

*Full Length Research Paper*

# **Simulation study of damage detection in steel shear frame buildings using impulse response and wavelet analysis**

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Accepted 13 April, 2012

**Structural frames, demonstrate both elastic and plastic behaviour depending on their load. At early stages of earthquake loading structural systems remain in the elastic phase, but when the loading increases, plastic hinges form and affect the structural response. Determining the location and time of this damage is critical. The aim of previous methods such as Fourier, short time Fourier or wavelet transform was to find the time and location of hinge formation. This was conducted by changing response space, with the use of vibration properties of a system like vibration frequency or vibration modes of the structure. However, these methods are not very effective in early warning applications since the vibration properties do not change significantly at the beginning of plastic hinge formation. In this research, the capability of wavelet transform in extracting local signal information will be used to determine the time and location of plastic hinge formation. These signals are recorded responses of the structure under seismic excitation. The obtained results indicate that the proposed method can effectively determine the time and location of damage without employing complicated methods and concepts.**

**Key words:** Damage detection, plastic hinge, impulse response, shear frame.

## **INTRODUCTION**

An estimation of damage in the building frames has two main targets: (I) determination of stability and serviceability of structures after the earthquake, (II) producing a priority scheme and timetable to repair the damaged parts. Also, as these estimations are often based on vibration responses of a system, substantive changes in response - e.g. natural frequency changes - are often the basis for damage detection. These changes are functions of intensity and location of induced damage. This means that less damage at the closer distance can have similar effects as more damage at farther away

which makes detection more complicated. In any case, relative damage estimation is also useful because it can be the basis for choosing strategies to repair damaged areas. These behavioural changes are usually rooted in the change of material behaviour in one or several structural elements and will weaken element(s) and consequently the system as a whole.

Modelling of damage is also an important point in determining structural behaviour. In proper modelling, damage should be formed in the loading process and increase with loading increments. In other words, it should be in agreement with real damage in the structure. The formation of plastic hinges in a specific section has these properties, so it can be a simple and appropriate model to track the behaviour changes in a section of the

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structural element.

Plastic hinge behaviour is expressed in terms of force-displacement curves, e.g. moment-curvature. Having moment-curvature relations for a specific member, one can determine the level of plastic rotation capacity. Angular displacements of the plastic hinges at the ends of the beams and columns in a frame are important in the nonlinear dynamic analysis because they represent damage to the structure (Hui et al., 2004). A damage index based on the same idea was presented by Campbell et al. (2008), which is a quantitative parameter for estimating structural damage and damage in a member.

Jankovic and Stojadinovic (2008) provided a damage index based on joint maximum plastic rotation for positive and negative rotation. The plastic ductility damage index is at the centre of the standards' attention such as FEMA (2000) because of simplicity in calculation and tangible physical concept (Powell and Allahabadi, 1987). The main idea is that, if a given type of damage changes a linear system into a nonlinear system, then any observed manifestation of nonlinearity serves to indicate that damage is present (Farrar et al., 2007).

A method of structural damage detection called Local Damage Factor (LDF) was presented by Shanshan et al. (2006), which is capable of determining the presence, severity, and location of structural damage at the same time. This method is based on auto-correlation and cross-correlation of the entire structure response and local structure response. A study regarding the development of a damage detection indicator for civil engineering structures was performed by Zabel (2005) which is based on energy components of wavelet decompositions of measured signals' impulse response and transmissibility functions.

There has also been some research using wavelet analysis to locate discontinuity caused by damage using local analysis of the signal (Ovanesova and Sua'rez, 2004). In this method, the response needs to be obtained only at the regions where it is suspected that the damage may be present. Wavelet analysis could detect the place of pre-embedded damage in the structure.

