Scientific Research and Essays Vol. 7(36), pp. 3186-3193, 13 September, 2012
Available online at http://www.academicjournals.org/SRE
DOI: 10.5897/SRE12.335
ISSN 1992-2248 ©2012 Academic Journals

Full Length Research Paper

Estimation of the Rayleigh distribution parameters with presence of k outliers generated from Gamma distribution

Afshin Ghanizadeh

Department of Statistics, Kermanshah Branch, Islamic Azad University, Kermanshah, Iran.
E-mail: afshinghanizadeh@yahoo.com.

Accepted 3 August, 2012

The moment, maximum likelihood and mixture of the estimators of parameters of the Rayleigh distribution are derived with presence of k outliers generated from Gamma distribution. These estimators are compared empirically when all the parameters are unknown; their Bias and MSE are investigated with the help of numerical technique. We have shown that these estimators are asymptotically unbiased. At the end, we conclude that mixture estimators are better than the maximum likelihood and moment estimators.

Key words: Rayleigh distribution, mixture estimators, outliers, Newton-Raphson method.

INTRODUCTION

Rayleigh distribution is a special case of the Weibull distribution. It has been used to study the scattering of radiation, wind speeds or to make certain transformation. The Rayleigh distribution is a suitable model for life testing experiments and clinical studies which age with time as its hazard rate $H(t)=t/\theta$ is a linear function of time. Polovko (1968) and Dyer and Whisenand (1973) demonstrated the importance of this distribution in electro vacuum devices and communication engineering. Ariyawansa and Templeton (1984) have also discussed some of the applications. Howlader and Hossian (1995) obtained Bayesian estimators for the scale parameter and the reliability function in the case of type-II censored sampling. The origin and order aspects of this distribution can be found in Siddiqui (1962) and Hirai (1966 and 1972). Abd Elfattah et al. (2006) studied the efficiency of the maximum likelihood estimates of the parameter under three cases, namely, type-I, type-II and progressive type-II censored sampling schemes.

The probability density function of the Rayleigh distribution with parameter of θ is given by:

$$f\left(x,\theta\right)=\frac{2x}{\theta}e^{-\frac{x^2}{\theta}}\ ;\qquad 0\le t<\infty,\quad \theta>0 \quad .\qquad (1)$$

The maximum likelihood estimator (MLE) of θ is $\frac{\sum_{i=1}^{n}x_i^2}{n}$ with large sample variance $\frac{\theta^2}{4n}$. The reliability function $R(t)$ and the failure rate function (or hazard function) $H(t)$ at miss time t are:

$$R\left(t\right)=e^{-\frac{t^2}{\theta}}\ ;\qquad 0\le t<\infty,\quad \theta>0$$

$$H\left(t\right)=\frac{2t}{\theta}\ ;\qquad 0\le t<\infty,\quad \theta>0 \ .$$

Consider spread from a point source, for example, which might be a small plot of plants. During favourable weather conditions, the plants release their pollen and it disperses according to exponential distribution with distance from the source. However, in less favourable condition, light, rain or mist, not only are the plants less likely to release pollen, but that which is released still falls with a uniform distribution. Dixit et al. (1996), considered the above example in the context of spread of disease amongst plants of viral spores such as barley yellow mosaic dwarf virus (BYMDV).

According to Dixit et al. (1996), we assume that a set of random variables ($X_1, X_2, ..., X_n$) represent the distance of an infected sampled plant from a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed according to the Rayleigh distribution. The other observations out of *n* random variables (say *k*) are present because aphids which are known to be carriers of BYMDV have passed the virus into the plants when the aphids feed on the sap. These *k* aphids are considered to be exponentially distributed, that is a special case of the Gamma distribution, however in this paper we will consider these *k* aphids to be distributed by the Gamma distribution.

