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Bounds for the norm of lower triangular matrices on the Cesàro weighted sequence space

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Abstract
This paper is concerned with the problem of finding bounds for the norm of lower triangular matrix operators from $l_p(w)$ into $c_p(w)$, where $c_p(w)$ is the Cesàro weighted sequence space and (w_n) is a non-negative sequence. Also this problem is considered for lower triangular matrix operators from $c_p(w)$ into $l_p(w)$, and the norms of certain matrix operators such as Cesàro, Nörlund and weighted mean are computed.

Keywords: norm; lower triangular matrix; Nörlund matrix; weighted mean matrix; weighted sequence space

1 Introduction

Let $p \geq 1$ and ω denote the set of all real-valued sequences. The space l_p is the set of all real sequences $x = (x_n) \in \omega$ such that

$$\|x\|_p = \left(\sum_{n=1}^{\infty} |x_n|^p\right)^{1/p} < \infty.$$

If $w = (w_n) \in \omega$ is a non-negative sequence, we define the weighted sequence space $l_p(w)$ as follows:

$$l_p(w) := \left\{x = (x_n) \in \omega : \sum_{n=1}^{\infty} w_n |x_n|^p < \infty\right\},$$

with norm $\|\cdot\|_{p,w}$, which is defined in the following way:

$$\|x\|_{p,w} = \left(\sum_{n=1}^{\infty} w_n |x_n|^p\right)^{1/p}.$$

Let $C = (c_{n,k})$ denote the Cesàro matrix. We recall that the elements $c_{n,k}$ of the matrix C are given by

$$c_{n,k} = \begin{cases} \frac{1}{n} & \text{for } 1 \leq k \leq n, \\ 0 & \text{for } k > n. \end{cases}$$

The sequence space defined by

$$c_p(w) = \{(x_n) \in \omega : Cx \in l_p(w)\}$$
$$= \left\{(x_n) \in \omega : \sum_{n=1}^{\infty} w_n \left|\frac{1}{n}\sum_{i=1}^{n} x_i\right|^p < \infty\right\}$$

is called the Cesàro weighted sequence space, and the norm $\|\cdot\|_{p,w,c}$ of the space is defined by

$$\|x\|_{p,w,c} = \left(\sum_{n=1}^{\infty} w_n \left|\frac{1}{n}\sum_{i=1}^{n} x_i\right|^p\right)^{1/p}.$$

The Cesàro sequence spaces were studied in [1], where $w_n = 1$ for all n. It is significant that in the special case $w_n = 1$, we have $l_p(w) = l_p$ and $c_p(w) = c_p$.

Let (w_n) be a non-negative sequence and $A = (a_{n,k})$ be a lower triangular matrix with non-negative entries. In this paper, we shall consider the inequality of the form

$$\|Ax\|_{p,w,c} \le U\|x\|_{p,w},$$

and the inequality of the form

$$\|Ax\|_{p,w} \le U\|x\|_{p,w,c},$$

where $x = (x_n)$ is a non-negative sequence. The constant U does not depend on x, and we seek the smallest possible value of U. We write $\|A\|_{p,w,c}$ for the norm of A as an operator from $l_p(w)$ into $c_p(w)$, $\|A\|_{p,c}$ for the norm of A as an operator from l_p into c_p, $\|A\|_{c,p,w}$ for the norm of A as an operator from $c_p(w)$ into $l_p(w)$, $\|A\|_{c,p}$ for the norm of A as an operator from c_p into l_p, $\|A\|_{p,w}$ for the norm of A as an operator from $l_p(w)$ into itself and $\|A\|_p$ for the norm of A as an operator from l_p into itself.

The problem of finding the norm of a lower triangular matrix on the sequence spaces l_p and $l_p(w)$ has been studied before in [2–8]. In the study, we will expand this problem for matrix operators from $l_p(w)$ into $c_p(w)$ and matrix operators from $c_p(w)$ into $l_p(w)$, and we consider certain matrix operators such as Cesàro, Nörlund and weighted mean. The study is an extension of some results obtained by [3, 7].

