International Journal of the Physical Sciences Vol. 6(8), pp. 1920-1928, 18 April, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.301
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

The effects of support size on the vibration of the point supported plate

C. Demir* and S. Burak Izmirli

Department of Mechanical Engineering, Faculty of Mechanical Engineering, Yildiz Technical University, Yildiz 34349, Istanbul, Turkey.

Accepted 21 March, 2011

The fundamental frequencies of plate are important for engineering designs. Free vibration characteristics of isotropic rectangular plates with point, spring and viscoelastic supports have been studied by several researchers. However, the practical implementation of these boundary conditions in application is not at all trivial. In this study, the effects of the size of the support hole area on the fundamental frequencies of the plate have been analyzed numerically and experimentally. The process involves numerical modeling using finite elements and ANSYS software is used for the analysis of plates. It has advantages such as simplicity in use, applicability to various boundary conditions which solution is not available analytically. The convergence analysis and the analysis for comparison purposes are carried out for the elastic and point supported boundary conditions. For the rigid point supported plate with the specified size of the support area, the experimental study has been conducted and its results compared with the numerical results. The results show that the effects of support size might be considered in engineering vibration analysis of point supported plates.

Key words: Plate, natural frequency, modal analysis, point support, support area.

INTRODUCTION

The dynamic behavior of plates has been the subject of intensive study for many years due to its great importance in many engineering applications, such as design of aircraft, ships, earthquake resistance structures, bridges, hydraulic structures, containers, missiles, instruments, machine parts, printed circuit boards, solar panels and other structures prone to vibration. Plates with mixed boundary conditions are key components in civil and mechanical engineering and industrial design. In their structural designs, vibration control is the serious problem that is often encountered in the post fabrication stage.

Large amount of research has been done on the vibration of rectangular plates. Theoretical studies are based on different methods, namely, finite difference method, Ritz method, Rayleigh-Ritz method, finite element method, Lagrange multiplier method, Galerkin's method and others. In most of the studies, natural frequencies of the rectangular plates are presented for various boundary conditions. Some of the studies also include mode shapes of the plates (Rao et al., 1973; Slizard, 1974; Narita, 1984). A comprehensive study on the free vibration of rectangular thin plates has been presented by Leissa (1973). Frequency parameters for twenty-one different cases which involve the possible combinations of clamped, simply-supported and free edge conditions are presented. The Ritz method is employed with 36 terms containing the products of beam functions to analyze different cases. The effects of changing Poisson's ratio are also studied in the same work.

The vibration of orthotropic rectangular plates having viscoelastic point supports symmetrically located on its diagonals has been analyzed by Kocatürk et al. (2005). The plate was under the effect of a sinusoidally varying force at its center. The influence of the mechanical properties, the damping of the supports and the locations of point supports on the steady-state response of the viscoelastically point-supported rectangular plates have

*Corresponding author. E-mail: cihandem@gmail.com, cdemir@yildiz.edu.tr.

been investigated numerically for the concentrated load at the center for various values of the mechanical properties characterizing the anisotropy of the plate material, damping and location of the supports for a certain stiffness value of the supports.

Kim and Dickinson (1987) has studied on the flexural vibration of rectangular thin plates with point supports. Orthogonally generated polynomial functions with the Lagrange multiplier method is used in this study. The analysis has been applied to the plates having a combination of clamped, simply supported or free edges with arbitrarily located point supports.

Various possible practical implementations of the simply supported boundary condition have been investigated by Aglietti and Cunningham (2002). A suitable simply support condition has been selected and tested. The results from a forced vibration test are compared with those obtained from theory in order to verify the relative accuracy of the approach.

Kocaturk and Ilhan (2005) analyzed vibration of orthotropic rectangular plates having viscoelastic point supports at the corners. They used the Lagrange's equations to examine the free vibration characteristics and steady state response to a sinusoidally varying force affecting at the center of a viscoelastically point-supported orthotropic elastic plate of rectangular shape. They investigated the influence of the mechanical properties, and of the damping of the supports to the mode shapes and to the steady state response of the viscoelastically point-supported rectangular plates.