Recently some research has been conducted to extract damage caused by earthquake loading directly from stories' seismic responses (Todorovska and Trifunac, 2009; Raufi and Bahar, 2010; Todorovska and Trifunac, 2008; Lynn et al., 1997; Bisht, 2005; Safak and Hudnut, 2006). These responses can be measured by using instruments planted on structures and can be acceleration, velocity or displacement responses. Processing of these recorded signals can reveal some information regarding the time and location of damage. A study on the acceleration responses of a six-storey concrete building in the Southern California area has shown that wavelet analysis of the recorded responses has useful information about the time and location of damage (Todorovska, Trifunac, 2009). On the other hand, different response components do not have the same amount of

information about the formation of plastic hinges, so choosing a specific response which is more sensitive to changes due to nonlinear behaviour of the structure is also an important issue. Research conducted by Raufi and Bahar (2010) demonstrated that nodal rotational response can be a good indicator of plastic hinge location.

Wave travel has also been used to determine damage location (Todorovska and Trifunac, 2008). The travel time can be computed having the seismic response in different stories. The main concept of this method is the reduction of wave transferring velocity due to damage. Time Delay (TD) in inter-storey propagation indicates local damage, where TD in wave propagation from the basement to top floor indicates global damage.

In this research, the acceleration response of different stories has been selected for processing. It will be shown that by having an impulse response function of a linear structure in the time domain, useful information can be obtained about the time and location of plastic hinges formed in case of a real earthquake.

## Impulse response function

A discrete linear system is a digital implementation of a linear time-invariant system (LTI). Its input is a vector representing the sampled input signal and its output is a vector of the same size as the input, representing the sampled output signal. Any sampled signal is just a series of scaled impulses whose amplitudes are the instantaneous amplitudes of the original analogue signal which occur regularly at the sampling instants. Thus if the input signal (ground acceleration) is just a series of impulses considering that the system obeys the principle of superposition; by knowing the system's response to impulses, the output (response) of the system to any input signal can be calculated using convolution (Lynn et al., 1997).

Impulse Response Function (IFR) has been used in damage detection. It can be employed as a basis to determine the damage state of the system. The structural response of a Linear Time Invariant system in the time domain, that is,  $x(t)$  is related to an excitation  $f(t)$  by the system's impulse response function  $h(t)$  (Lynn et al., 1997) Equation (1):

$$x(t) = \int_{-\infty}^t h(t-\tau)f(\tau)d\tau \quad (1)$$

Where  $x(t)$  is the time series of response,  $f(\tau)$  is the time series of excitation and  $h(t)$  is impulse response function (IRF). In a discrete-time case for LTI, Equation (1) can be written as:

$$x(t) = \sum_0^{\infty} f(\tau)h(t-\tau)\Delta\tau \quad (2)$$

Impulse response has been used in determination of dynamic properties of a system like estimation of vibration frequency and mode shapes. The impulse response function of a multi-storey building can be generated from structural analysis or ambient noise. This can be used to study (1) modal parameters of the building, (2) wave propagation inside the building, (3) estimates of the quality factor (which relates the natural frequency and amplitude decay) associated with the normal modes, and (4) predictions of building response to scenario ground motions of moderately sized earthquakes (Prieto et al., 2010). These obtained dynamic properties can be used in determining damage in structural systems by analysing the response in the frequency domain by means of continuous wavelet transform using the morlet mother wavelet (Bisht, 2005). However, this method cannot capture the changes in behavioural stages with no significant changes in vibration properties of the system, that is, the initiation of damage. On the other hand, change in natural frequencies, as a commonly used criterion for damage detection, is not always a reliable indicator of damage (Safak and Hudnut, 2006).

Impulse response can also be expressed in the frequency domain. Building response can be calculated by deconvolving the waves which are achieved from records of all stories either with records of the basement or with records of the top floor of the building. The frequency domain form of deconvolution is as Equation (3) (Snieder and Safak, 2006):

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} \quad (3)$$

Where  $H(j\omega)$  is the frequency response of the system—that is, the Fourier transform of the system's impulse response function;  $F(j\omega)$  and  $X(j\omega)$  are the Fourier transform of inputs and output of the system, respectively.

