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Rotating Disk Flow With Heat Transfer of a Non Newtonian Fluid in Porous Medium

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Abstract

The steady flow of an incompressible viscous non-Newtonian fluid above an infinite rotating disk in a porous medium is studied with heat transfer. Numerical solutions of the nonlinear differential equations which govern the hydrodynamics and energy transfer are obtained. The effect of the porosity of the medium and the characteristics of the non-Newtonian fluid on the velocity and temperature distributions is considered.

Key Words: Rotating disk flow, Heat Transfer, Porous medium, Non-Newtonian fluid.

1. Introduction

Pioneering study of fluid flow due to an infinite rotating disk was carried by von Karman in 1921 [1]. von Karman gave a formulation of the problem and then introduced his famous transformations which reduced the governing partial differential equations to ordinary differential equations. Cochran [2] obtained asymptotic solutions for the steady hydrodynamic problem formulated by von Karman. Benton [3] improved Cochran's solutions and solved the unsteady problem. The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen [4] for a variety of Prandtl numbers in the steady state. Sparrow and Gregg [5] studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Later Attia [6] extended the problem discussed in [4, 5] to the unsteady state in the presence of an applied uniform magnetic field. The steady flow of a non-Newtonian fluid due to a rotating disk with uniform suction was considered by Mithal [7]. Then, Attia [8] extended the problem to the transient state with heat transfer.

In the present work, the steady laminar flow of an incompressible viscous non-Newtonian fluid due to uniform rotation of a disk of infinite extent in a porous medium is studied with heat transfer. The flow in the porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [9–11]. The temperature of the disk is maintained at a constant value. The governing nonlinear differential equations are integrated numerically using the finite difference approximations. The effect of the porosity of the medium and the characteristics of the non-Newtonian fluid on the steady flow and heat transfer is presented and discussed.

2. Basic Equations

Let the disk lie in the plane $z = 0$ and the space $z > 0$ is occupied by a viscous incompressible fluid. The motion is due to the rotation of an insulated disk of infinite extent about an axis perpendicular to its plane with constant angular speed ω through a porous medium where the Darcy model is assumed [11]. Otherwise the fluid is at rest under pressure p_∞. The disk is maintained at a constant temperature T_w. The non-Newtonian fluid considered in the present paper is that for which the stress tensor τ^i_j is related to the rate of strain tensor e^i_j as [7]

$$\tau^i_j = 2\mu e^i_j + 2\mu_c e^i_k e^k_j - p\delta^i_j, e^i_j = 0,$$

where p denots the pressure, μ is the coefficient of viscosity and μ_c is the coefficient of cross viscosity. The equations of steady motion are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\rho(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}) + \frac{\mu}{K1}u = \frac{\partial \tau^r_r}{\partial r} + \frac{\partial \tau^z_r}{\partial z} + \frac{\tau^r_r - \tau^\varphi_\varphi}{r} \tag{2}$$

$$\rho(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}) + \frac{\mu}{K1}v = \frac{\partial \tau^r_\varphi}{\partial r} + \frac{\partial \tau^z_\varphi}{\partial z} + \frac{2\tau^r_\varphi}{r} \tag{3}$$

$$\rho(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}) + \frac{\mu}{K1}w = \frac{\partial \tau^r_z}{\partial r} + \frac{\partial \tau^z_z}{\partial z} + \frac{\tau^r_z}{r}, \tag{4}$$

where u, v, w are velocity components in the directions of increasing r,φ, z, respectively, P denotes the pressure, ρ is the density of the fluid, and $K1$ is the Darcy permeability [9–11]. We introduce von Karman transformations [1]

$$u = r\omega F, \quad v = r\omega G, \quad w = \sqrt{\omega\nu}H, \quad z = \sqrt{v/\omega}\zeta, \quad p - p_\infty = -\rho v\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation; F, G, H and P are non-dimensional functions of ζ; and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$. With these definitions, equations (1)–(4) take the form