We assume that the random variables are (X_1 , X_2 , … , X_n) such that *k* of them are distributed with p.d.f $g(x, \alpha, \theta)$,

$$g(x, \alpha, \theta) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\theta}}, \quad x > 0, \quad \alpha, \theta > 0 \tag{2}$$

and the remaining (*n-k*) random variables are distributed with p.d.f $f(x, \theta)$, the Rayleigh distribution, as mentioned in Equation (1).

In "Methods of Estimation", first we have obtained the joint distribution of ($X_1, X_2, ..., X_n$) and also the marginal distribution of X in the presence of *k* outliers. After that, we deal with the methods of estimation such as the methods of moment, maximum likelihood and mixture of the estimators of moment and maximum likelihood in estimating α and θ. Thereafter, we compare the Bias and MSE of both the estimates empirically.

METHODS OF ESTIMATION

Here, we deal with the methods of moment and maximum likelihood in estimating α and θ; thereafter mixture of the estimators of moment and maximum likelihood are derived.

The joint distribution of $(X_1, X_2, ..., X_n)$ in the presence of *k* outliers can be expressed as:

$$f(x_1, x_2, ..., x_n; \alpha, \theta) = \frac{1}{c(n,k)} \prod_{i=1}^{n} f(x_i; \theta) \cdot \sum_{\underline{A}} \left[\prod_{r=1}^{k} \frac{g(x_{A_r}; \alpha, \theta)}{f(x_{A_r}; \theta)} \right]$$

$$= \frac{2^{n-k}}{c(n,k)(\Gamma(\alpha))^k \theta^{n+(\alpha-1)k}} \prod_{i=1}^{n} x_i e^{\frac{-\sum_{i=1}^{n} x_i^2}{\theta}} \sum_{\underline{A}} \left[\prod_{r=1}^{k} (x_{A_r})^{\alpha-2} e^{\frac{-\sum_{r=1}^{k}(x_{A_r} - x_{A_r}^2)}{\theta}} \right] \tag{3}$$

where $c(n,k) = n!/((n-k)!k!)$ and $\sum_{\underline{A}} = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \cdots \sum_{A_k=A_{k-1}+1}^{n}$. (Dixit, 1989; Dixit and Nasiri, 2001; Nasiri and Pazira, 2010).

It is easy to show that the marginal distribution of X is

$$f(x; \alpha, \theta) = \frac{k\, x^{\alpha-1}}{n\, \Gamma(\alpha)\, \theta^{\alpha}} e^{-\frac{x}{\theta}} + \frac{2(n-k)x}{n\, \theta} e^{-\frac{x^2}{\theta}} \tag{4}$$

Method of moment

Consider, $m_i' = \frac{1}{n} \sum_{j=1}^{n} X_j^i \;, \; i = 1, 2$ and let $b = \frac{k}{n}$ and $\bar{b} = \frac{n-k}{n}$.

Under (4)

$$m_1' = b\alpha\theta + \frac{\bar{b}\sqrt{\theta\pi}}{2} \tag{5}$$

From (5), we have

$$b\alpha t^2 + \frac{\bar{b}\sqrt{\pi}}{2} t - m_1' = 0$$

where

$$t = \sqrt{\theta}$$

if $\Delta = \frac{\pi}{4}\bar{b}^2 + 4b\,\alpha m_1'$ is non-negative then the roots are real. Therefore

$$\sqrt{\theta} = \frac{-\frac{\sqrt{\pi}}{2}\bar{b} + \sqrt{\frac{\pi}{4}\bar{b}^2 + 4b\,\alpha m_1'}}{2b\,\alpha}$$

and then

$$\hat{\theta} = \left[\frac{-\sqrt{\pi}\bar{b} + \sqrt{\pi\bar{b}^2 + 16b\,\hat{\alpha}\, m_1'}}{4b\,\hat{\alpha}} \right]^2 \tag{6}$$