In this research, wavelet transform of the difference between the linear response obtained from impulse excitation of the structural system and nonlinear response obtained from a random excitation, that is, earthquake excitation, will be used to detect the time and location of probable damage in the system. More detailed descriptions will be presented in the methodology section.

## Wavelet

Wavelet is a rather new tool for detecting damage. It is a group of mathematical functions that are used to break down a signal to its frequency components. The resolution of each component is equal to the “scale”. Wavelet transform analysis of a function is based on wavelet functions. Wavelet functions have a limited

bandwidth in time and frequency domain. These functions are available as asymmetrical, symmetrical inverted, real or virtual. Wavelets (which are known as daughter wavelets) are transferred and scaled samples from a mother function.

The relationship between wavelet transforms and filter bank and the possibility of doing a fast wavelet analysis are the advantages of wavelet transform. Wavelet coefficients contain much information about the contents of the signal.

Equations (4) to (8) show the main equations of the wavelet transform method. Using a selected analysis or mother wavelet function  $\psi(t)$ , the continuous wavelet transform of signal  $f(t)$  is defined as (Misiti et al., 2007):

$$C(a, b) = \int_{-\infty}^{+\infty} f(t) \psi_{a,b}(t) dt \quad (4)$$

$$\psi_{a,b}(t) = a^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right) \quad (5)$$

And discrete wavelet transform can be defined as:

$$C(a, b) = c(j, k) = \sum_{n \in \mathbb{Z}} f(n) \psi_{j,k}(n) \quad (6)$$

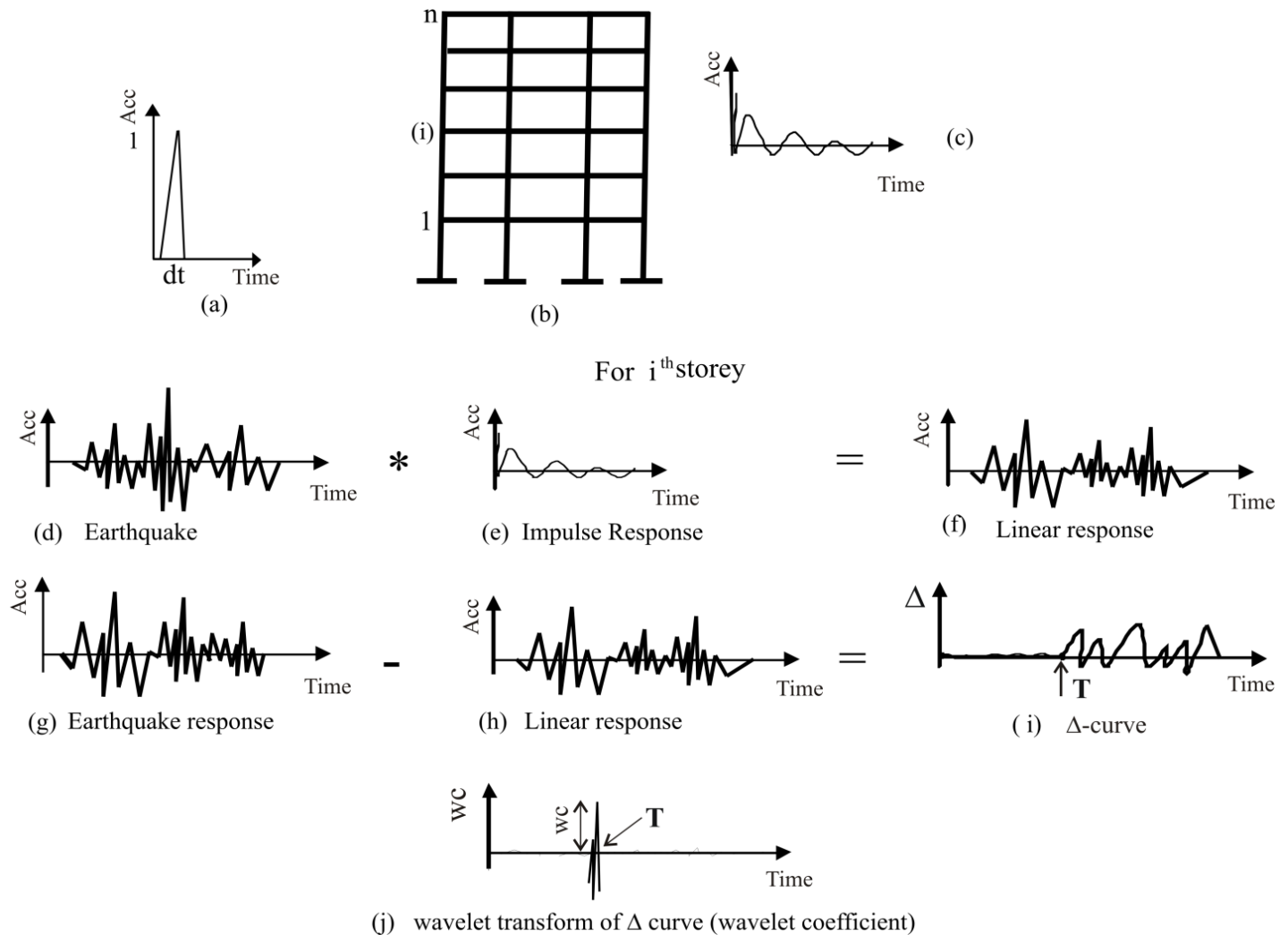
In which  $\psi_{j,k}$  is a discrete wavelet defined as follows (sometimes also called binary analysis):