$$\frac{dH}{d\zeta} + 2F = 0 \tag{5}$$

$$\frac{d^2F}{d\zeta^2} - H\frac{dF}{d\zeta} - F^2 + G^2 - MF - \frac{1}{2}K((\frac{dF}{d\zeta})^2 + 3(\frac{dG}{d\zeta})^2 + 2F\frac{d^2F}{d\zeta^2}) = 0 \tag{6}$$

$$\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG - MG + K(\frac{dF}{d\zeta}\frac{dG}{d\zeta} - F\frac{d^2G}{d\zeta^2}) = 0 \tag{7}$$

$$\frac{d^2H}{d\zeta^2} - H\frac{dH}{d\zeta} - MH - \frac{7}{2}K\frac{dH}{d\zeta}\frac{d^2H}{d\zeta^2} + \frac{dP}{d\zeta} = 0, \tag{8}$$

where $M = \nu/\omega K1$ is the porosity parameter. The boundary conditions for the velocity problem are given by

$$\zeta = 0, \quad F = 0, \quad G = 1, \quad H = 0, \tag{9a}$$

$$\zeta \to \infty, \quad F \to 0, \quad G \to 0, \quad P \to 0. \tag{9b}$$

Equation (9a) indicates the no-slip condition of viscous flow applied at the surface of the disk. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in equation (9a). The above system of equations (5)–(7), with the prescribed boundary conditions given by equations (9), are sufficient to solve for the three components of the flow velocity. Equation (8) can be used to solve for the pressure distribution if required.

Heat transfer takes place due to the difference in temperature between the wall and the ambient fluid. The energy equation without the dissipation terms takes the form [4–5]

$$\rho c_p(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}) - k(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}) = 0, \tag{10}$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to T_∞ where T_∞ is the temperature of the ambient fluid. In terms of the non-dimensional variable θ = $(T\text{-}T_\infty)/(T_w\ \text{-}T_\infty)$, and using von Karman transformations, equation (11) takes the form

$$\frac{1}{\Pr}\frac{d^2\theta}{d\zeta^2} + H\frac{d\theta}{d\zeta} = 0, \tag{11}$$

where Pr is the Prandtl number, $Pr = c_p\mu/k$. The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1, \quad \theta(\infty) = 0. \tag{12}$$

The system of non-linear ordinary differential equations (5)–(7) and (11) is solved under the conditions given by equations (9) and (12) for the three components of the flow velocity and temperature distribution, using the Crank-Nicolson method [12]. The resulting system of difference equations has to be solved in the infinite domain $0 < \zeta < \infty$. A finite domain in the ζ-direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_\infty = 12$.

3. Results and Discussion

Figures 1–3 show the influence of the non-Newtonian parameter K on the steady state velocity profiles G, F, and H, respectively, for the porosity parameter $M = 0$ and 1. Figures 1–3 indicate the restraining effect of the porosity of the medium on the flow velocity in the three directions. Increasing the porosity parameter M decreases G, F, the magnitude of H and the thickness of the boundary layer. Figure 1 indicates that increasing K increases G for all ζ. Figure 2 indicates that increasing K decreases F for small and moderate ζ. However, for large values of ζa crossover point that depends on K appears in F-ζcharts whereas increasing K more increases F. Figure 3 shows that increasing the parameter K increases the resistance for the incoming axial flow and consequently increases the axial velocity component H for all ζ due to the influence of K on reducing F. A crossover point in H-ζcharts is also shown in Figure 3, which indicates that for large values of K, the axial flow component H decreases with increasing K for small values of ζ. It is of interest to see the effect of the parameter M in the suppression of the crossover in F-ζas well as H-ζ charts as depicted in Figures. 2 and 3. An interesting effect for the parameters K and M appears in Figures 2 and 3 that for non-zero values of K, that increasing M reverses the direction of the velocity components F and H for all ζ.