2 The norm of matrix operators from $l_p(w)$ into $c_p(w)$

In this section, we tend to compute the bounds for the norm of lower triangular matrix operators from $l_p(w)$ into $c_p(w)$. In particular, we apply our results for lower triangular matrix operators from l_p into c_p, when $w_n = 1$ for all n.

Throughout this paper, let $A = (a_{n,k})$ be a matrix with non-negative real entries i.e., $a_{n,k} \ge 0$, for all n, k. This implies that $\|Ax\|_{p,w,c} \le \|A|x|\|_{p,w,c}$, and hence the non-negative sequences are sufficient to determine the norm of A. We say that $A = (a_{n,k})$ is lower triangular if $a_{n,k} = 0$ for $n < k$. A non-negative lower triangular matrix is called a summability matrix if $\sum_{k=1}^{n} a_{n,k} = 1$ for all n.

We first state some lemmas from [3, 7], which are needed for our main result. Set $\xi^+ = \max(\xi, 0)$ and $\xi^- = \min(\xi, 0)$ and $p^* = p/(p-1)$.

Lemma 2.1 ([3], Lemma 2.1) *Let a and x be two non-negative sequences, then for all n,*

$$\sum_{k=1}^{n} a_k x_k \le \left\{ \max_{1\le k\le n} \frac{1}{n-k+1} \sum_{j=k}^{n} x_j \right\} \sum_{k=1}^{n} (n-k+1)(a_k - a_{k-1})^+ .$$

Lemma 2.2 ([3], Lemma 2.2) *Let $N \ge 1$, and let a and x be two non-negative sequences. If $x_N \ge x_{N+1} \ge \cdots \ge 0$ and $x_n = 0$ for $n < N$, then*

$$\sum_{k=1}^{n} a_k x_k \ge \left(\frac{1}{n} \sum_{j=1}^{n} x_j \right) \left\{ n a_N + \frac{n}{n-N+1} \sum_{k=N+1}^{n} (n-k+1)(a_k - a_{k-1})^- \right\}$$

for all n.

Lemma 2.3 ([7], Lemma 1.4) *Let $p > 1$ and $w = (w_n)$ be a decreasing sequence with non-negative entries and $\sum_{n=1}^{\infty} \frac{w_n}{n}$ be divergent. Let $N \ge 1$ and the matrix $C_N = (c_{n,k}^N)$ be with the following entries:*

$$c_{n,k}^N = \begin{cases} \frac{1}{n+N-1} & \text{for } n \ge k, \\ 0 & \text{for } n < k. \end{cases}$$

Then $\|C_N\|_{p,w}$ is determined by non-negative decreasing sequences and $\|C_N\|_{p,w} = p^*$.

Note that C_1 is the well-known Cesàro matrix.

Lemma 2.4 ([7], Lemma 1.5) *If $p > 1$ and x and w are two non-negative sequences and also w is decreasing, then*

$$\sum_{j=1}^{\infty} w_j \max_{1\le i\le j} \left(\frac{1}{j-i+1} \sum_{k=i}^{j} x_k \right)^p \le \left(p^*\right)^p \sum_{k=1}^{\infty} w_k x_k^p .$$

We set $a_{0,0} = 0$ and $a_{n,0} = 0$ for $n \ge 1$ and

$$M_A = \sup_{n\ge 1} \left\{ \sum_{k=1}^{n} \frac{n-k+1}{n} \left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1} \right)^+ \right\},$$

$$m_A = \sup_{N\ge 1} \inf_{n\ge N} \left\{ \sum_{i=N}^{n} a_{i,N} + \frac{1}{n-N+1} \sum_{k=N+1}^{n} (n-k+1) \left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1} \right)^- \right\}.$$

We are now ready to present the main result of this section.