Aksu and Felemban (1992) used a method based on the variational procedure in conjunction with a finite difference energy method for the frequency analysis of corner point supported midline plates. One quarter of the plate is examined in the study due to the symmetry. In the numerical examples, the frequency parameters and nodal patterns are determined for three mode types and the effects of transverse shear deformation and rotary inertia are analyzed.

Free vibration of orthotropic rectangular plates having elastically point supports at the corners is analyzed by Kocatürk et al. (2003). The classical Ritz method is used for solving the problem. The influence of the mechanical properties on the mode shapes of the elastically point supported rectangular orthotropic plates is investigated numerically for various values of aspect ratios, stiffness parameters of point supports and mechanical properties characterizing the anisotropy of the plate material. The eigenvalues are tabulated for a wide range of orthotrophy parameters, stiffness parameters of point supports, and two aspect ratios for four mode families.

Grossi et al. (1997) generated antisymmetric mode for elastically restrained rectangular plates with located circular holes by using Rayleigh-Ritz method.

Avalos and Laura (2003) determined lower natural frequencies of a plate with a concentric square hole with free edges. The outer boundary of plate is either clamped or simply supported. They solved the problem with the Rayleigh-Ritz method and the finite element method and the results obtain were compared.

Liew et al. (2003) analyzed the free vibration of plates with internal discontinuities due to central cut-outs. They divided L-shape element into appropriate sub-domains. The energy function was derived by assembling the strain and kinetic energy contributions of each sub-domain. They extracted the vibration frequencies and mode shapes of the plate by using Ritz procedure.

Ulz and Semercigil (2008) explored the possibility of making a rectangular cut-out in a plate for the purposes of employing this cut-out as a dynamic vibration absorber by using the finite element method software (ANSYS).

Engineers and scientists have dealt with this kind of elements subjected to repeated loads for many years. In practice, plate-type elements are mounted as the point supported, spring supported, etc. These attachments are considered as discrete elements and plate is connected to springs or base such as an infinite small point in theoretical studies.

However, the size of the supports like point, springs, etc., which a plate has or is connected with holes, can seriously affect the free vibration properties of that plate. So, in certain cases, the dimension of the support hole and the washer contact area should be taken into the consideration of these attachments. Hence, in this study, the free vibrations of a plate which carries four support holes with/without considering washer contact area are treated. We examined the effects of changing diameter of the support hole and the washer attachment upon the frequencies and mode shapes.

The purposes of this work is to present reasonably accurate results for free vibration frequencies of point supported boundary conditions for thin rectangular plates and to investigate the effect of the size of support area on the natural frequencies. The study is largely based on numerical and experimental procedures of finite elements and modal analysis. The validity of the support effects was tested by comparing the numerical result with the experimental results for the chosen size of the diameter of the support.

ANALYSIS

Although theoretical analysis is valuable for providing basic understanding, in general, the analytical solution of the problems of vibration of rectangular plates with discontinuous boundary conditions is not easy. Therefore, numerical computation is one of the most important approaches for obtaining full solutions for theoretical analysis and engineering design. ANSYS 10.0 software (2007) which has advantages such as simplicity in use, applicability to various boundary conditions is used for the analysis of plates and generating the natural frequencies of plate. A four node two dimensional structural shell element shell63 was used from ANSYS's library.

The element options have been chosen for the plate to make pure bending displacement. So the problems considered are within the framework of the Kirchhoff- Love hypothesis. For an isotropic thin plate, potential energy of bending in Cartesian coordinates is given as

$$U=\frac{1}{2}D\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left[\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)\right]\right]dxdy \qquad (1)$$

$$D=\frac{Eh^3}{12(1-\upsilon^2)}$$

The kinetic energy of the plate is

$$T=\frac{\rho h}{2}\int_{-\frac{a}{2}}^{\frac{a}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}}\left(\frac{dw}{dt}\right)^2 dxdy \qquad (2)$$

$w(x,y)$: Flexural displacement; *D*: Flexural rigidity; ρ: Mass density; *h*: Thickness of the plate; *a*: Length of the plate; *b*: Width of the plate.