Figure 4 shows the function of non-Newtonian parameter K on the steady state temperature profile θ for the porosity parameter $M = 0$ and 1 and for $Pr = 0.7$. Increasing M or the parameter K increases the temperatureθ as a result of the effect of the porosity or the non-Newtonian behavior in preventing the fluid at near-ambient temperature from reaching the surface of the disk. Consequently, increasing M or K increases the temperature as well as the thermal boundary layer thickness.

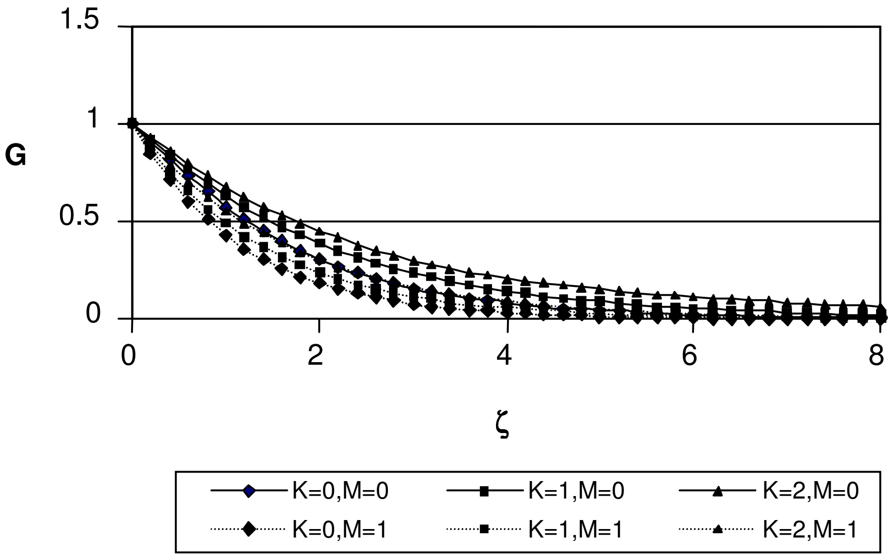

Figure 1. Effect of the porosity parameter M and the non-Newtonian parameter K on the profile of G.

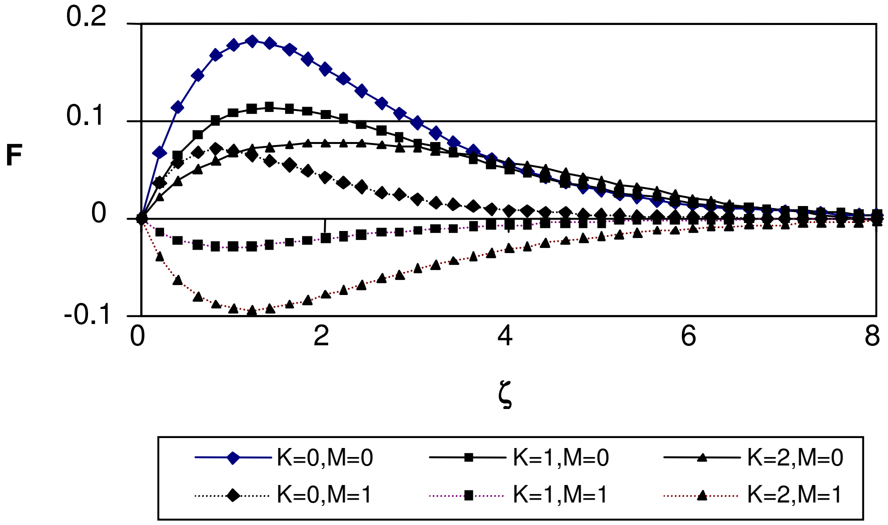

Figure 2. Effect of the porosity parameter M and the non-Newtonian parameter K on the profile of F.