Theorem 2.5 *Suppose that $p > 1$ and $w = (w_n)$ is a decreasing sequence with non-negative entries. If $A = (a_{n,k})$ is a lower triangular matrix with non-negative entries, then we have the following statements.*

(i) *$\|A\|_{p,w,c} \le p^* M_A$. Moreover, if $M_A < \infty$, then A is a bounded matrix operator from $l_p(w)$ into $c_p(w)$.*
(ii) *If $\sum_{n=1}^{\infty} \frac{w_n}{n}$ is divergent and $(\frac{w_n}{w_{n+1}})$ is decreasing, then $\|A\|_{p,w,c} \ge p^* m_A$.*

Therefore if $w = (w_n)$ is a decreasing sequence with non-negative entries and $(\frac{w_n}{w_{n+1}})$ is decreasing and $\sum_{n=1}^{\infty} \frac{w_n}{n} = \infty$, then

$$p^* m_A \leq \|A\|_{p,w,c} \leq p^* M_A.$$

In particular, if $w_n = 1$ for all n and if $M_A < \infty$, then A is a bounded matrix operator from l_p into c_p and $p^* m_A \leq \|A\|_{p,c} \leq p^* M_A$.

Proof (i) Let (x_n) be a non-negative sequence. By using Lemma 2.1, we get

$$\begin{aligned}
&\sum_{k=1}^{n} \left(\frac{1}{n} \sum_{i=k}^{n} a_{i,k} \right) x_k \\
&\quad \leq \left\{ \max_{1 \leq k \leq n} \frac{1}{n-k+1} \sum_{j=k}^{n} x_j \right\} \sum_{k=1}^{n} \frac{n-k+1}{n} \left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1} \right)^{+} \\
&\quad \leq M_A \max_{1 \leq k \leq n} \left\{ \frac{1}{n-k+1} \sum_{j=k}^{n} x_j \right\}.
\end{aligned}$$

By applying Lemma 2.4, we deduce that

$$\begin{aligned}
\sum_{n=1}^{\infty} w_n \left(\sum_{k=1}^{n} \left(\frac{1}{n} \sum_{i=k}^{n} a_{i,k} \right) x_k \right)^p &\leq M_A^p \sum_{n=1}^{\infty} w_n \max_{1 \leq k \leq n} \left(\frac{1}{n-k+1} \sum_{j=k}^{n} x_j \right)^p \\
&\leq \left(p^* M_A \right)^p \sum_{k=1}^{\infty} w_k x_k^p.
\end{aligned}$$

(ii) We have $m_A = \sup_{N \geq 1} \beta_N$, where

$$\beta_N = \inf_{n \geq N} \left\{ \sum_{i=N}^{n} a_{i,N} + \frac{1}{n-N+1} \sum_{k=N+1}^{n} (n-k+1) \left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1} \right)^{-} \right\}.$$

Let $N \geq 1$, so that $\beta_N \geq 0$. If $y = (y_n)$ is a decreasing sequence with non-negative entries and $\|y\|_{p,w} = 1$, we set $x_1 = x_2 = \cdots = x_{N-1} = 0$ and

$$x_{n+N-1} = \left(\frac{w_n}{w_{n+N-1}} \right)^{1/p} y_n$$

for all $n \geq 1$. So $\|x\|_{p,w} = \|y\|_{p,w} = 1$, and from Lemma 2.2 it follows that

$$\begin{aligned}
\|A\|_{p,w,c}^p &\geq \sum_{n=1}^{\infty} w_n \left(\sum_{k=1}^{n} \left(\frac{1}{n} \sum_{i=k}^{n} a_{i,k} \right) x_k \right)^p \\
&\geq \beta_N^p \sum_{n=1}^{\infty} w_n \left(\frac{1}{n} \sum_{j=1}^{n} x_j \right)^p \\
&= \beta_N^p \sum_{n=1}^{\infty} w_{n+N-1} \left(\frac{1}{n+N-1} \sum_{j=1}^{n} x_{j+N-1} \right)^p
\end{aligned}$$

$$= \beta_N^p \sum_{n=1}^{\infty} w_{n+N-1}\left(\frac{1}{n+N-1}\sum_{j=1}^{n}\left(\frac{w_j}{w_{j+N-1}}\right)^{1/p} y_j\right)^p$$
$$\geq \beta_N^p \|C_N y\|_{p,w,}^p.$$

By Lemma 2.3, we conclude that $\|A\|_{p,w,c} \geq p^*\beta_N$, so

$$\|A\|_{p,w,c} \geq p^* m_A. \qquad \square$$

In what follows we assume that $w = (w_n)$ is a decreasing sequence with non-negative entries and $(\frac{w_n}{w_{n+1}})$ is decreasing and $\sum_{n=1}^{\infty} \frac{w_n}{n} = \infty$.