$$\psi_{j,k}(n) = 2^{-j/2} \psi(2^{-j}n - k) \quad (7)$$

It can be explained that continuous transform of the signal in the full domain of  $C(a, b)$  is highly redundant for a certain choice of the mother wavelet. It is possible to describe the scaling and shifting factors as:

$$a = 2^j, b = 2^j k \quad (j, k) \in \mathbb{Z}^2 \quad (8)$$

Selection of a proper wavelet function is the first step in wavelet analysis. The choice depends on the desired issue and can have a considerable effect on the results. In this study, discrete wavelet transform (DWT) employing bior 6.8 (Todorovska and Trifunac, 2005; Ovanesova and Sua'ez, 2004; Raghu Prasad et al., 2006) is used. In DWT, the scale parameter ‘a’ is chosen as  $a = 2^j$  where  $j$  is an integer value  $j \in \mathbb{Z}$  and  $j = \log_2(a)$ . For a function  $f(t) \in L^2$ -space with a Fourier transform  $F(\omega)$ , a change in scale factor  $j$  is followed by a change in the scale of frequency domain given by  $a = 2^j$ . Signal decomposition in wavelet analysis is carried-out by projecting the signal into a subspace of the wavelet's basis functions at different scales and their transmission.



**Figure 1.** Overview of the proposed method (a) impulse load (b) system (frame) (c) impulse response function in time domain (d) general excitation (e)  $i^{\text{th}}$  storey impulse response (f) linear response (g)  $i^{\text{th}}$  storey earthquake response (h) linear response (same as “f”) (i) Difference curve ( $\Delta$ -curve) (j) wavelet transform of difference curve.

It is important to explain that wavelet transform scaling factors are related to the frequency content. “First wavelet detail” is obtained when the mother wavelet fits the original signal at the smallest scale and the remainder is called “approximation”.