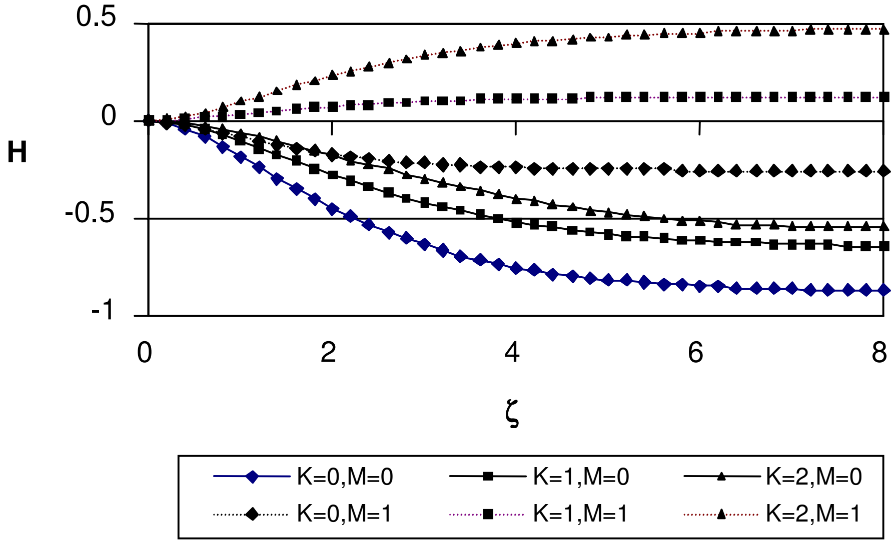

Figure 3. Effect of the porosity parameter M and the non-Newtonian parameter K on the profile of H.

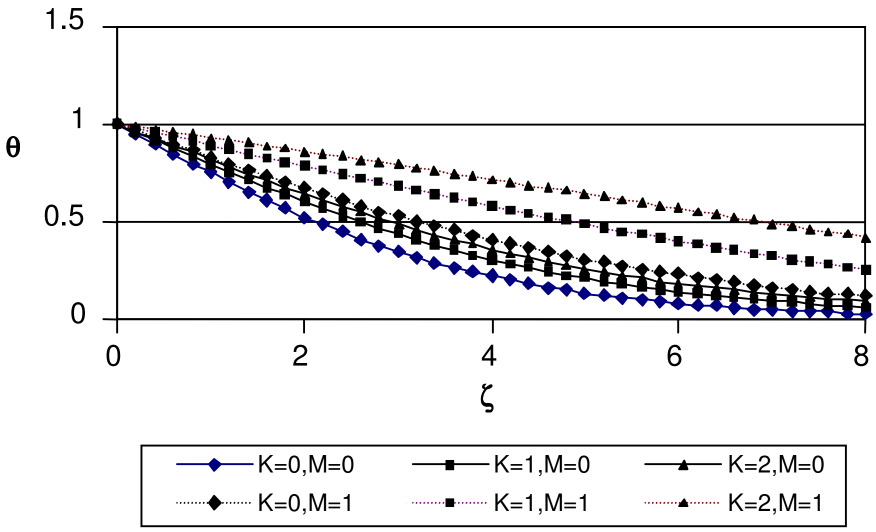

Figure 4. Effect of the porosity parameter M and the non-Newtonian parameter K on the profile of θ for $Pr = 0.7$.

4. Conclusions

The steady flow of a non-Newtonian fluid induced by a rotating disk with heat transfer in a porous medium was studied. The results indicate the restraining effect of the porosity on the flow velocities and the thickness of the boundary layer. On the other hand, increasing the porosity parameter increases the temperature and thickness of the thermal boundary layer. It is of interest to see the combined effect of the porosity of the medium and the non-Newtonian fluid characteristics on reversing the direction of the radial and the axial components of velocity. Also, it is interesting to see the effect of the porosity parameter in the suppression of the crossover in the radial and axial velocity profiles occurring for large values of the non-Newtonian fluid characteristics.

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