At first we bring a corollary of Theorem 2.5 for a lower triangular matrix $A = (a_{n,k})$. The rows of C_1A are increasing, where C_1 is the Cesàro matrix and

$$(C_1A)_{n,k} = \sum_{i=1}^{\infty} c_{n,i}^1 a_{i,k} = \frac{1}{n}\sum_{i=k}^{n} a_{i,k}, \quad (n,k = 1,2,\ldots).$$

Corollary 2.6 *Suppose that* $p > 1$ *and* $A = (a_{n,k})$ *is a non-negative lower triangular matrix that* $\sum_{i=k-1}^{n} a_{i,k-1} \leq \sum_{i=k}^{n} a_{i,k}$ *for* $1 < k \leq n$. *Then*

$$\|A\|_{p,w,c} = p^* \sup_{n\geq 1} a_{n,n}.$$

In particular, $\|I\|_{p,w,c} = p^*$, *where I is the identity matrix.*

Proof Since the finite sequence $(\sum_{i=k}^{n} a_{i,k})_{k=1}^{n}$ is increasing for each n, we have

$$\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)^+ = \sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}$$

for $1 \leq k \leq n$. Hence

$$M_A = \sup_{n\geq 1}\left\{\sum_{k=1}^{n}\frac{n-k+1}{n}\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)\right\}$$
$$= \sup_{n\geq 1}\frac{1}{n}\sum_{k=1}^{n}\sum_{i=k}^{n} a_{i,k} \leq \sup_{n\geq 1} a_{n,n}.$$

Moreover,

$$\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)^- = 0 \quad (1 \leq k \leq n)$$

and

$$m_A = \sup_{N\geq 1}\inf_{n\geq N}\sum_{i=N}^{n} a_{i,N} = \sup_{n\geq 1} a_{n,n}.$$

Hence, according to Theorem 2.5, we obtain the desired result. $\square$

Example 2.7 Let $A = (a_{n,k})$ be defined by

$$a_{n,k} = \begin{cases} \frac{1}{n^2} & \text{for } k < n, \\ \frac{2n-1}{n} & \text{for } k = n, \\ 0 & \text{for } k > n. \end{cases}$$

Since the finite sequence $(\sum_{i=k}^{n} a_{i,k})_{k=1}^{n}$ is increasing for each n and $\sup_{n\geq 1} a_{n,n} = 2$, by Corollary 2.6, we have $\|A\|_{p,w,c} = 2p^*$.

Now, in the second case, we state some corollaries of Theorem 2.5 for a lower triangular matrix A, where the rows of C_1A are decreasing.

Corollary 2.8 *Suppose that* $p > 1$ *and* $A = (a_{n,k})$ *is a lower triangular matrix with* $\sum_{i=k-1}^{n} a_{i,k-1} \geq \sum_{i=k}^{n} a_{i,k}$ *for* $1 < k \leq n$. *Then*

$$p^*\left(\inf_{n\geq 1} \frac{1}{n} \sum_{k=1}^{n} \sum_{i=k}^{n} a_{i,k}\right) \leq \|A\|_{p,w,c} \leq p^*\left(\sup_{n\geq 1} \sum_{i=1}^{n} a_{i,1}\right).$$

In particular, for summability matrices the left-hand side of the above inequality reduces to p^*.

Moreover, if the right-hand side of the above inequality is finite, then A *is a bounded matrix operator from* $l_p(w)$ *into* $c_p(w)$.