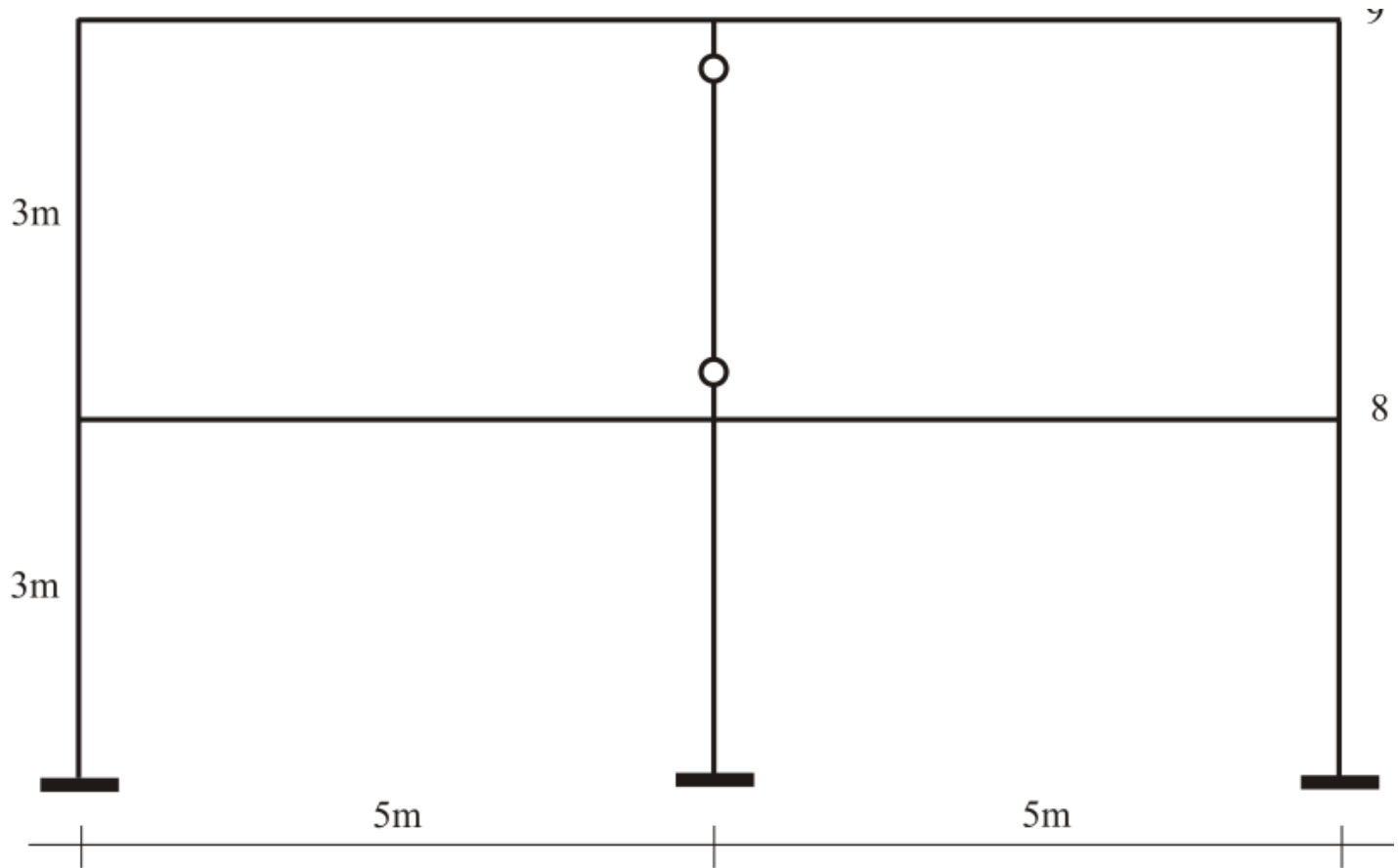
## METHODOLOGY

Earthquake time history can be recorded by using proper instruments in the site. Furthermore, it is possible to record the response of different stories employing special sensors. Supposing linear behaviour, these responses can be related in the linear response phase. In this research, the input signal is the ground acceleration and recorded acceleration (acc.) response at the selected stories is considered as the output signal.