The displacement of any point on the element is expressed in the nodal displacements which are given by

$$\{q\}^e=\{w_1,b\theta_{x1},a\theta_{y1},w_2,b\theta_{x2},a\theta_{y2},w_3,b\theta_{x3},a\theta_{y3},w_4,b\theta_{x4},a\theta_{y4}\} \qquad (3)$$

The boundary conditions are used to find coefficients of shape functions. The shape functions are given by

$$[N_W]=\{N_w^1,N_{\theta x}^1,N_{\theta y}^1,N_w^2,N_{\theta x}^2,N_{\theta y}^2,N_w^3,N_{\theta x}^3,N_{\theta y}^3,N_w^4,N_{\theta x}^4,N_{\theta y}^4\} \qquad (4)$$

$w(x,y)$ is approximated by space-dependent interpolation functions and nodal displacements.

$$\{w(x,y)\}^e=[N]^e\{q\}^e \qquad (5)$$

By substituting derivatives of Equation (5) into Equations (1) and (2) and by using Lagrange's equations (Equation 6), the stiffness matrix and mass matrix are derived for the element by using energy theorems (Equation 7). The stiffness matrix and mass matrix of the assemblage of elements are generated.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)+\left(\frac{\partial U}{\partial q_i}\right)=Q_i$$
$$i=1,\ 2,\ 3,......,12 \qquad (6)$$

Where the overdot stands for the partial derivative with respect to time.

$$[M]^e\{\ddot{q}\}^e+[K]^e\{q\}^e=\{0\}^e \qquad (7)$$

The modal analysis is applied to the idealized structure; a set of simultaneous algebraic equations can be formed, the solution of which gives all the natural frequencies and mode shapes. The time-dependent generalized functions can be expressed as follows:

$$q(t)=q_{\max}e^{i\,\omega\,t} \qquad (8)$$

In Equation (8), $q_{\max}$ is a complex variable containing a phase angle. By using Equation (8), the following set of linear algebraic equations is obtained which can be expressed in the following matrix form

$$[K]\{q\}-\omega^2[M]\{q\}=\{0\} \qquad (9)$$

Where $[K]$, $[M]$ are stiffness and mass matrices of the assemblage of elements, respectively.

Figure 1 shows the finite element mesh model of the plate. The mesh has been generated by using quadrilateral elements with the mapped meshing control. Point supported boundary conditions can be modeled by coupling all degrees of freedoms of the slave nodes located on the edge of support circle with the master node where is center of the point support circle. The master nodes' degrees of the freedom have been fixed. We applied the constraints with / without the washer to show its effects on the natural frequencies. We coupled the nodes of washer contact area with the master node to show the effect of the washer (Figures 2a and 2b). We simulated the washer effects only for the experimental condition. We derived first eleven natural frequencies of the plate by using subspace mode extraction method.

The elastic point support of the plate has been modeled by using the Combin14 spring damper element from ANSYS' library.

The following non-dimensional parameters were introduced for comparison with the results of the other studies (Narita, 1984 and Kocaturk et al., 2003).

$$\mathrm{K}=\frac{k}{b}\frac{a^3}{D}\ ;\ \lambda^2=\frac{\rho h\omega^2 a^4}{D} \qquad (10)$$

k: Stiffness coefficient of support; *K*: Dimensionless stiffness coefficient; *λ*: Dimensionless frequency parameter

NUMERICAL RESULTS

In the present study, rigid point supported plate at the corner is selected in order to investigate the convergence tendency of the method. The frequency parameters which are calculated by using different mesh sizes are tabulated in Table 1. The results of 0.005 m element size are used to compare with the results of the studies of Kocaturk et al. (2003) and Narita (1984).

The effects of the point support can also be seen in the elastic point supported and viscoelastic point supported plates. Therefore, the frequency parameters of the elastic point supported plate from the studies of Kocaturk et al. (2003) are compared with the finite element model results for the stiffness constant values K = 10, 100, 1000; respectively in Table 2. The natural frequencies were converted into dimensionless form by using Equation (10) because of the comparison of the results of the others.