Proof Since the finite sequence $(\sum_{i=k}^{n} a_{i,k})_{k=1}^{n}$ is decreasing for each n, we have

$$\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)^{+} = 0 \quad (1 < k \leq n),$$

and $(\sum_{i=1}^{n} a_{i,1} - \sum_{i=0}^{n} a_{i,0})^{+} = \sum_{i=1}^{n} a_{i,1}$. Hence $M_A = \sup_{n\geq 1} \sum_{i=1}^{n} a_{i,1}$. Moreover,

$$\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)^{-} = \sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1},$$

for $1 < k \leq n$, so

$$\begin{aligned} m_A &= \sup_{N\geq 1} \inf_{n\geq N} \left\{\sum_{i=N}^{n} a_{i,N} + \frac{1}{n-N+1} \sum_{k=N+1}^{n} (n-k+1)\left(\sum_{i=k}^{n} a_{i,k} - \sum_{i=k-1}^{n} a_{i,k-1}\right)\right\} \\ &= \sup_{N\geq 1} \inf_{n\geq N} \frac{1}{n-N+1} \sum_{k=N}^{n} \sum_{i=k}^{n} a_{i,k} \\ &\geq \inf_{n\geq 1} \frac{1}{n} \sum_{k=1}^{n} \sum_{i=k}^{n} a_{i,k}. \end{aligned}$$

Therefore, by Theorem 2.5, we prove the desired result. □

The two examples of Corollary 2.8 are given as follows.

Example 2.9 Suppose that $\alpha \geq 2$ and the matrix $A = (a_{n,k})$ is defined by

$$a_{n,k} = \begin{cases} \frac{1}{n^\alpha} & \text{for } n \geq k, \\ 0 & \text{for } n < k. \end{cases}$$

Since $\sum_{i=k}^n a_{i,k} = \sum_{i=k}^n \frac{1}{i^\alpha}$ and $\sum_{i=k-1}^n a_{i,k-1} \geq \sum_{i=k}^n a_{i,k}$ for $1 < k \leq n$, we have $0 \leq \|A\|_{p,w,c} \leq p^*\zeta(\alpha)$, where $\zeta(\alpha) = \sum_{n=1}^\infty \frac{1}{n^\alpha}$.

Example 2.10 Suppose that the matrix $A = (a_{n,k})$ is defined by

$$a_{n,k} = \begin{cases} \frac{1}{n(n+1)} & \text{for } n \geq k, \\ 0 & \text{for } n < k. \end{cases}$$

Since $\sum_{i=k}^n a_{i,k} = \sum_{i=k}^n \frac{1}{i(i+1)}$, by Corollary 2.8, we have $0 \leq \|A\|_{p,w,c} \leq p^*$.

We apply the above corollary to the following two special cases.

Let (a_n) be a non-negative sequence with $a_1 > 0$, and $A_n = a_1 + \cdots + a_n$. The Nörlund matrix $N_a = (a_{n,k})$ is defined as follows:

$$a_{n,k} = \begin{cases} \frac{a_{n-k+1}}{A_n} & \text{for } 1 \leq k \leq n, \\ 0 & \text{for } k > n. \end{cases}$$

Also the weighted mean matrix $M_a = (a_{n,k})$ is defined by

$$a_{n,k} = \begin{cases} \frac{a_k}{A_n} & \text{for } 1 \leq k \leq n, \\ 0 & \text{for } k > n. \end{cases}$$

Corollary 2.11 *Suppose that $p > 1$ and $N_a = (a_{n,k})$ is the Nörlund matrix and (a_n) is an increasing sequence. Then*

$$p^* \leq \|N_a\|_{p,w,c} \leq p^*\left(\sup_{n\geq 1}\sum_{i=1}^n \frac{a_i}{A_i}\right).$$

Proof Since N_a is a summability matrix and $\sum_{i=1}^n a_{i,1} = \sum_{i=1}^n \frac{a_i}{A_i}$, by applying Corollary 2.8, we have the desired result. □

Corollary 2.12 *Suppose that $p > 1$ and $M_a = (a_{n,k})$ is the weighted mean matrix and (a_n) is a decreasing sequence. Then*

$$p^* \leq \|M_a\|_{p,w,c} \leq p^* a_1\left(\sup_{n\geq 1}\sum_{i=1}^n \frac{1}{A_i}\right).$$

Proof Since M_a is a summability matrix and $\sum_{i=1}^n a_{i,1} = \sum_{i=1}^n \frac{a_1}{A_i}$, by Corollary 2.8, the proof is obvious. □

Finally, in the third case, if the rows of C_1A are neither increasing nor decreasing, we present the following theorem.