Figure 1 presents steps required in the proposed methodology. Figure 1(a) is the impulse load and Figures 1(b) and (c) show the structural frame and impulse acceleration response. Having the

base excitation shown in Figure 1(d) and impulse response of the system in the  $i^{\text{th}}$  storey as in Figure 1(e), linear response can be calculated using convolution shown in Figure 1(f). If the behaviour remains elastic there will be little or no difference between this response and the earthquake response recorded in a specific storey as shown in Figure 1(g). But once a hinge is formed and the structure enters the nonlinear phase, the recorded response at stories and the response computed from the convolution will no longer be the same and the difference will be greater at the time of plastic hinge formation. In other words, the difference curve ( $\Delta$ -curve) which is obtained by subtracting the earthquake response and impulse response has two distinguishable parts in which the first part has very small amplitudes relative to the second part as shown in Figure 1(i). In ideal problems, the first part will be equal to zero but in more realistic problems it will not be exactly zero which leaves the distinction of these two parts to the engineer’s judgment. In order to avoid subjectivity, wavelet analysis can be used to determine the distinction point of these two parts.

Figure 1(j) shows the wavelet transforms of the difference curve ( $\Delta$ -curve). In Figure 1, T is the time of damage and WC is Wavelet



**Figure 2.** Schematic representation of the main damage in 2-bay, 2-storey frame and nodal numbers and location of plastic hinge formation.

Coefficient. Larger wavelet coefficients for a specific storey in comparison with values associated with other stories show that damage location is closer to that specific storey.

It is worth mentioning that impulse response functions are needed to be computed once for the structure at hand and can be used for damage assessment of any earthquake which may happen to the structure in the future.

## NUMERICAL ANALYSIS AND RESULTS

### Example 1: 2-storey 2-bay steel frame

In this example, 2-storey 2-bay plane steel frame presented in Figure 2, has been analysed using the OpenSees program (McKenna et al., 2000) under Round Valley record excitation. Figure 3 shows the normalized earthquake excitation registered at Round Valley, California, November 23, 1984. This excitation is used for analysis of the frame with PGA scaled to 1 g.

A uniform gravity load equal to 1650 kg/m is applied to the spans. These frame models have been designed using AISC ASD (1989) and IPE 300 for beams and 240 for the columns steel profiles with 2400 kg/cm<sup>2</sup> yield

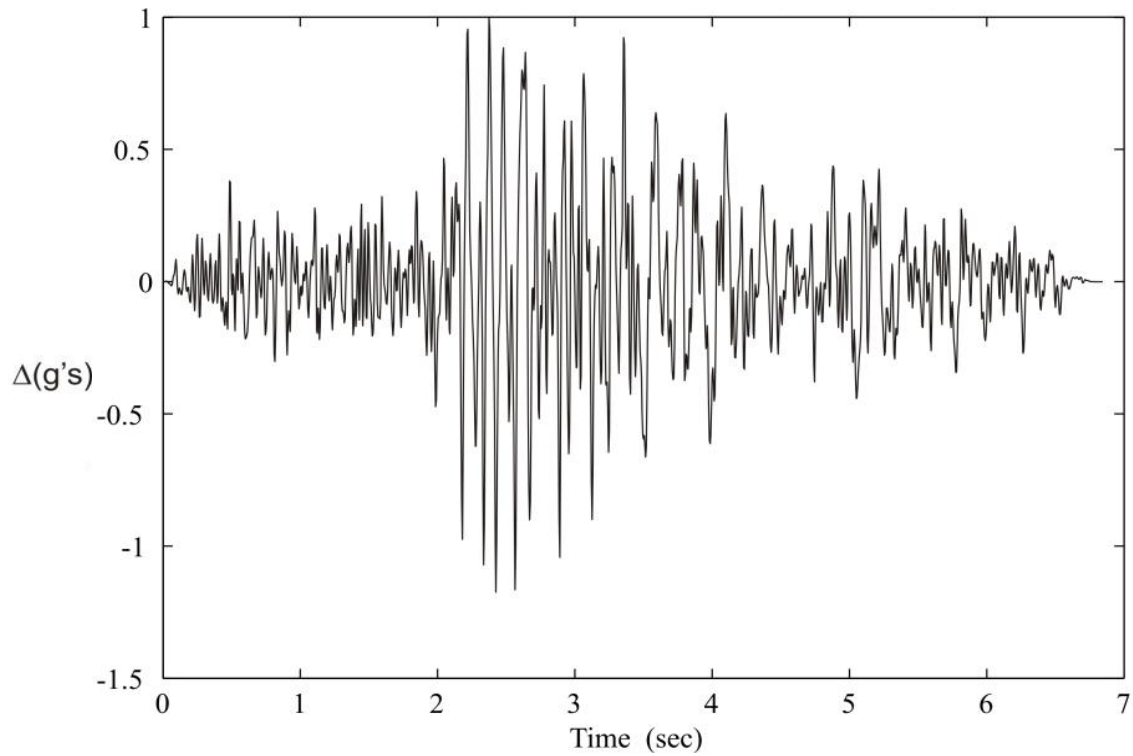
strength and ultimate strength of 3500 kg/cm<sup>2</sup>. In this example, as shown in Figure 2, the first and second hinges are developed at  $T=4.14$  s in the second storey. Plastic hinges were obtained from inelastic dynamic analyses.

