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On some inequalities of a certain class of analytic functions

Mohsan Raza[1*], Sarfraz Nawaz Malik[1] and Khalida Inayat Noor[2]

*Correspondence: mohsan976@yahoo.com
[1]Department of Mathematics, GC University Faisalabad, Faisalabad, Pakistan
Full list of author information is available at the end of the article

Abstract
The aim of this paper is to study the properties of a subclass of analytic functions related to p-valent Bazilevic functions by using the concept of differential subordination. We investigate some results concerned with coefficient bounds, inclusion results, radius problem, covering theorem, angular estimation of a certain integral operator, and some other interesting properties.
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1 Introduction and preliminaries

Let A_p be the class of analytic functions

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n \quad \bigl(p \in \mathbb{N} = \{1,2,3,\ldots\}\bigr), \tag{1.1}$$

defined in the open unit disc $E = \{z : |z| < 1\}$. A function $f \in A_p$ is a p-valent starlike function of order ρ if and only if

$$\operatorname{Re} \frac{zf'(z)}{f(z)} > \rho, \quad 0 \le \rho < p, z \in E.$$

This class of functions is denoted by $S_p^*(\rho)$. It is noted that $S_p^*(0) = S_p^*$. If $f(z) \in A_p$ satisfies

$$\left|\arg \frac{zf'(z)}{pf(z)}\right| < \frac{\pi\eta}{2} \tag{1.2}$$

for some $\eta \in (0,1]$ and for all $z \in E$, then the function f is called strongly starlike p-valent of order η in E. We denote this class by $\tilde{S}_p^*(\eta)$. Let $f_1(z)$ and $f_2(z)$ be analytic in E. We say $f_1(z)$ is subordinate to $f_2(z)$, written $f_1 \prec f_2$ or $f_1(z) \prec f_2(z)$, if there exists a Schwarz function $w(z)$, $w(0) = 0$, and $|w(z)| < 1$ in E, then $f_1(z) = f_2(w(z))$. A function f in A_p is said to belong to the class $S_p^*[A,B]$, $-1 \le B < A \le 1$, if and only if

$$\frac{zf'(z)}{f(z)} \prec p\frac{1+Az}{1+Bz}, \quad z \in E.$$

For $p = 1$, we obtain the class $S^*[A,B]$ of Janowski starlike functions. Janowski functions have extensively been studied by several researchers; see for example [1–3]. It is clear that

$f \in S_p^*[A,B]$ if and only if

$$\left|\frac{zf'(z)}{f(z)} - p\frac{1-AB}{1-B^2}\right| < p\frac{A-B}{1-B^2} \quad (-1 < B < A \leq 1, z \in E), \tag{1.3}$$

and

$$\operatorname{Re}\frac{zf'(z)}{f(z)} > p\frac{1-A}{2} \quad (B = -1, z \in E).$$

A function $f \in A_p$ is p-valent Bazilevic function of type (α, β) and order ρ if and only if

$$\operatorname{Re}\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta} > \rho, \quad 0 \leq \rho < p, z \in E,$$

where $\alpha \geq 0$, $\beta \in \mathbb{R}$ and $g \in S_p^*$. For $p = 1$ and $\rho = 0$, this class was introduced by Bazilevic and these functions are univalent for $\alpha \geq 0$, $\beta \in \mathbb{R}$. This class of functions is studied by many authors; for some details, see [4–11].

Using this concept, we generalize and define a subclass of p-valent Bazilevic functions of type (α, β) as follows.

Definition 1.1 A function $f \in M_p(\alpha, \beta, \mu, A, B)$ if it satisfies the condition

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\\&\quad + \mu\left\{1 + \frac{zf''(z)}{f'(z)} + (\alpha + i\beta - 1)\frac{zf'(z)}{f(z)} - \alpha\frac{zg'(z)}{g(z)} - ip\beta\right\} \prec p\frac{1+Az}{1+Bz}, \quad z \in E,\end{aligned}$$

where $\alpha \geq 0$, $\mu > 0$, $g \in S_p^*$, $-1 \leq B < A \leq 1$ and β is any real.

We have the following special cases.

(i) For $\beta = 0$, we have the subclass of Bazilevic functions defined by Patel [12].

(ii) For $\beta = 0$, $p = 1$, $g(z) = z$, $A = 1 - 2\rho$, $B = -1$, we obtain the subclass of Bazilevic functions defined in [13].

For $A = 1 - 2\rho$, $B = -1$, we have the following subclass of analytic functions.

Definition 1.2 A function $f \in B_p(\alpha, \beta, \rho)$ if it satisfies the condition

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\\&\quad + \mu\left\{1 + \frac{zf''(z)}{f'(z)} + (\alpha + i\beta - 1)\frac{zf'(z)}{f(z)} - \alpha\frac{zg'(z)}{g(z)} - ip\beta\right\} \prec p\frac{1+(1-2\rho)z}{1-z},\end{aligned}$$

where $\alpha \geq 0$, $\mu > 0$, $g \in S_p^*$, $0 \leq \rho < 1$, β is any real and $z \in E$. In other words, a function $f \in B_p(\alpha, \beta, \rho)$ if it satisfies the condition

$$\begin{aligned}&\operatorname{Re}\frac{1}{p}\left[\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\right.\\&\quad \left. + \mu\left\{1 + \frac{zf''(z)}{f'(z)} + (\alpha + i\beta - 1)\frac{zf'(z)}{f(z)} - \alpha\frac{zg'(z)}{g(z)} - ip\beta\right\}\right] > \rho, \quad z \in E.\end{aligned}$$

We need the following definition and lemmas which will be used in our main results.