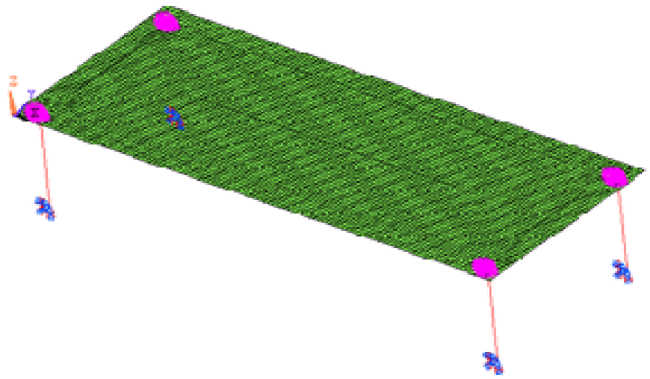
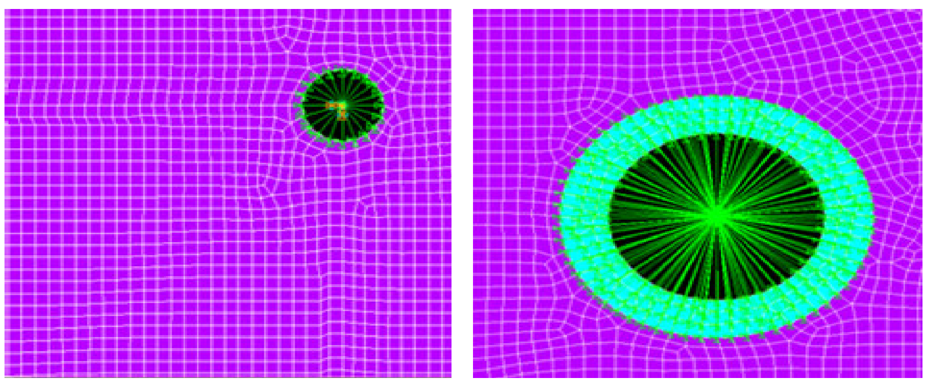


Figure 1. Finite element model of the point supported plate.



a) b)

Figure 2. The constraint of the point support for a) without washer b) with washer.

Table 1. The convergence studies of frequency parameters of the point supported plate.

Frequency parameter	Element size [m]					Kocaturk and Ilhan (2003)	Narita (1984)
	0.1	0.05	0.02	0.01	0.005		
λ_1	7.164515	7.12485	7.113158	7.111455	7.111027	7.1109	7.11089
λ_2	15.85921	15.79371	15.77408	15.7712	15.77048	15.7702	-
λ_3	15.85921	15.79371	15.77408	15.7712	15.77048	15.7702	-
λ_4	19.60853	19.59966	19.59672	19.59628	19.59617	19.5961	19.5961
λ_5	39.00519	38.59374	38.45878	38.43842	38.43327	38.4316	-
λ_6	44.3835	44.37464	44.37058	44.36986	44.36967	44.3696	44.3696
λ_7	50.92708	50.53321	50.40295	50.38327	50.37829	50.3767	-
λ_8	50.92708	50.53321	50.40295	50.38327	50.37829	50.3767	-
λ_9	69.86979	69.43293	69.29313	69.27237	69.26715	69.2654	-
λ_{10}	80.4462	80.39917	80.3685	80.36284	80.36134	80.3609	-
λ_{11}	80.4462	80.39918	80.36851	80.36285	80.36135	80.3609	-

Table 2. The frequency parameters of the spring supported plate for variation of support stiffness and comparison with other studies.

Frequency parameter	Dimensionless spring coefficient					
	K = 10		K = 100		K = 1000	
	Present study	Kocatürk and İlhan (2003)	Present study	Kocatürk and İlhan (2003)	Present study	Kocatürk and İlhan (2003)
λ_1	4.767318	4.7673	6.723623	6.7235	7.069539	7.0694
λ_2	9.155279	9.1552	14.5174	14.5172	15.63388	15.6336
λ_3	9.155279	9.1552	14.5174	14.5172	15.63388	15.6336
λ_4	19.59617	19.5961	19.59617	19.5961	19.59617	19.5961
λ_5	20.62005	20.6198	33.37032	33.3691	37.82697	37.8253
λ_6	29.61804	29.6181	40.61976	40.6198	43.9561	43.9561
λ_7	38.6128	38.6122	47.59727	47.5958	50.09099	50.0894
λ_8	38.6128	38.6122	47.59728	47.5958	50.09099	50.0894
λ_9	62.78086	62.7807	69.26715	69.2654	69.26715	69.2654
λ_{10}	62.78086	62.7807	72.41892	72.4189	79.3364	79.336
λ_{11}	66.42131	-	72.41892	72.4189	79.3364	79.336

The natural frequencies of the point supported plate were investigated with the variation of the diameter of support hole. The reason to select this type of plate is to test effects of boundary conditions; support hole and washer diameters. We tabulated the results for the variation of the diameter between 0 - 15 × 10^{-3} m in Table 3.