The time and location of damage can be calculated using the proposed method. The difference between linear and nonlinear time history response ( $\Delta$ -curve), computed using finite element analysis, is given in Figure 4.

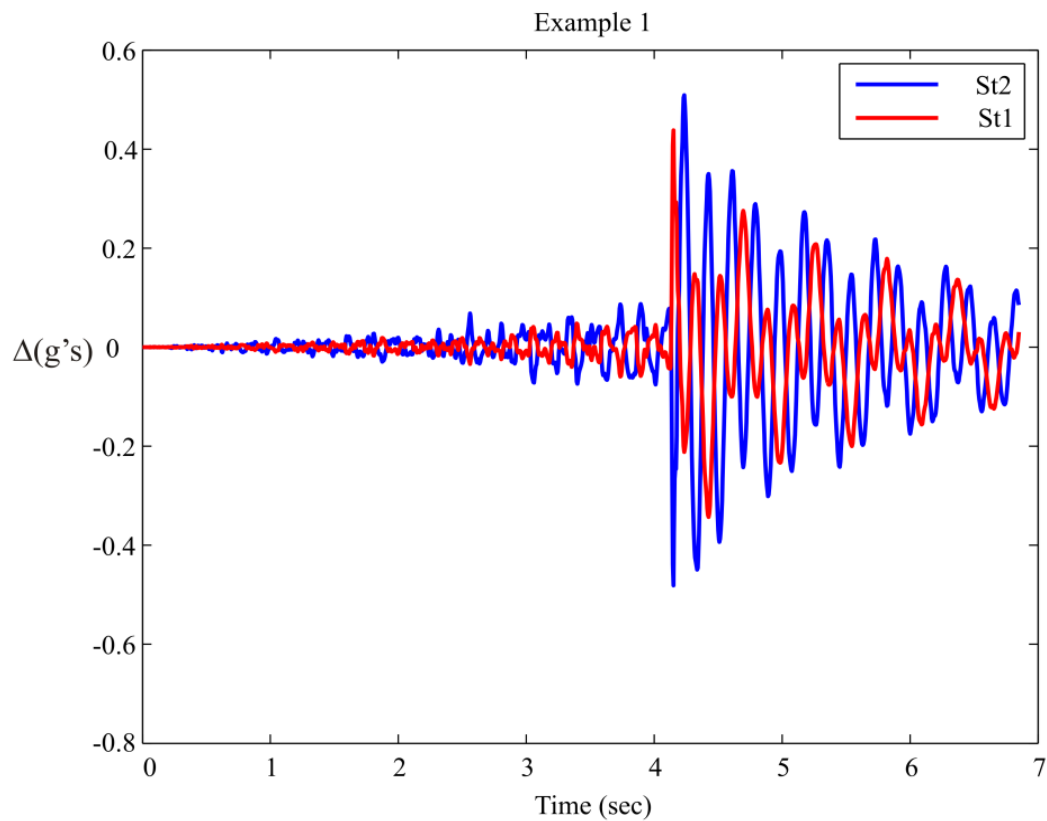
As shown in Figures 5 and 6, the first level of discrete wavelet decomposition of the difference curve ( $\Delta$ -curve) clearly detects the time of plastic hinge formation. Extraction of local signal information by wavelet transform can be clearly observed in these figures.