Definition 1.3 Let $\Psi : \mathbb{C}^2 \times E \to \mathbb{C}$ be analytic in a domain D and h be univalent in E. If p is analytic in E with $(p(z), zp'(z); z) \in D$ when $z \in E$, then we say that p satisfies a first-order differential subordination if

$$\Psi\bigl(p(z), zp'(z); z\bigr) \prec h(z), \quad z \in E. \tag{1.4}$$

The univalent function q is called dominant of the differential subordination (1.4) if $p \prec q$ for all p satisfies (1.4). If $\tilde{q} \prec q$ for all dominants of (1.4), then we say that $\tilde{q}$ is the best dominant of (1.4).

Lemma 1.4 ([14]) *If* $-1 \leq B < A \leq 1$, $\lambda > 0$ *and the complex number* γ *satisfies* $\operatorname{Re}\{\gamma\} \geq -\lambda(1-A)/(1-B)$, *then the differential equation*

$$q(z) + \frac{zq'(z)}{\lambda q(z) + \gamma} = \frac{1+Az}{1+Bz}, \quad z \in E,$$

has a univalent solution in E given by

$$q(z) = \begin{cases} \frac{z^{\lambda+\gamma}(1+Bz)^{\lambda(A-B)/B}}{\lambda\int_0^z t^{\lambda+\gamma-1}(1+Bt)^{\lambda(A-B)/B}\,dt} - \frac{\gamma}{\lambda}, & B \neq 0, \\ \frac{z^{\lambda+\gamma}e^{\lambda Az}}{\lambda\int_0^z t^{\lambda+\gamma-1}e^{\lambda At}\,dt} - \frac{\gamma}{\lambda}, & B = 0. \end{cases}$$

If $h(z) = 1 + c_1 z + c_2 z^2 + \cdots$ *is analytic in E and satisfies*

$$h(z) + \frac{zh'(z)}{\lambda h(z) + \gamma} \prec \frac{1+Az}{1+Bz}, \quad z \in E,$$

then

$$h(z) \prec q(z) \prec \frac{1+Az}{1+Bz},$$

and $q(z)$ *is the best dominant.*

Lemma 1.5 ([15]) *Let* ε *be a positive measure on* $[0,1]$. *Let g be a complex-valued function defined on* $E \times [0,1]$ *such that* $g(\cdot,t)$ *is analytic in E for each* $t \in [0,1]$ *and* $g(z,\cdot)$ *is* ε*-integrable on* $[0,1]$ *for all* $z \in E$. *In addition, suppose that* $\operatorname{Re} g(z,t) > 0$, $g(-r,t)$ *is real and* $\operatorname{Re}\{1/g(z,t)\} \geq 1/g(-r,t)$ *for* $|z| \leq r < 1$ *and* $t \in [0,1]$. *If* $g(z) = \int_0^1 g(z,t)\,d\varepsilon(t)$, *then* $\operatorname{Re}\{1/g(z)\} \geq 1/g(-r)$.

Lemma 1.6 ([16, Chapter 14]) *Let* a_1, b_1 *and* $c_1 \neq 0, -1, -2, \ldots$ *be complex numbers. Then, for* $\operatorname{Re} c_1 > \operatorname{Re} b_1 > 0$,

(i) $${}_2F_1(a_1, b_1, c_1; z) = \frac{\Gamma(c_1)}{\Gamma(c_1 - b_1)\Gamma(b_1)} \int_0^1 t^{b_1-1}(1-t)^{c_1-b_1-1}(1-tz)^{-a_1}\,dt,$$

(ii) $${}_2F_1(a_1, b_1, c_1; z) = {}_2F_1(b_1, a_1, c_1; z),$$

(iii) $${}_2F_1(a_1, b_1, c_1; z) = (1-z)^{-a_1}\,{}_2F_1\left(a_1, c_1 - b_1, c_1; \frac{z}{z-1}\right).$$

Lemma 1.7 ([17]) *Let* $-1 \le B_1 \le B_2 < A_2 \le A_1 \le 1$. *Then*

$$\frac{1+A_2 z}{1+B_2 z} \prec \frac{1+A_1 z}{1+B_1 z}.$$

Lemma 1.8 ([18]) *Let F be analytic and convex in E. If $f, g \in A_p$ and $f, g \prec F$, then*

$$\mu f + (1-\mu) g \prec F, \quad 0 \le \mu \le 1.$$

Lemma 1.9 ([19]) *Let* $f(z) = \sum_{k=0}^{\infty} a_k z^k$ *be analytic in E and* $F(z) = \sum_{k=0}^{\infty} b_k z^k$ *be analytic and convex in E. If $f \prec F$, then*

$$|a_k| \le |b_1| \quad (k \in \mathbb{N}).$$

Lemma 1.10 ([20]) *Let* $h(z) = 1 + d_1 z + d_2 z^2 + \cdots$ *be analytic in E and $h(z) \neq 0$ in E. If there exists a point $z_0 \in E$ such that* $|\arg h(z)| < \frac{\pi}{2}\eta$ $(|z| < |z_0|)$ *and* $|\arg h(z_0)| = \frac{\pi}{2}\eta$ $(0 < \eta \le 1)$, *then we have* $\frac{z_0 h'(z_0)}{h(z_0)} = ik\eta$, *where*

$$\begin{cases} k \ge \frac{1}{2}(x + \frac{1}{x}), & \textit{when } \arg h(z_0) = \frac{\pi}{2}\eta, \\ k \le -\frac{1}{2}(x + \frac{1}{x}), & \textit{when } \arg h(z_0) = -\frac{\pi}{2}\eta, \end{cases}$$

and $(h(z_0))^{1/\eta} = \pm ix$ $(x > 0)$.