Theorem 2.13 *Suppose that $p > 1$ and $A = (a_{n,k})$ is a non-negative lower triangular matrix. If A is a bounded matrix operator from $l_p(w)$ into itself, then A is a bounded matrix operator from $l_p(w)$ into $c_p(w)$ and*

$$\|A\|_{p,w,c} \leq p^* \|A\|_{p,w}.$$

Proof We have

$$\begin{aligned}\|Ax\|_{p,w,c}^p &= \sum_{n=1}^{\infty} w_n \left| \frac{1}{n} \sum_{k=1}^{n} \sum_{j=1}^{k} a_{k,j} x_j \right|^p \\ &= \sum_{n=1}^{\infty} w_n \left| \sum_{j=1}^{n} (C_1 A)_{n,j} x_j \right|^p = \left\| (C_1 A) x \right\|_{p,w}^p.\end{aligned}$$

Hence, by Lemma 2.3, we conclude that $\|A\|_{p,w,c} = \|C_1 A\|_{p,w} \leq p^* \|A\|_{p,w}$. □

We apply the above theorem to the following two Nörlund and weighted mean matrices.

Corollary 2.14 ([7], Corollary 1.3) *Suppose that $p > 1$ and $N_a = (a_{n,k})$ is the Nörlund matrix and (a_n) is a decreasing sequence with $a_n \downarrow \alpha$ and $\alpha > 0$. Then*

$$\|N_a\|_{p,w} = p^*.$$

Corollary 2.15 *Suppose that $p > 1$ and $N_a = (a_{n,k})$ is the Nörlund matrix and (a_n) is a decreasing sequence with $a_n \downarrow \alpha$ and $\alpha > 0$. Then*

$$\|N_a\|_{p,w,c} \leq \left(p^*\right)^2.$$

Proof By applying Theorem 2.13 and Corollary 2.14, we have the desired result. □

Corollary 2.16 ([7], Corollary 1.4) *Suppose that $p > 1$ and $M_a = (a_{n,k})$ is the weighted mean matrix and (a_n) is an increasing sequence with $a_n \uparrow \alpha$ and $\alpha < \infty$. Then*

$$\|M_a\|_{p,w} = p^*.$$

Corollary 2.17 *Suppose that $p > 1$ and $M_a = (a_{n,k})$ is the weighted mean matrix and (a_n) is an increasing sequence with $a_n \uparrow \alpha$ and $\alpha < \infty$. Then*

$$\|M_a\|_{p,w,c} \leq \left(p^*\right)^2.$$

Proof By using Theorem 2.13 and Corollary 2.16, the proof is clear. □

3 The norm of matrix operators from $c_p(w)$ into $l_p(w)$

In this section, we compute the bounds for the norm of lower triangular matrix operators from $c_p(w)$ into $l_p(w)$. In particular, when $w_n = 1$ for all n, the bounds for the norm of lower triangular matrix operators from c_p into l_p are deduced. Moreover, we apply our results for Cesàro, Nörlund and weighted mean matrices.

We begin with a proposition which is needed to prove the main theorem of this section.

Proposition 3.1 ([6], *Proposition* 5.1). *Let $p > 1$ and $w = (w_n)$ be a decreasing sequence with non-negative entries, and let C_1 be the Cesàro matrix. Then we have $\|C_1\|_{p,w} \le p^*$.*

Theorem 3.2 *Suppose that $p > 1$ and $w = (w_n)$ is a sequence with non-negative entries and $A = (a_{n,k})$ is a lower triangular matrix with non-negative entries. We have*

$$\frac{1}{p^*}\|A\|_{p,w} \le \|A\|_{c,p,w} \le \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big).$$

Moreover, if the right-hand side of the above inequality is finite, then A is a bounded matrix operator from $c_p(w)$ into $l_p(w)$. In particular, if $w_n = 1$ for all n, then we have

$$\frac{1}{p^*}\|A\|_{p} \le \|A\|_{c,p} \le \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big).$$

