of cosines in inequality (2.3), we have the following inequality, which is a general characteristic of the spaces with nonpositive curvature [26]:

$$\langle \exp_x^{-1} y, \exp_x^{-1} z \rangle + \langle \exp_y^{-1} x, \exp_y^{-1} z \rangle \geq d^2(x, y). \tag{2.4}$$

From the properties of the exponential map, we have the following known result.

Lemma 2.4 ([26]) *Let* $x_0 \in M$ *and* $\{x_n\} \subset M$ *such that* $x_n \longrightarrow x_0$. *Then the following assertions hold.*

(i) *For any* $y \in M$,

$$\exp_{x_n}^{-1} y \longrightarrow \exp_{x_o}^{-1} y \quad \textit{and} \quad \exp_y^{-1} x_n \longrightarrow \exp_y^{-1} x_o.$$

(ii) *If* $\{v_n\}$ *is a sequence such that* $v_n \in T_{x_n}M$ *and* $v_n \longrightarrow v_0$, *then* $v_0 \in T_{x_0}M$.

(iii) *Given the sequences* $\{u_n\}$ *and* $\{v_n\}$ *satisfying* $u_n, v_n \in T_{x_n}M$, *if* $u_n \longrightarrow u_0$ *and* $v_n \longrightarrow v_0$, *with* $u_0, v_0 \in T_{x_0}M$, *then*

$$\langle u_n, v_n \rangle \longrightarrow \langle u_0, v_0 \rangle.$$

A subset $K \subseteq M$ is said to be convex if for any two points $x, y \in K$, the geodesic joining x and y is contained in K, that is, if $\gamma : [a, b] \longrightarrow M$ is a geodesic such that $x = \gamma(a)$ and $y = \gamma(b)$, then $\gamma((1-t)a + tb) \in K$, $\forall t \in [0, 1]$. From now on, $K \subseteq M$ will denote a nonempty, closed and convex set, unless explicitly stated otherwise.

A real-valued function f defined on K is said to be convex, if for any geodesic γ of M, the composition function $f \circ \gamma : R \longrightarrow R$ is convex, that is,

$$(f \circ \gamma)\big(ta + (1-t)b\big) \leq t(f \circ \gamma)(a) + (1-t)(f \circ \gamma)(b), \quad \forall a, b \in R, t \in [0,1].$$

The subdifferential of a function $f : M \longrightarrow R$ is the set-valued mapping $\partial f : M \longrightarrow 2^{TM}$ defined as

$$\partial f(x) = \big\{u \in T_x M : \langle u, \exp_x^{-1} y\rangle \leq f(y) - f(x), \forall y \in M\big\}, \quad \forall x \in M,$$

and its elements are called subgradients. The subdifferential $\partial f(x)$ at a point $x \in M$ is a closed and convex (possibly empty) set. Let $D(\partial f)$ denote the domain of ∂f defined by

$$D(\partial f) = \big\{x \in M : \partial f(x) \neq \emptyset\big\}.$$

The existence of subgradients for convex functions is guaranteed by the following proposition, see [29].

Lemma 2.5 ([26, 29]) *Let M be a Hadamard manifold and $f : M \longrightarrow R$ be convex. Then for any $x \in M$, the subdifferential $\partial f(x)$ of f at x is nonempty. That is, $D(\partial f) = M$.*

For a given bifunction $F(\cdot,\cdot) : K \times K \longrightarrow R$, we consider the problem of finding $u \in K$ such that

$$F(u,v) \geq 0, \quad \forall v \in K, \tag{2.5}$$

which is called the equilibrium problem on Hadamard manifolds. This problem was considered by Colao *et al.* [3]. They proved the existence of a solution of the problem (2.5) using the KKM maps. Colao *et al.* [3] have given an example of the equilibrium problem defined in an Euclidean space whose set K is not a convex set, so it cannot be solved using the technique of Blum and Oettli [2]. However, if one can reformulate the equilibrium problem on a Riemannian manifold, then it can be solved. This shows the importance of considering these problems on Hadamard manifolds. Noor *et al.* [19, 20] have used the auxiliary principle technique to suggest and analyze an implicit method for solving the equilibrium problems on a Hadamard manifold. For the applications, formulation and other aspects of equilibrium problems in the linear setting, see [2, 5, 7–18, 22].