Lemma 1.11 *Let $g \in S^*[A,B]$. Then the function*

$$G(z) = \left[\frac{c+\alpha+i\beta}{z^{c+i\beta}} \int_0^z t^{c+i\beta-1} g^{\alpha}(t)\, dt\right]^{\frac{1}{\alpha}} \tag{1.5}$$

belongs to $S^*[A,B]$ for $c \ge -\alpha \frac{1-A}{1-B}$.

Proof is straightforward by using Lemma 1.4.

Throughout this paper, $\alpha \ge 0$, $\beta \in \mathbb{R}$, $\mu > 0$, and $-1 \le B < A \le 1$ unless otherwise stated.

2 Main results

Theorem 2.1 *If $f \in M_p(\alpha, \beta, \mu, A, B)$, then*

$$\frac{zf'(z)}{pf(z)} \left(\frac{f(z)}{g(z)}\right)^{\alpha} \left(\frac{f(z)}{z^p}\right)^{i\beta} \prec q(z), \tag{2.1}$$

where $q(z) = \frac{\mu}{pQ(z)}$ *and*

$$Q(z) = \begin{cases} \int_0^1 t^{\frac{p}{\mu}-1} (\frac{1+Bzt}{1+Bz})^{\frac{p}{\mu}(A-B)/B}\, dt, & B \neq 0, \\ \int_0^1 t^{\frac{p}{\mu}-1} e^{\frac{p}{\mu}(t-1)Az}\, dt, & B = 0. \end{cases} \tag{2.2}$$

In hypergeometric function form,

$$q(z) = \begin{cases} [{}_2F_1(1, \frac{p}{\mu}(1 - \frac{A}{B}); \frac{p}{\mu}+1; \frac{Bz}{Bz+1})]^{-1}, & B \neq 0, \\ [{}_1F_1(1, \frac{p}{\mu}+1; -\frac{p}{\mu}Az)]^{-1}, & B = 0, \end{cases} \tag{2.3}$$

and if $A < -\frac{\mu B}{p}$, $-1 \leq B < 0$, *then* $M_p(\alpha,\beta,\mu,A,B) \subset B_p(\alpha,\beta,\rho)$, *where*

$$\rho = p\left\{{}_2F_1\left(1, \frac{p}{\mu}\left(1-\frac{A}{B}\right); \frac{p}{\mu}+1; \frac{B}{B-1}\right)\right\}^{-1}. \tag{2.4}$$

This result is best possible.

Proof Let

$$h(z) = \frac{zf'(z)}{pf(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta},$$

where $h(z)$ is analytic in E with $h(0) = 1$. Differentiating logarithmically, we obtain

$$\begin{aligned} &\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta} + \mu\left\{1 + \frac{zf''(z)}{f'(z)} + (\alpha + i\beta - 1)\frac{zf'(z)}{f(z)} - \alpha\frac{zg'(z)}{g(z)} - ip\beta\right\} \\ &\quad = ph(z) + \frac{\mu z h'(z)}{h(z)} \prec p\frac{1+Az}{1+Bz}. \end{aligned} \tag{2.5}$$

Using Lemma 1.4 for $\lambda = \frac{p}{\mu}$ and $\gamma = 0$, we have

$$h(z) \prec q(z) \prec \frac{1+Az}{1+Bz},$$

where $q(z)$ is given in (2.3) and is the best dominant of (2.5). Next, in order to prove $M_p(\alpha,\beta,\mu,A,B) \subset B_p(\alpha,\beta,\rho)$, we show that $\inf_{|z|<1}\{\operatorname{Re} q(z)\} = q(-1)$. Now, we set $a = \frac{p}{\mu}(B-A)/B$, $b = \frac{p}{\mu}$ and $c = \frac{p}{\mu} + 1$, then it is clear that $c > b > 0$; therefore, for $B \neq 0$ it follows from (2.2) by using Lemma 1.6 that

$$\begin{aligned} Q(z) &= (1+Bz)^a \int_0^1 t^{b-1}(1+Btz)^{-a}\,dt \\ &= \frac{\Gamma(b)}{\Gamma(c)}\,{}_2F_1\left(1, a, c; \frac{Bz}{Bz+1}\right). \end{aligned} \tag{2.6}$$

To prove that $\inf_{|z|<1}\{\operatorname{Re} q(z)\} = q(-1)$, we need to show that

$$\operatorname{Re}\left\{1/Q(z)\right\} \geq 1/Q(-1).$$

Since $A < -\frac{\mu B}{p}$ with $-1 \leq B < 0$ implies that $c > a > 0$, therefore, by using Lemma 1.6, (2.6) yields

$$Q(z) = \int_0^1 g(z,t)\,d\varepsilon(t),$$

where

$$\begin{aligned} g(z,t) &= \frac{1+Bz}{1+(1-t)Bz} \quad (0 \leq t \leq 1), \\ d\varepsilon(t) &= \frac{\Gamma(b)}{\Gamma(a)\Gamma(c-a)}t^{a-1}(1-t)^{c-a-1}\,dt, \end{aligned}$$

which is a positive measure on $[0,1]$. For $-1 \leq B < 0$ it is clear that $\operatorname{Re} g(z,t) > 0$ and $g(-r,t)$ is real for $0 \leq |z| \leq r < 1$ and $t \in [0,1]$. Also,

$$\operatorname{Re}\left\{\frac{1}{g(z,t)}\right\} = \operatorname{Re}\left\{\frac{1+(1-t)Bz}{1+Bz}\right\} \geq \frac{1-(1-t)Br}{1-Br} = \frac{1}{g(-r,t)}$$

for $|z| \leq r < 1$. Therefore, using Lemma 1.5, we have

$$\operatorname{Re}\{1/Q(z)\} \geq 1/Q(-r).$$

Now, letting $r \to 1^-$, it follows

$$\operatorname{Re}\{1/Q(z)\} \geq 1/Q(-1).$$

Therefore, $M_p(\alpha,\beta,\mu,A,B) \subset B_p(\alpha,\beta,\rho)$. □

For $\beta = 0$, we have the following result proved in [12].