Since from (5) we also have

$$\hat{\alpha} = \frac{m_1'}{b\,\hat{\theta}} - \frac{\bar{b}\sqrt{\pi}}{2b\sqrt{\hat{\theta}}} \tag{7}$$

then we can replace $\hat{\alpha}$ in (6) to obtain $\hat{\theta}$

$$\left[\frac{-\sqrt{\pi}\,\bar{b}\,\hat{\theta}+\sqrt{\pi\bar{b}^{2}\hat{\theta}^{2}+16m_1'^{2}\,\hat{\theta}-8\bar{b}\,m_1'\sqrt{\pi\,\hat{\theta}^{3}}}}{4m_1'-2\bar{b}\sqrt{\pi\,\hat{\theta}}}\right]^{2}-\hat{\theta}=0 \qquad (8)$$

with solving (8), $\hat{\theta}$ is obtained. Then we replace $\hat{\theta}$ in (7) to find $\hat{\alpha}$. Thus, we can obtain the moment estimator of θ by using Equation (8) and the moment estimator of α by using Equation (7).

Here, we shall show that $\hat{\alpha}$ and $\hat{\theta}$ are asymptotically unbiased estimators. Let $W_1=\frac{1}{n}\sum_{i=1}^{n}X_i$, then $m_1'=W_1$. Here, we can write $\hat{\theta}$ as a function of W_1,

$$\hat{\theta}=h(W_1) \qquad (9)$$

Let,

$$E(W_1)=\mu=b\alpha\theta+\frac{\bar{b}\sqrt{\theta\pi}}{2} \qquad (10)$$

Expand the function $h(w_1)$ around μ by Taylor series,

$$\hat{\theta}=h(W_1)=h(\mu)+(W_1-\mu)\frac{\partial h}{\partial W_1}\Big|_{W_1=\mu}+\ldots \qquad (11)$$

then

$$E(\hat{\theta})=h(\mu)=\left(\frac{-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^{2}+16b\,\alpha\left(b\alpha\theta+\frac{\bar{b}\sqrt{\theta\pi}}{2}\right)}}{4b\,\alpha}\right)^{2}$$

$$=\left(\frac{-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^{2}+16b^{2}\alpha^{2}\theta+8b\,\bar{b}\,\alpha\sqrt{\theta\pi}}}{4b\,\alpha}\right)^{2}$$

$$=\left(\frac{-\sqrt{\pi}\bar{b}+\sqrt{\left[\sqrt{\pi}\bar{b}+4b\,\alpha\sqrt{\theta}\right]^{2}}}{4b\,\alpha}\right)^{2}$$

$$=\left(\frac{4b\,\alpha\sqrt{\theta}}{4b\,\alpha}\right)^{2}$$

$$=\theta \qquad (12)$$

By using (10), let $W_1=\sum_{i=1}^{n}X_i$, $W_2=\hat{\theta}$, $E(W_1)=\mu$, and $E(W_2)=\theta$.

$$\hat{\alpha}=h(W_1,W_2)=h(\mu,\theta)+(W_1-\mu)\frac{\partial h}{\partial W_1}\Big|_{W_1=\mu,W_2=\theta}+(W_2-\theta)\frac{\partial h}{\partial W_2}\Big|_{W_1=\mu,W_2=\theta}+\ldots$$

According to the previous procedure

$$E(\hat{\alpha})=\frac{\mu}{b\,\theta}-\frac{\bar{b}\sqrt{\pi}}{2b\sqrt{\theta}}$$

$$=\frac{b\alpha\theta+\frac{\bar{b}\sqrt{\theta\pi}}{2}}{b\,\theta}-\frac{\bar{b}\sqrt{\pi}}{2b\sqrt{\theta}}$$

$$=\alpha. \qquad (13)$$

Hence, $\hat{\alpha}$ and $\hat{\theta}$ are asymptotically unbiased.