The changes of the natural frequencies of the point supported plate may be based on the mass reduction arose from the cut outs of the support. To decide which one is the more effective, the natural frequencies of the plate with/without the support holes were derived for completely free support condition. Results can be seen in Table 4.

EXPERIMENTAL STUDIES

The natural frequencies of point supported plate were measured by using transfer function method. The mode shapes from experimental results were derived to compare the results which were derived from ANSYS. The Data Physics data acquisition card, B&K pulse hammer and accelerometer were used in the experiments. The evaluation of the data and graphical representation has been realized through the signal ACE software.

The plate is dimensioned 0.625 m by 0.240 m and the thickness of the plate is 0.0015 m. The plate is supported with four steel bars to form point supported condition. The support hole diameters are 6.8 × 10^{-3}m. The washer diameters are 9.8 × 10^{-3}m. The material of the plate is spring steel, modulus of elasticity E = 1.8 × 10^{11} N m^{-2}, Poisson’s ratio ν = 0.3, mass density ρ = 7995 kgm^{-3}. The experimental setup can be seen in Figure 3.

Frequency response function and Coherence – frequency graphics of the point supported plate were obtained from ACE software which is shown in Figure 4.

The finite element simulation results were compared with the experimental results for point supported plate with / without washer as shown in Table 5.

The experimental results were used to derive mode shapes. A good agreement between numerical simulations and experimental data of the point supported plate is shown in Figure 5.

CONCLUSIONS

In this study, the good agreement was acquired between the results from literature and the finite element simulation results with the experimental results, which can be seen in Tables 1, 2, 5 and Figure 4.

It is seen from Table 3 that if the size of the support hole diameter chosen is very small, then the fundamental natural frequency of the plate will be affected very seriously. The natural frequencies increase with the

Table 3. The natural frequencies of the point supported plate for variation of the diameter of the support holes.

Natural frequency [Hz]	Variation of diameter of the support hole [m]								
	0	0.01×10^{-3}	0.1×10^{-3}	1×10^{-3}	3×10^{-3}	6×10^{-3}	9×10^{-3}	12×10^{-3}	15×10^{-3}
f_1	9.7984	16.766	16.807	17.887	18.772	19.597	20.314	20.988	21.642
f_2	35.296	38.519	38.548	39.442	40.234	41.001	41.685	42.349	43.014
f_3	39.191	48.587	48.680	50.960	52.975	54.917	56.644	58.307	59.958
f_4	78.772	85.148	85.215	87.264	89.144	90.999	92.668	94.299	95.944
f_5	83.655	95.219	95.411	99.160	102.70	106.19	109.33	112.38	115.44
f_6	109.26	137.24	137.68	144.94	147.49	149.11	150.30	151.28	152.15
f_7	116.88	141.66	141.84	145.28	151.70	155.38	158.46	161.50	164.57
f_8	135.97	144.99	145.11	148.62	151.98	158.24	163.93	169.24	174.31
f_9	165.97	174.37	174.72	182.06	190.32	199.50	205.10	208.24	211.30
f_{10}	167.08	186.60	186.77	192.66	197.46	201.66	208.31	217.16	226.07
f_{11}	209.65	220.30	220.48	225.65	230.87	236.29	241.26	246.17	251.17
f_{12}	221.05	222.49	222.69	227.30	233.34	241.20	249.84	259.56	270.51

Table 4. The natural frequencies of the plate for with / without the support holes.