Corollary 2.2 *If $f \in M_p(\alpha,\mu,A,B)$, then*

$$\frac{zf'(z)}{pf(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha} \prec \begin{cases} [{}_2F_1(1, \frac{p}{\mu}(1-\frac{A}{B}); \frac{p}{\mu}+1; \frac{Bz}{Bz+1})]^{-1}, & B \neq 0, \\ [{}_1F_1(1, \frac{p}{\mu}+1; -\frac{p}{\mu}Az)]^{-1}, & B = 0, \end{cases}$$

and if $A < -\frac{\mu B}{p}$, $-1 \leq B < 0$, then $M_p(\alpha,\mu,A,B) \subset B_p(\alpha,\rho)$, where

$$\rho = p\left\{{}_2F_1\left(1, \frac{p}{\mu}\left(1-\frac{A}{B}\right); \frac{p}{\mu}+1; \frac{B}{B-1}\right)\right\}^{-1}.$$

This result is best possible.

For $p = 1$, we have the class $M_1(\alpha,\beta,\mu,A,B) = M(\alpha,\beta,\mu,A,B)$. We denote the class of functions $f \in A$, having Taylor series representation of the form

$$f(z) = z + \sum_{k=n+1}^{\infty} a_k z^k$$

and satisfying the condition

$$\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z}\right)^{i\beta} + \mu\left\{1 + \frac{zf''(z)}{f'(z)} + (\alpha + i\beta - 1)\frac{zf'(z)}{f(z)} - \alpha\frac{zg'(z)}{g(z)} - i\beta\right\} \prec \frac{1+Az}{1+Bz}, \quad z \in E, \tag{2.7}$$

by $M^*(\alpha,\beta,\mu,A,B)$, where $g(z) = z + \sum_{k=n+1}^{\infty} b_k z^k$ such that $\operatorname{Re}\frac{zg'(z)}{g(z)} > 0$. Now, we derive the following result for the class $M^*(\alpha,\beta,\mu,A,B)$.

Theorem 2.3 *Let $f \in M^*(\alpha,\beta,\mu,A,B)$. Then*

$$|a_{n+1}| \leq \frac{(A-B)+\alpha(n+1)(1+\mu n)}{|\alpha+i\beta+n|(1+\mu n)}. \tag{2.8}$$

Proof Since $f \in M^*(\alpha,\beta,\mu,A,B)$, therefore,

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z}\right)^{i\beta}\\&\quad+\mu\left\{1+\frac{zf''(z)}{f'(z)}+(\alpha+i\beta-1)\frac{zf'(z)}{f(z)}-\alpha\frac{zg'(z)}{g(z)}-i\beta\right\}\prec\frac{1+Az}{1+Bz}.\end{aligned}$$

Now, using the fact that $f(z)=z+\sum_{k=n+1}^{\infty}a_kz^k$ and $g(z)=z+\sum_{k=n+1}^{\infty}b_kz^k$, we obtain

$$1+\left\{(\alpha+i\beta+n)(1+\mu n)a_{n+1}-\alpha(1+\mu n)b_{n+1}\right\}z^n+\cdots\prec\frac{1+Az}{1+Bz}.$$

By a well-known result due to Janowski and Lemma 1.9, we have

$$\left|(\alpha+i\beta+n)(1+\mu n)a_{n+1}-\alpha(1+\mu n)b_{n+1}\right|\leq A-B.$$

By the triangle inequality, we obtain

$$\left|(\alpha+i\beta+n)(1+\mu n)a_{n+1}\right|-\left|\alpha(1+\mu n)b_{n+1}\right|\leq A-B.$$

Using the coefficient bound for the class S^*, we have the required result. □

For $\beta=0$ and $g(z)=z$, we have the following result proved in [13].

Corollary 2.4 *Let $f \in M(\alpha,\mu,A,B)$. Then*

$$|a_{n+1}| \leq \frac{(A-B)}{(\alpha+n)(1+\mu n)}.$$

For $\beta=0$, $p=1$, $g(z)=z$, $A=1-2\rho$ and $B=-1$, we have the following result proved in [21].