Method of maximum likelihood

From (3), the likelihood of ($X_1, X_2, ..., X_n$) is

$$L(x_1,x_2,...,x_n;\alpha,\theta)=\frac{2^{n-k}\prod_{i=1}^{n}x_i\,e^{\frac{-\sum_{i=1}^{n}x_i^{2}}{\theta}}}{c(n,k)\left(\Gamma(\alpha)\right)^{k}\,\theta^{n+(\alpha-1)k}}\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^{2})}{\theta}}\right] \qquad (14)$$

If $L(\alpha,\theta)=\ln[L(\underline{x};\alpha,\theta)]$, then

$$L(\alpha,\theta)\propto-\frac{\sum_{i=1}^{n}x_i^{2}}{\theta}-(n+(\alpha-1)k)\ln\theta-k\,\ln\left(\Gamma(\alpha)\right)+\ln\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^{2})}{\theta}}\right]\right)$$

To solve for our MLEs of α and θ we take the derivative of the log likelihood ($L(\alpha,\theta)$) with respect to each parameter set the partial derivatives equal to zero and solve for $\hat{\alpha}$ and $\hat{\theta}$:

$$\frac{\partial L(\alpha,\theta)}{\partial \alpha} = -k\ \ln\theta - k\ \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} + \frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]} \overset{set}{=} 0 \qquad (15)$$

where $\Gamma'(\alpha) = \frac{\partial}{\partial\alpha}\Gamma(\alpha)$,

$$\frac{\partial L(\alpha,\theta)}{\partial \theta} = \frac{\sum_{i=1}^{n} x_i^2}{\theta^2} - \frac{n+(\alpha-1)k}{\theta} + \frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{\theta^2\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]} \overset{set}{=} 0 \qquad (16)$$

There is no closed-from solution to this system of equations, so we will solve for $\hat{\alpha}$ and $\hat{\theta}$ iteratively, using the Newton-Raphson method, a tangent method for root finding. In our case we will estimate $\beta = (\alpha,\theta)$ iteratively:

$\hat{\beta}_{i+1} = \hat{\beta}_i - G^{-1}g$; i=0, 1, 2, 3,… (17)

where *g* is the vector of normal equations for which we want

$$g = [g_1 \quad g_2]$$

with

$$g_1 = \frac{\Gamma'(\alpha)}{\Gamma(\alpha)} + \ln\theta - \frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{k\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]} \qquad (18)$$

$$g_2 = -\frac{\sum_{i=1}^{n} x_i^2}{k\,\theta^2} + \frac{n+(\alpha-1)k}{k\,\theta} - \frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{k\,\theta^2\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k} x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]} \qquad (19)$$

and G is the matrix of second derivatives

$$G=\begin{bmatrix}\frac{dg_1}{d\alpha} & \frac{dg_1}{d\theta}\\ \frac{dg_2}{d\alpha} & \frac{dg_2}{d\theta}\end{bmatrix} \quad (20)$$

Where

$$\frac{dg_1}{d\alpha}=\left(\frac{\Gamma''(\alpha)}{\Gamma(\alpha)}-\left(\frac{\Gamma'(\alpha)}{\Gamma(\alpha)}\right)^2\right)-\frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)^2 e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{k\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}$$
$$+\frac{\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)^2}{k\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)^2} \quad (21)$$

$$\frac{dg_1}{d\theta}=\frac{dg_2}{d\alpha}=\frac{1}{\theta}-\frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{k\theta^2\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}$$
$$+\frac{\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)}{k\theta^2\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)^2} \quad (22)$$

$$\frac{dg_2}{d\theta}=\frac{2\sum_{i=1}^{n}x_i^2}{k\theta^3}-\frac{n+(\alpha-1)k}{k\theta^2}-\frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)^2 e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}{k\theta^4\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]}$$

$$+\frac{\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)\times\Delta}{k\,\theta^4\left(\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\right]\right)^2} \quad (23)$$

where

$$\Delta=\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\theta}}\left(2\theta+\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)\right)\right]$$

Here, the initial solution β_0 should be selected from (6) and (8). The Newton-Raphson algorithm converges, as our estimates of α and θ change by less than a tolerated amount with each successive iteration, to $\hat{\alpha}$ and $\hat{\theta}$.