Natural frequency [Hz]	Experimental plate without support hole (free-free-free-free boundary condition)	Experimental plate with support hole (free-free-free-free boundary condition)
	19.431	**19.457**
f_1	31.089	31.152
f_2	54.001	54.052
f_3	66.470	66.603
f_4	105.85	105.89
f_5	110.38	110.58
f_6	137.32	137.47
f_7	148.41	148.58
f_8	166.62	166.86
f_9	179.66	179.83
f_{10}	189.05	189.3
f_{11}	237.85	238.09

increasing diameter of the support area. However, increasing diameter does not affect the natural frequencies so much for the small size of the support area that is the variation of the diameter at 0.01×10^{-3} -

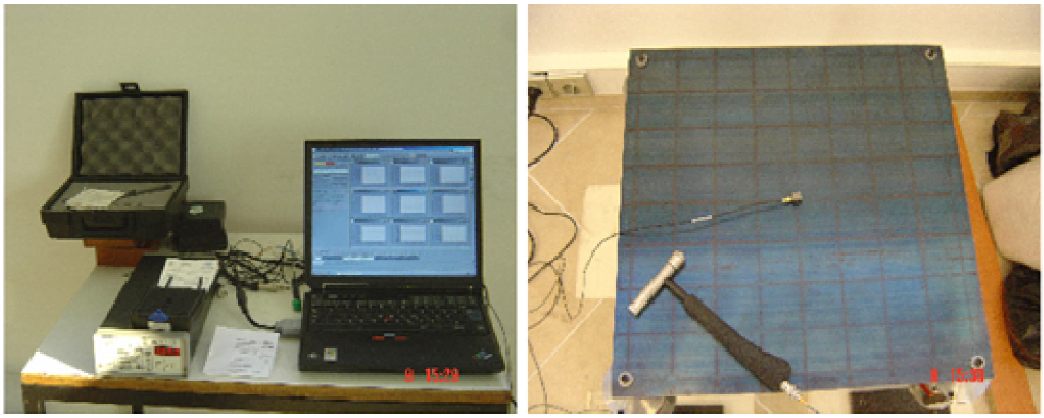


Figure 3. The experimental setup.