If $F(u,v) = \langle Tu, \exp_u^{-1} v\rangle$, where $T : K \longrightarrow TM$ is a single-valued vector filed, then problem (2.5) is equivalent to finding $u \in K$ such that

$$\langle Tu, \exp_x^{-1} v\rangle \geq 0, \quad \forall v \in K, \tag{2.6}$$

which is called the variational inequality on Hadamard manifolds. Nemeth [8], Colao *et al.* [3], Tang *et al.* [28] and Noor and Noor [19] studied variational inequalities on a Hadamard manifold from different points of view. In the linear setting, variational inequalities have been studied extensively; see [2, 6, 9–22, 27, 30, 31] and the references therein.

Definition 2.1 A bifunction $F(\cdot,\cdot)$ is said to be partially relaxed strongly monotone if and only if there exists a constant $\alpha > 0$ such that

$$F(u,v) + F(v,z) \leq \alpha d^2(z,u), \quad \forall u,v,z \in K.$$

We note that if $z = u$, then partially relaxed strongly monotonicity reduces to the monotonicity of the bifunction $F(\cdot,\cdot)$.

3 Main results

We now use the auxiliary principle technique of Glowinski *et al.* [6] to suggest and analyze an implicit iterative method for solving the equilibrium problems (2.5).

For a given $u \in K$ satisfying (2.5), consider the problem of finding $w \in K$ such that

$$\rho F(u,v) + \langle \exp_u^{-1} w, \exp_w^{-1} v \rangle \geq 0, \quad \forall v \in K, \tag{3.1}$$

which is called the auxiliary equilibrium problem on Hadamard manifolds. We note that if $w = u$, then w is a solution of (2.5). This observation enables us to suggest and analyze the following implicit method for solving the equilibrium problems (2.5). This is the main motivation of this paper.

Algorithm 3.1 For a given u_0, compute the approximate solution by the iterative scheme

$$\rho F(u_n,v) + \langle \exp_{u_n}^{-1} u_{n+1}, \exp_{u_{n+1}}^{-1} v \rangle \geq 0, \quad \forall v \in K. \tag{3.2}$$

Algorithm 3.1 is called the explicit iterative method for solving the equilibrium problem on the Hadamard manifold.

If K is a convex set in R^n, then Algorithm 3.1 collapses to

Algorithm 3.2 For a given $u_0 \in K$, find the approximate solution u_{n+1} by the iterative scheme

$$\rho F(u_n,v) + \langle u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K,$$

which is known as the explicit method for solving the equilibrium problem. For the convergence analysis of Algorithm 3.2, see [12, 15, 16].

If $F(u,v) = \langle Tu, \exp_u^{-1} v \rangle$, where T is a single valued vector filed $T : K \longrightarrow TM$, then Algorithm 3.1 reduces to the following implicit method for solving the variational inequalities.

Algorithm 3.3 For a given $u_0 \in K$, compute the approximate solution u_{n+1} by the iterative scheme

$$\langle \rho Tu_n + (\exp_{u_n}^{-1} u_{n+1}), \exp_{u_{n+1}}^{-1} v \rangle \geq 0, \quad \forall v \in K.$$

For $M = R^n$, Algorithm 3.3 reduces to

Algorithm 3.4 For a given $u_0 \in K$, compute the approximate solution u_{n+1} by the iterative scheme

$$\langle \rho T u_n + u_{n+1} - u_n, v - u_{n+1} \rangle \geq 0, \quad \forall v \in K,$$

which can be written in the following equivalent form.

Algorithm 3.5 For a given $u_0 \in K$, compute the approximate solution u_{n+1} by the iterative scheme

$$u_{n+1} = P_K[u_n - \rho T u_n], \quad n = 0, 1, 2, \ldots,$$

which is known as the projection method. For the convergence analysis and its applications, see [10, 11].