Corollary 2.5 *Let f satisfy the condition*

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{z}\right)^{\alpha}+\mu\left[1+\frac{zf''(z)}{f'(z)}+(1-\alpha)\left(1-\frac{zf'(z)}{f(z)}\right)\right]\right\}>\rho.$$

Then

$$|a_{n+1}| \leq \frac{2(1-\rho)}{(n+\alpha)(1+\mu n)}.$$

Theorem 2.6 *For $\mu_2 \geq \mu_1 \geq 0$ and $-1 \leq B_1 \leq B_2 < A_2 \leq A_1 \leq 1$,*

$$M_p(\alpha,\beta,\mu_2,A_2,B_2) \subset M_p(\alpha,\beta,\mu_1,A_1,B_1).$$

Proof Let $f \in M_p(\alpha, \beta, \mu_2, A_2, B_2)$. Then

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\\&\quad+\mu_2\left\{1+\frac{zf''(z)}{f'(z)}+(\alpha+i\beta-1)\frac{zf'(z)}{f(z)}-\alpha\frac{zg'(z)}{g(z)}-ip\beta\right\}\prec p\frac{1+A_2z}{1+B_2z}.\end{aligned}$$

Since $-1 \le B_1 \le B_2 < A_2 \le A_1 \le 1$, therefore by Lemma 1.7, we have

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\\&\quad+\mu_2\left\{1+\frac{zf''(z)}{f'(z)}+(\alpha+i\beta-1)\frac{zf'(z)}{f(z)}-\alpha\frac{zg'(z)}{g(z)}-ip\beta\right\}\prec p\frac{1+A_1z}{1+B_1z}.\end{aligned}$$

Hence, we have $f \in M_p(\alpha, \beta, \mu_2, A_1, B_1)$. For $\mu_2 = \mu_1 \ge 0$, we have the required result. When $\mu_2 > \mu_1 \ge 0$, Theorem 2.1 implies that

$$\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\prec p\frac{1+A_1z}{1+B_1z}.$$

Now

$$\begin{aligned}&\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\\&\qquad+\mu_1\left\{1+\frac{zf''(z)}{f'(z)}+(\alpha+i\beta-1)\frac{zf'(z)}{f(z)}-\alpha\frac{zg'(z)}{g(z)}-ip\beta\right\}\\&\quad=\left(1-\frac{\mu_1}{\mu_2}\right)\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}+\frac{\mu_1}{\mu_2}\left\{\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta}\right.\\&\qquad\left.+\mu_2\left\{1+\frac{zf''(z)}{f'(z)}+(\alpha+i\beta-1)\frac{zf'(z)}{f(z)}-\alpha\frac{zg'(z)}{g(z)}-ip\beta\right\}\right\}.\end{aligned}$$

Using Lemma 1.8, we have the required result. □

For $\beta = 0$, we have the following result.

Corollary 2.7 *For $\mu_2 \ge \mu_1 \ge 0$ and $-1 \le B_1 \le B_2 < A_2 \le A_1 \le 1$,*

$$M_p(\alpha, \mu_2, A_2, B_2) \subset M_p(\alpha, \mu_1, A_1, B_1).$$

This result is proved in [13].

For $\beta = 0$, $p = 1$, $g(z) = z$, $A = 1 - 2\rho$ and $B = -1$, we have the class $M(\alpha, \mu, \rho)$ defined as

$$\operatorname{Re}\left[\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{z}\right)^{\alpha}+\mu\left\{1+\frac{zf''(z)}{f'(z)}-\frac{zf'(z)}{f(z)}+\alpha\left(\frac{zf'(z)}{f(z)}-1\right)\right\}\right]>\rho \tag{2.9}$$

for $z \in E$. Now have the following result for the class $M(\alpha, \mu, \rho)$ proved in [21].

Corollary 2.8 *For* $\alpha \geq 0$, $\mu_2 \geq \mu_1 \geq 0$ *and* $1 > \rho_2 \geq \rho_1 \geq 0$,

$$M(\alpha, \mu_2, \rho_2) \subset M(\alpha, \mu_1, \rho_1).$$

Theorem 2.9 *Let* $f \in A_p$ *satisfy*

$$\operatorname{Re}\left\{\frac{f(z)}{z^p}\right\} > 0 \quad \textit{and} \quad \left|\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta} - p\right| < \sigma p, \quad 0 < \sigma \leq 1,$$

for $g \in S_p^*$. *Then* f *is* p-*valent convex in* $|z| < R_{\alpha,\beta,\sigma}$, *where*

$$R_{\alpha,\beta,\sigma} = \Big(\big(2|1-\alpha-i\beta| + 2\alpha p + \sigma\big) - \sqrt{\big(2|1-\alpha-i\beta| + 2\alpha p + \sigma\big)^2 - 4p(2\alpha p - p - \sigma)}\Big)/\big(2(2\alpha p - p - \sigma)\big). \tag{2.10}$$

Proof Let

$$h(z) = \frac{zf'(z)}{pf(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z^p}\right)^{i\beta} - 1,$$

where $h(z)$ is analytic in E with $h(0) = 0$ and $|h(z)| < 1$. By using the Schwarz lemma, we get

$$h(z) = \sigma z\psi(z),$$

where $\psi(z)$ is analytic in E with $|\psi(z)| < 1$. Differentiating logarithmically, we have

$$1 + \frac{zf''(z)}{f'(z)} = (1-\alpha-i\beta)\frac{zf'(z)}{f(z)} + \alpha\frac{zg'(z)}{g(z)} + \frac{\sigma z(z\psi'(z) + \psi(z))}{1 + \sigma z\psi(z)} + ip\beta.$$

Since $\operatorname{Re}\{\frac{f(z)}{z^p}\} > 0$, therefore,

$$\frac{zf'(z)}{f(z)} = p + \frac{z\varphi'(z)}{\varphi(z)}, \quad \operatorname{Re}\varphi(z) > 0.$$

This implies that

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \geq (1-\alpha)p + \alpha\operatorname{Re}\frac{zg'(z)}{g(z)} - |1-\alpha-i\beta|\left|\frac{z\varphi'(z)}{\varphi(z)}\right| - \sigma\left|\frac{z(z\psi'(z) + \psi(z))}{1 + \sigma z\psi(z)}\right|.$$