Mixture of method of moment and maximum likelihood

Read (1981) proposed the methods, which avoid the difficulty of complicated equations. According to Read (1981), replacement of some, but not all, of the equations in the system of likelihood may make it more manageable. From (6), we have

$$\hat{\theta}=\left[\frac{-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^2+16b\,\hat{\alpha}\,m_1'}}{4b\,\hat{\alpha}}\right]^2 \quad (24)$$

and

$$\Psi(\hat{\alpha})+\ln\hat{\theta}-\frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\hat{\alpha}-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\hat{\theta}}}\right]}{k\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\hat{\alpha}-2}\right)e^{\frac{-\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^2)}{\hat{\theta}}}\right]}=0 \quad (25)$$

Where $\Psi(\alpha)=\frac{\frac{\partial}{\partial\alpha}\Gamma(\alpha)}{\Gamma(\alpha)}$. From (24) and (25)

Table 1. Bias and MSE $\hat{\alpha}$ for: $\alpha = 2$, $\theta = 0.5$ and $k = 1$.

n	Bias			MSE		
	MLE	Mom	Mix	MLE	Mom	Mix
10	0.383	-0.011	-0.689	1.257	1.634	0.961
20	0.339	-0.009	-0.541	0.976	1.143	0.747
30	0.270	-0.005	-0.449	0.562	0.825	0.448
40	0.225	-0.003	-0.391	0.518	0.665	0.402
50	0.207	-0.003	-0.278	0.427	0.562	0.377
100	0.138	-0.001	-0.192	0.194	0.287	0.179
200	0.097	-0.000	-0.130	0.083	0.109	0.056

Table 2. Bias and MSE $\hat{\theta}$ for: $\alpha = 2$, $\theta = 0.5$ and $k = 1$.

n	Bias			MSE		
	MLE	Mom	Mix	MLE	Mom	Mix
10	0.982	0.126	0.250	0.151	0.469	0.112
20	0.740	0.082	0.147	0.096	0.189	0.080
30	0.580	0.089	0.111	0.075	0.120	0.066
40	0.506	0.078	0.118	0.064	0.091	0.057
50	0.427	0.065	0.108	0.054	0.072	0.049
100	0.390	0.024	0.096	0.051	0.066	0.046
200	0.278	0.012	0.060	0.029	0.035	0.019

$$\Psi(\alpha)+2\ln\left(-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^{2}+16b\,\alpha m_1'}\right)-2\ln(4b\,\alpha)-\frac{\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)\left(\sum_{r=1}^{k}\ln x_{A_r}\right)e^{-\frac{(4b\,\alpha)^2\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^{2})\right)}{\left(-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^{2}+16b\,\alpha m_1'}\right)^2}}\right]}{k\sum_{\underline{A}}\left[\left(\prod_{r=1}^{k}x_{A_r}^{\alpha-2}\right)e^{-\frac{(4b\,\alpha)^2\left(\sum_{r=1}^{k}(x_{A_r}-x_{A_r}^{2})\right)}{\left(-\sqrt{\pi}\bar{b}+\sqrt{\pi\bar{b}^{2}+16b\,\alpha m_1'}\right)^2}}\right]}=0 \qquad (26)$$

with solving (26), $\hat{\alpha}$ is obtained. Then we replace $\hat{\alpha}$ in (24) to find $\hat{\theta}$.