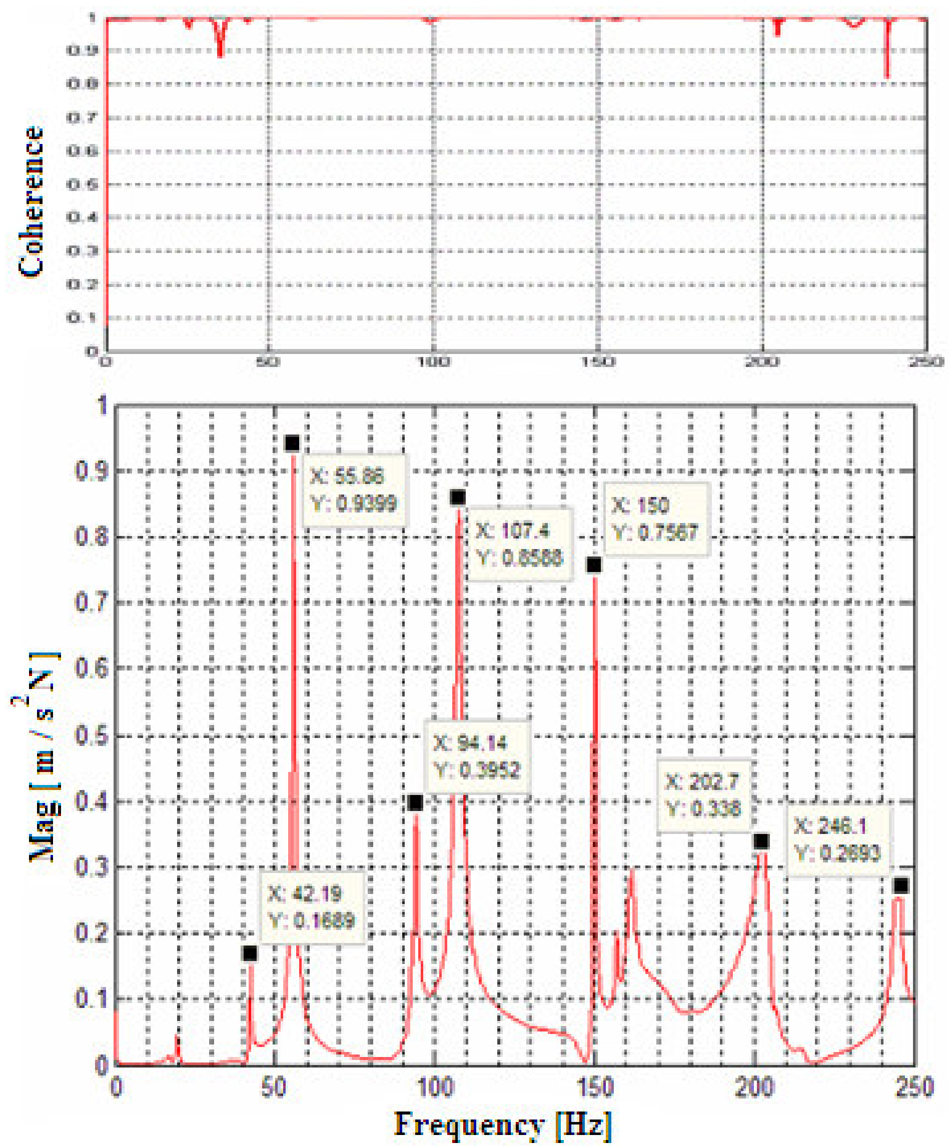


Figure 4. Frequency response function and Coherence – frequency graphics of the point supported plate.

Table 5. The comparison the natural frequencies obtained from ANSYS with the results of the experiment.

Natural frequency [Hz]	ANSYS simulation results		Experimental results
	Included only support effect (support hole diameter 6.8 mm)	Included also washer effect (washer diameter 9.8 mm)	
*f*1	19.792	20.521	19.92
*f*2	41.186	41.887	42.19
*f*3	55.383	57.154	55.86
*f*4	91.448	93.164	94.14
*f*5	107.04	110.27	107.42
*f*6	149.45	150.61	150.00
*f*7	156.20	159.39	157.03
*f*8	159.79	165.58	161.72
*f*9	201.83	206.07	202.73
*f*10	202.61	211.00	205.86
*f*11	237.62	242.75	244.53
*f*12	243.39	252.66	246.10