Now, using the well-known results for classes S_p^*, P and the Schwarz function [22], we have

$$\begin{aligned}\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} &\geq (1-\alpha)p + \alpha p\frac{1-r}{1+r} - |1-\alpha-i\beta|\frac{2r}{(1-r^2)} - \frac{\sigma r}{(1-r)} \quad (0 < \sigma \leq 1)\\ &= \frac{(2\alpha p - p - \sigma)r^2 - (2|1-\alpha-i\beta| + 2\alpha p + \sigma)r + p}{(1-r^2)}.\end{aligned}$$

Let $P(r) = (2\alpha p - p - \sigma)r^2 - (2|1-\alpha-i\beta| + 2\alpha p + \sigma)r + p$. Since $p \in \mathbb{N}$ and $0 < \sigma \leq 1$, therefore, $P(0) = p > 0$ and $P(1) = -2(|1-\alpha-i\beta| + \sigma) < 0$. It follows that the root lies in $(0,1)$. This implies that $\operatorname{Re}\{1 + \frac{zf''(z)}{f'(z)}\} > 0$ if $r < R_{\alpha,\beta,\sigma}$, where $R_{\alpha,\beta,\sigma}$ is given by (2.10). □

For $\sigma = 1$ and $\beta = 0$, we have the following result which is proved in [12].

Corollary 2.10 *Let $f \in A_p$ satisfy*

$$\operatorname{Re}\left\{\frac{f(z)}{z^p}\right\} > 0 \quad \text{and} \quad \left|\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha} - p\right| < p,$$

$g \in S_p^*$. Then f is p-valent convex in $|z| < R_\alpha$, where

$$R_\alpha = \frac{3 + 2\alpha(p-1) - \sqrt{(3 + 2\alpha(p-1))^2 - 4p(2\alpha p - p - 1)}}{2(2\alpha p - p - 1)}.$$

Theorem 2.11 *Let $f \in A_p$ satisfy*

$$\left|\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha} - p\right| < \sigma p,$$

for $g \in S_p^*$. Then, for $\alpha > 0$, f is p-valent $\frac{1}{\alpha}$-convex in $|z| < R_{\alpha,\sigma}$, where

$$R_{\alpha,\sigma} = \frac{2\alpha p + \sigma - \sqrt{(2\alpha p + \sigma)^2 - 4\alpha p(\alpha p - \sigma)}}{2(\alpha p - \sigma)}. \tag{2.11}$$

Proof Let

$$h(z) = \frac{zf'(z)}{pf(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha} - 1,$$

where $h(z)$ is analytic in E with $h(0) = 0$ and $|h(z)| < 1$. By using the Schwarz lemma, we get

$$h(z) = \sigma z\psi(z),$$

where $\psi(z)$ is analytic in E with $|\psi(z)| < 1$. Differentiating logarithmically, we have

$$\frac{1}{\alpha}\left(1 + \frac{zf''(z)}{f'(z)}\right) + \left(1 - \frac{1}{\alpha}\right)\frac{zf'(z)}{f(z)} = \frac{zg'(z)}{g(z)} + \frac{\sigma}{\alpha}\frac{z(z\psi'(z) + \psi(z))}{1 + \sigma z\psi(z)}.$$

This implies that

$$\operatorname{Re}\left\{\frac{1}{\alpha}\left(1 + \frac{zf''(z)}{f'(z)}\right) + \left(1 - \frac{1}{\alpha}\right)\frac{zf'(z)}{f(z)}\right\} \geq \operatorname{Re}\frac{zg'(z)}{g(z)} - \frac{\sigma}{\alpha}\left|\frac{(z\psi'(z) + \psi(z))}{1 + \sigma z\psi(z)}\right|.$$

Now, using the well-known results for classes S_p^*, and the Schwarz function, we have

$$\begin{aligned}\operatorname{Re}\frac{1}{p}\left\{\frac{1}{\alpha}\left(1 + \frac{zf''(z)}{f'(z)}\right) + \left(1 - \frac{1}{\alpha}\right)\frac{zf'(z)}{f(z)}\right\} &\geq \frac{1-r}{1+r} - \frac{\sigma r}{\alpha p(1-r)} \quad (0 < \sigma \leq 1)\\ &= \frac{(\alpha p - \sigma)r^2 - (2\alpha p + \sigma)r + \alpha p}{\alpha(1-r^2)}.\end{aligned}$$

Let $Q(r) = (\alpha p - \sigma)r^2 - (2\alpha p + \sigma)r + \alpha p$. Then for $p \in \mathbb{N}$, $\alpha > 0$ and $0 < \sigma \leq 1$, $Q(0) = \alpha p > 0$ and $Q(1) = -2\sigma < 0$. It shows that the root lies in $(0,1)$. This implies that $\operatorname{Re}\{\frac{1}{\alpha}(1 + \frac{zf''(z)}{f'(z)}) + (1 - \frac{1}{\alpha})\frac{zf'(z)}{f(z)}\} > 0$ if $r < R_{\alpha,\sigma}$, where $R_{\alpha,\sigma}$ is given by (2.11). □

Theorem 2.12 *Let $f \in M(\alpha, \beta, \mu, A, B)$. Then E is mapped by f on a domain that contains the disc $|w| < R_{\alpha,\beta,\mu} \in (0,1)$, where*

$$R_{\alpha,\beta,\mu} = \frac{|\alpha + i\beta + 1|(1+\mu)}{2|\alpha + i\beta + 1|(1+\mu) + (A-B) + 2\alpha(1+\mu)}. \tag{2.12}$$

Proof Let w_0 be any complex number such that $f(z) \neq w_0$. Then

$$\frac{w_0 f(z)}{w_0 - f(z)} = z + \left(a_2 + \frac{1}{w_0}\right)z^2 + \cdots,$$

is univalent in E, so that

$$\left|a_2 + \frac{1}{w_0}\right| \leq 2.$$

Therefore,

$$\left|\frac{1}{w_0}\right| - |a_2| \leq 2.$$

Hence,

$$w_0 \geq \frac{|\alpha + i\beta + 1|(1+\mu)}{2|\alpha + i\beta + 1|(1+\mu) + (A-B) + 2\alpha(1+\mu)} = R_{\alpha,\beta,\mu}.$$ □

For $\beta = 0$, $g(z) = z$, $A = 1 - 2\rho_1$, $B = -1$, we have the following result proved in [13].