Proof Suppose that $x \in c_p(w)$

$$\begin{aligned}
\|Ax\|_{p,w}^p &= \sum_{n=1}^{\infty} w_n \left|\sum_{k=1}^{n} a_{n,k}x_k\right|^p \\
&\le \sum_{n=1}^{\infty} w_n \left(\sup_{1\le k\le n} a_{n,k} \sum_{k=1}^{n} x_k\right)^p \\
&\le \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big)^p \sum_{n=1}^{\infty} w_n \left(\frac{1}{n}\sum_{k=1}^{n} x_k\right)^p \\
&= \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big)^p \|x\|_{p,w,c}^p.
\end{aligned}$$

Hence

$$\frac{\|Ax\|_{p,w}}{\|x\|_{p,w,c}} \le \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big)$$

and

$$\|A\|_{c,p,w} \le \sup_{n\ge 1}\Big(n \sup_{1\le k\le n} a_{n,k}\Big).$$

On the other hand, Proposition 3.1 concludes that $\|x\|_{p,w,c} = \|C_1x\|_{p,w} \le p^*\|x\|_{p,w}$, so

$$\frac{\|Ax\|_{p,w}}{\|x\|_{p,w,c}} \ge \frac{1}{p^*}\frac{\|Ax\|_{p,w}}{\|x\|_{p,w}}.$$

Therefore $\frac{1}{p^*}\|A\|_{p,w} \le \|A\|_{c,p,w}$, and the proof is complete. □

Corollary 3.3 *If $p > 1$, then the generalized Cesàro matrix C_N is bounded from $c_p(w)$ into $l_p(w)$ and*

$$\|C_N\|_{c,p,w} = 1.$$

Proof Since

$$\sup_{n\geq 1}\Bigl(n \sup_{1\leq k\leq n} a_{n,k}\Bigr) = \sup_{n\geq 1} \frac{n}{n+N-1} = 1,$$

by using Lemma 2.3 and Theorem 3.2, the proof is obvious. □

We apply the above theorem to the following two special cases.

Corollary 3.4 *Suppose that $p > 1$ and $N_a = (a_{n,k})$ is the Nörlund matrix and (a_n) is a decreasing sequence with $a_n \downarrow \alpha$ and $\alpha > 0$. Then*

$$1 \leq \|N_a\|_{c,p,w} \leq a_1 \sup_{n\geq 1} \frac{n}{A_n}.$$

Proof By Theorem 3.2 and Corollary 2.14, the proof is clear. □

Example 3.5 Let $\alpha > 0$ and $a_n = \alpha + \frac{1}{n^{\alpha+1}}$ for all n. The sequence (a_n) is decreasing and $a_n \downarrow \alpha$ and also $a_1 \sup_{n\geq 1} \frac{n}{A_n} = 1 + \frac{1}{\alpha}$. So

$$1 \leq \|N_a\|_{c,p,w} \leq 1 + \frac{1}{\alpha}.$$

Specially $\|N_a\|_{c,p,w} \to 1$, when $\alpha \to \infty$.

Corollary 3.6 *Suppose that $p > 1$ and $M_a = (a_{n,k})$ is the weighted mean matrix and (a_n) is an increasing sequence with $a_n \uparrow \alpha$ and $\alpha < \infty$. Then*

$$1 \leq \|M_a\|_{c,p,w} \leq \sup_{n\geq 1} \frac{na_n}{A_n}.$$

Proof By using Theorem 3.2 and Corollary 2.16, the proof is obvious. □

Example 3.7 Let $a_n = 1 - \frac{1}{(n+1)^2}$ for all n. The sequence (a_n) is increasing and $a_n \uparrow 1$ and also

$$\sup_{n\geq 1} \frac{na_n}{A_n} = \frac{3a_3}{A_3} \simeq 1.091.$$

So

$$1 \leq \|M_a\|_{c,p,w} \leq 1.091.$$

4 Conclusions

In the present study, we considered the problem of finding bounds for the norm of lower triangular matrix operators from $l_p(w)$ into $c_p(w)$ and from $c_p(w)$ into $l_p(w)$. Moreover, we computed the norms of certain matrix operators such as Cesàro, Nörlund and weighted mean, and we extended some results of [3, 7].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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