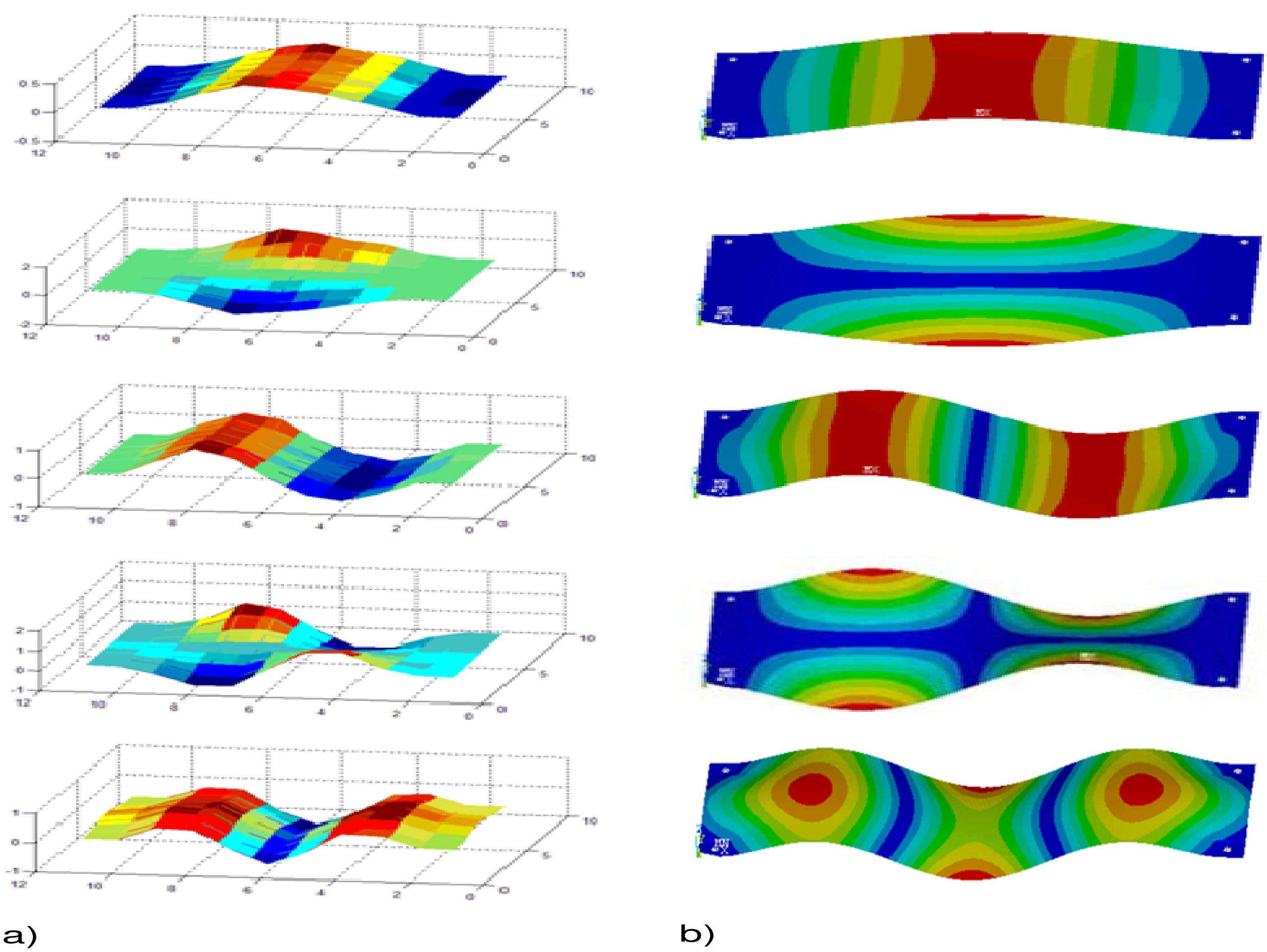


Figure 5. Comparison of the first five mode shapes a) Experimental results b) ANSYS simulation.

0.1×10^{-3} m as shown in Table 3.

The support condition is more effective than the mass reduction arose from cut outs on the natural frequencies of the point supported plate. It can be seen from Tables 3 and 4.

It is seen from the results of ANSYS simulation with the washer (Table 5), the washer does not affect the variation of the natural frequencies too much.

The results show that in the practical applications, the support hole effects should be considered for point spring or viscoelastic support conditions.

REFERENCES

Aglietti GS, Cunningham PR (2002). Is a simple support really that simple?. J. Sound Vib., 257(2): 321-335.

Aksu G, Felemban MB (1992). Frequency analysis of corner point supported mindlin plates by a finite difference energy method. J. Sound Vib., 158(3): 531-544.

ANSYS release 10.0(2007). ANSYS Inc..

Avalos DR, Laura PAA (2003). Transverse vibrations of simply supported rectangular plates with two rectangular cutouts. J. Sound Vib., 267(4): 967-977.

Grossi RO, Arenas BV, Laura PAA (1997). Free vibration of rectangular plates with circular openings. Ocean Eng., 24(1): 19-24.

Kim CS, Dickinson SM (1987). The flexural vibration of rectangular plates with point supports. J. Sound Vib., 117(2): 249-261.

Kocaturk T, Ilhan N (2003). Free Vibration Analysis of Elastically Point Supported Orthotropic Plates. J. Yildiz Techn. Uni., 1: 104-115.

Kocatürk T, Sezer S, Demir C (2005). The effects of support location on the steady-state response of viscoelastically point- supported rectangular orthotropic plates. Arc. Appl. Mech., 75: 58-67.

Kocatürk T, İlhan N (2005). Determination of the steady state response of viscoelastically corner point-supported rectangular orthotropic plates. Comput. & Struct., 83: 23-24.

Leissa AW (1973). The free vibrations of rectangular plates. J. Sound Vib., 31: 257–293.

Liew KM, Kitipornchai S, Leung AYT, Lim CW (2003) Analysis of the free vibration of rectangular plates with central cut-outs using the discrete Ritz method. Int. J. Mech. Sci., 45(5): 941-959.

Narita Y (1984). Note on vibrations of point supported rectangular plates. J. Sound Vib., 93: 593-597.

Rao GV, Raju IS, Amba-Rao CL (1973). Vibrations of point supported plates. J. Sound Vib., 29: 387-391.

Slizard R (1974). Theory and Analysis of Plate. Prentice-Hall. London.

Ulz MH, Semercigil SE (2008). Vibration control for plate-like structures using strategic cut-outs. J. Sound Vib., 309(1-2): 246-261.