Corollary 2.13 *Let $f \in M(\alpha, \mu, \rho)$. Then E is mapped by f on a domain that contains the disc $|w| < R_{\alpha,\mu}$, where*

$$R_{\alpha,\beta,\mu} = \frac{(1+\alpha)(1+\mu)}{2(1+\alpha)(1+\mu) + 2(1-\rho_1)}.$$

Theorem 2.14 *Let $\alpha > 0$, $c \geq -\alpha\frac{1-A}{1-B}$ and let $f \in A$. If*

$$\left|\arg\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z}\right)^{i\beta} - \rho\right| < \frac{\pi}{2}\upsilon \quad (0 < \upsilon \leq 1, 0 \leq \rho < 1), \tag{2.13}$$

for some $g \in S^*$, then

$$\left|\arg\frac{zF'(z)}{F(z)}\left(\frac{F(z)}{G(z)}\right)^{\alpha}\left(\frac{F(z)}{z}\right)^{i\beta} - \rho\right| < \frac{\pi}{2}\eta \quad (0 < \eta \leq 1), \tag{2.14}$$

where

$$F(z) = \left[\frac{c + \alpha + i\beta}{z^c}\int_0^z t^{c-1}f^{\alpha+i\beta}(t)\,dt\right]^{\frac{1}{\alpha+i\beta}}, \tag{2.15}$$

with

$$\upsilon = \begin{cases} \eta + \frac{2}{\pi}\tan^{-1}\left(\frac{(1+B)\eta\sin(\pi(1-t(\alpha,\beta,c,A,B))/2)}{|c+i\beta|(1+B)+\alpha(1+A)+\eta(1+B)\cos(\pi(1-t(\alpha,\beta,c,A,B))/2)}\right), & B \neq -1, \\ \eta, & B = -1, \end{cases} \tag{2.16}$$

and

$$t(\alpha,\beta,c,A,B) = \frac{2}{\pi}\sin^{-1}\frac{\alpha(A-B)+\beta(1-B^2)}{|c+i\beta|(1-B^2)+\alpha(1-AB)}. \tag{2.17}$$

Proof Since

$$F^{\alpha+i\beta}(z) = \frac{c+\alpha+i\beta}{z^c}\int_0^z t^{c-1}f^{\alpha+i\beta}(t)\,dt,$$

therefore,

$$z^{1-i\beta}F^{\alpha+i\beta-1}(z)F'(z) = \frac{1}{\alpha+i\beta}\left\{(c+\alpha+i\beta)z^{-i\beta}f^{\alpha+i\beta}(z) - cz^{-i\beta}F^{\alpha+i\beta}(z)\right\}.$$

Now using (1.5), we have

$$\frac{zF'(z)}{F(z)}\left(\frac{F(z)}{G(z)}\right)^{\alpha}\left(\frac{F(z)}{z}\right)^{i\beta} = \frac{\frac{1}{\alpha+i\beta}\{(c+\alpha+i\beta)z^c f^{\alpha+i\beta}(z) - cz^c F^{\alpha+i\beta}(z)\}}{(c+\alpha+i\beta)\int_0^z t^{c+i\beta-1}g^{\alpha}(t)\,dt}.$$

Let

$$h(z) = \frac{1}{1-\rho}\left(\frac{zF'(z)}{F(z)}\left(\frac{F(z)}{G(z)}\right)^{\alpha}\left(\frac{F(z)}{z}\right)^{i\beta} - \rho\right) = \frac{N(z)}{D(z)},$$

where $h(z)$ is analytic with $h(0) = 1$. Now

$$\begin{aligned} \frac{N'(z)}{D'(z)} &= \frac{1}{1-\rho}\left(\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^{\alpha}\left(\frac{f(z)}{z}\right)^{i\beta} - \rho\right) \\ &= h(z)\left\{1 + \frac{D(z)}{zD'(z)}\cdot\frac{zh'(z)}{h(z)}\right\}, \end{aligned}$$

where

$$\begin{aligned} N(z) &= \frac{1}{1-\rho}\left(\frac{1}{\alpha+i\beta}\left\{(c+\alpha+i\beta)z^c f^{\alpha+i\beta}(z) - cz^c F^{\alpha+i\beta}(z)\right\} - \rho z^{c+i\beta-1}G^{\alpha}(z)\right), \\ D(z) &= (c+\alpha+i\beta)\int_0^z t^{c+i\beta-1}g^{\alpha}(t)\,dt. \end{aligned}$$