In a similar way, one can obtain several iterative methods for solving the variational inequalities on the Hadamard manifold.

We now consider the convergence analysis of Algorithm 3.1, and this is the motivation of our next result.

Theorem 3.1 *Let $F(\cdot,\cdot)$ be a partially relaxed strongly monotone bifunction with a constant $\alpha > 0$. Let u_n be the approximate solution of the equilibrium problem* (2.5) *obtained from Algorithm* 3.1, *then*

$$d^2(u_{n+1}, u) \leq d^2(u_n, u) - (1 - \rho\alpha) d^2(u_{n+1}, u_n), \tag{3.3}$$

where $u \in K$ is a solution of the equilibrium problem (2.5).

Proof Let $u \in K$ be a solution of the equilibrium problem (2.5). Then

$$F(u, v) \geq 0, \quad \forall v \in K. \tag{3.4}$$

Taking $v = u_{n+1}$ in (3.4), we have

$$F(u, u_{n+1}) \geq 0. \tag{3.5}$$

Taking $v = u$ in (3.2), we have

$$\rho F(u_n, u) + \langle \exp_{u_n}^{-1} u_{n+1}, \exp_{u_{n+1}}^{-1} u \rangle \geq 0. \tag{3.6}$$

From (3.5) and (3.6), we have

$$\begin{aligned} \langle \exp_{u_{n+1}}^{-1} u_n, \exp_{u_{n+1}}^{-1} u \rangle &\leq \{F(u, u_{n+1}) + F(u_n, u)\} \\ &\leq \alpha\rho d^2(u_{n+1}, u_n), \end{aligned} \tag{3.7}$$

where we have used the fact that the bifunction $F(\cdot,\cdot)$ is partially relaxed strongly monotone with a constant $\alpha > 0$. For the geodesic triangle $\triangle(u_n, u_{n+1}, u)$, the inequality (3.7) can be written as

$$d^2(u_{n+1}, u) + d^2(u_{n+1}, u_n) - \langle \exp^{-1}_{u_{n+1}} u_n, \exp^{-1}_{u_{n+1}} u \rangle \leq d^2(u_n, u). \tag{3.8}$$

Thus, from (3.7) and (3.8), we obtained the inequality (3.3), the required result. □

Theorem 3.2 *Let $u \in K$ be solution of* (2.5), *and let u_{n+1} be the approximate solution obtained from Algorithm* 3.1. *If $\rho < \frac{1}{2\alpha}$, then $\lim_{n\longrightarrow\infty} u_{n+1} = u$.*

Proof Let $\hat{u}$ be a solution of (2.5). Then, from (3.3), it follows that the sequence $\{u_n\}$ is bounded and

$$\sum_{n=0}^{\infty}(1 - 2\alpha\rho)d^2(u_{n+1}, u_n) \leq d^2(u_0, u),$$

from which, we have

$$\lim_{n\longrightarrow\infty} d(u_{n+1}, u_n) = 0. \tag{3.9}$$

Let $\hat{u}$ be a cluster point of $\{u_n\}$. Then there exits a subsequence $\{u_{n_i}\}$ such that $\{u_{u_i}\}$ converges to $\hat{u}$. Replacing u_{n+1} by u_{n_i} in (3.2), taking the limit and using (3.9), we have

$$F(\hat{u}, v) \geq 0, \quad \forall v \in K.$$

This shows that $\hat{u} \in K$ solves (2.5) and

$$d^2(u_{n+1}, \hat{u}) \leq d^2(u_n, \hat{u}),$$

which implies that the sequence $\{u_n\}$ has a unique cluster point and $\lim_{n\longrightarrow\infty} u_n = \hat{u}$ is a solution of (2.5), the required result. □

4 Conclusion

The auxiliary principle technique is used to suggest and analyze an explicit method for solving the equilibrium problems on Hadamard manifolds. It is shown that the convergence analysis of this method requires only the partially relaxed strongly monotonicity. Some special cases are discussed. Results proved in this paper may stimulate research in this area.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally and significantly in writing this paper. Both authors read and approved the final manuscript.

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