NUMERICAL RESULTS AND DISCUSSION

In this paper, we have addressed the problem of estimating parameters of Rayleigh distribution in presence of *k* outliers. The moment, maximum likelihood and mixture estimators of the parameters are derived and have been shown that the moment estimators of the parameters are asymptotically unbiased estimators. In order to have some idea about Bias and Mean Square Error (MSE) of methods of moment, MLE and mixture of these estimators, we perform sampling experiments using a MATLAB. The results are given in Tables 1, 2, 3 and 4, for $\alpha = 2$, $\theta = 0.5$ and $k = 1\, and\ 2$, with samples size *n=10(10)50, 100 and 200*. We report the average estimates and the MSEs based on 1500 replications.

From Tables 1 to 4, we conjecture that the moment estimates are asymptotically unbiased. It is difficult to show analytically that mixture estimate of θ is asymptotically unbiased. But from simulation of study, Tables 2 and 4, we conjecture that mixture estimate of θ is asymptotically unbiased. On the other hand, for α, the

Table 3. Bias and MSE $\hat{\alpha}$ for: $\alpha = 2$, $\theta = 0.5$ *and* $k = 2$.

n	Bias			MSE		
	MLE	Mom	Mix	MLE	Mom	Mix
10	1.322	-0.105	-0.432	2.238	2.862	1.893
20	1.072	-0.090	-0.343	1.926	2.317	1.584
30	0.987	-0.066	-0.299	1.628	1.954	1.175
40	0.943	-0.079	-0.281	1.501	1.844	0.901
50	0.797	-0.070	-0.232	1.279	1.579	0.793
100	0.486	-0.017	-0.131	0.863	1.263	0.459
200	0.267	-0.009	-0.109	0.479	0.781	0.232

Table 4. Bias and MSE $\hat{\theta}$ for: $\alpha = 2$, $\theta = 0.5$ *and* $k = 2$.

n	Bias			MSE		
	MLE	Mom	Mix	MLE	Mom	Mix
10	1.687	0.128	0.230	0.257	0.393	0.177
20	1.209	0.102	0.197	0.173	0.237	0.130
30	0.938	0.101	0.150	0.149	0.189	0.120
40	0.973	0.086	0.151	0.118	0.144	0.099
50	0.856	0.083	0.099	0.115	0.136	0.099
100	0.531	0.058	0.082	0.089	0.105	0.071
200	0.490	0.033	0.067	0.049	0.070	0.036

moment and mixture estimators are underestimation, but the maximum likelihood estimator is overestimation; also for θ, all of the estimators are overestimation. The MSEs of any three estimators are tending to zero and when *n* increases then the MSEs decrease. Meanwhile, when *k* increases then the MSEs increase.

Tables 1, 2, 3 and 4 show that the mixture estimators have the smallest estimated MSEs as compared with the moment and maximum likelihood estimators. We strongly feel mixture estimator is better and easy to calculate than the maximum likelihood and moment estimations. Therefore, we conclude that mixture estimate should be used always.

ACKNOWLEDGEMENTS

Author would like to thank the editor and two anonymous reviewers for valuable comments which improved the presentation of the paper. This paper is derived from a research project which was supported by Islamic Azad University, Kermanshah Branch, Kermanshah, Iran.

REFERENCES

Dixit UJ (1989). Estimation of parameters of the gamma distribution in the presence of outliers. Commun. Stat-Theory Methods 18:3071-3085.

Dixit UJ, Moor KL, Barnett V (1996). On the estimation of the power of the scale parameter of the exponential distribution in the presence of outliers generated from uniform distribution. Metron 54:201-211.

Dixit UJ, Nasiri PF (2001). Estimation of parameters of the exponential distribution in the presence of outliers generated from uniform distribution. Metron 49(3-4):187-198.

Hirai AS (1966). The properties of the Rayleigh distribution and of estimating of its parameter. Unpublished thesis submitted to the University of Leicester.

Hirai AS (1972). Estimation of scale and location parameters, using quadratic coefficient. Aust. J. Stat. 14(3):20-24.

Nasiri PF, Pazira H (2010). Bayesian and non-Bayesian estimations on the Generalized Exponential distribution in the presence outliers. J. Stat. Theory Pract. 4(3):453-475.