Since $G \in S^*[A,B]$, therefore, we can write

$$\frac{zD'(z)}{D(z)} = c + i\beta + \alpha\frac{zG'(z)}{G(z)} = r_1 e^{i\frac{\pi}{2}\theta},$$

where

$$\begin{cases} |c+i\beta| + \alpha\frac{1-A}{1-B} < r_1 < |c+i\beta| + \alpha\frac{1+A}{1+B}, & B \neq -1, \\ -t(\alpha,\beta,c,A,B) < \theta < t(\alpha,\beta,c,A,B), & B \neq -1, \end{cases} \tag{2.18}$$

similarly

$$\begin{cases} |c+i\beta|+\alpha\frac{1-A}{2}<r_1<\infty, & B=-1, \\ -1<\theta<1, & B=-1. \end{cases} \tag{2.19}$$

Suppose that $h(z)\neq 0$ in E, there exists a point $z_0\in E$ such that $|\arg h(z)|<\frac{\pi}{2}\eta$ ($|z|<|z_0|$) and $|\arg h(z_0)|=\frac{\pi}{2}\eta$. Now, using Lemma 1.10, we have $\frac{zh'(z_0)}{h(z_0)}=ik\eta$. At first suppose that $h(z_0)=(ix)^\eta$ ($x>0$) for the case $B\neq -1$, we obtain

$$\begin{aligned}
&\arg\left(\frac{z_0f'(z_0)}{f(z_0)}\left(\frac{f(z_0)}{g(z_0)}\right)^\alpha\left(\frac{f(z_0)}{z_0}\right)^{i\beta}-\rho\right) \\
&\quad=\arg h(z_0)+\arg\left(1+\frac{1}{c+i\beta+\alpha\frac{zG'(z_0)}{G(z_0)}}\cdot\frac{z_0h'(z_0)}{h(z_0)}\right) \\
&\quad=\frac{\pi}{2}\eta+\arg\bigl(1+\bigl(r_1e^{i\frac{\pi}{2}\theta}\bigr)^{-1}ik\eta\bigr) \\
&\quad=\frac{\pi}{2}\eta+\tan^{-1}\left(\frac{k\eta\sin\pi(1-\theta)/2}{r_1+k\eta\cos\pi(1-\theta)/2}\right) \\
&\quad\geq\frac{\pi}{2}\eta+\tan^{-1}\left(\frac{\eta\sin\pi(1-t(\alpha,\beta,c,A,B))/2}{|c+i\beta|+\alpha\frac{1+A}{1+B}+\eta\cos\pi(1-t(\alpha,\beta,c,A,B))/2}\right) \\
&\quad=\frac{\pi}{2}\upsilon,
\end{aligned}$$

where υ and $t(\alpha,\beta,c,A,B)$ are given by (2.16) and (2.17) respectively. For $B=-1$, we have

$$\arg\left(\frac{z_0f'(z_0)}{f(z_0)}\left(\frac{f(z_0)}{g(z_0)}\right)^\alpha\left(\frac{f(z_0)}{z_0}\right)^{i\beta}-\rho\right)\geq\frac{\pi}{2}\eta,$$

which is a contradiction to the assumption of our theorem. Now we suppose that $h(z_0)=(-ix)^\eta$. For the case $B\neq -1$ using a similar method, we obtain

$$\arg\left(\frac{z_0f'(z_0)}{f(z_0)}\left(\frac{f(z_0)}{g(z_0)}\right)^\alpha\left(\frac{f(z_0)}{z_0}\right)^{i\beta}-\rho\right)\leq-\frac{\pi}{2}\upsilon,$$

and for $B=-1$, we have

$$\arg\left(\frac{z_0f'(z_0)}{f(z_0)}\left(\frac{f(z_0)}{g(z_0)}\right)^\alpha\left(\frac{f(z_0)}{z_0}\right)^{i\beta}-\rho\right)\leq-\frac{\pi}{2}\eta,$$

which is a contradiction to the assumption of our theorem. Hence, we have the proof. □

This kind of problem is also considered in [23]. For $\beta=0$, we have the following result proved in [24].

Corollary 2.15 *Let $\alpha>0$, $c\geq-\alpha\frac{1-A}{1-B}$ and let $f\in A$. If*

$$\left|\arg\left(\frac{zf'(z)}{f(z)}\left(\frac{f(z)}{g(z)}\right)^\alpha-\rho\right)\right|<\frac{\pi}{2}\upsilon\quad(0<\upsilon\leq 1),$$

then

$$\left|\arg\left(\frac{zF'(z)}{F(z)}\left(\frac{F(z)}{G(z)}\right)^{\alpha}-\rho\right)\right|<\frac{\pi}{2}\eta \quad (0<\eta\leq 1),$$

where

$$F(z)=\left[\frac{c+\alpha}{z^c}\int_0^z t^{c-1}f^{\alpha}(t)\,dt\right]^{\frac{1}{\alpha}},$$

with

$$\upsilon=\begin{cases}\eta+\frac{2}{\pi}\tan^{-1}\left(\frac{(1+B)\eta\sin(\pi(1-t(\alpha,\beta,c,A,B))/2)}{c(1+B)+\alpha(1+A)+\eta(1+B)\cos(\pi(1-t(\alpha,\beta,c,A,B))/2)}\right), & B\neq -1,\\ \eta, & B=-1,\end{cases}$$

and

$$t(\alpha,\beta,c,A,B)=\frac{2}{\pi}\sin^{-1}\frac{\alpha(A-B)}{c(1-B^2)+\alpha(1-AB)}.$$

Remark 2.16 By using the suitable choices of parameters c, α, A and B, we can find many results proved in the literature.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
MR, SNM and KIN jointly discussed and presented the ideas of this article. MR made the text file and all the communications regarding the manuscript. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, GC University Faisalabad, Faisalabad, Pakistan. [2]Department of Mathematics, COMSATS Institute of Information Technology, Islamabad, Pakistan.

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