To show the advantage of the difference curve ( $\Delta$ -curve) compared with transitional response in damage detection, results of wavelet transform on a difference curve and a horizontal response function for the above example is compared in Figures 5 and 6.

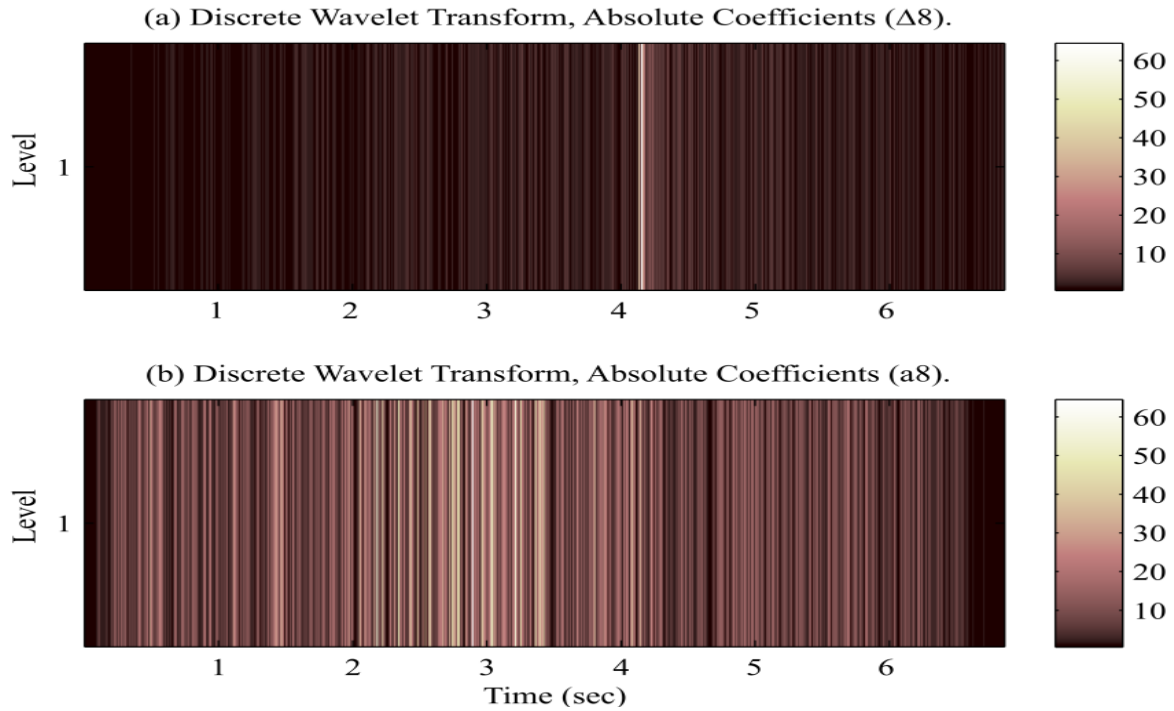
In Figures 5 and 6, the colour at each point is associated to the magnitude of the wavelet coefficients. Lighter colours correspond to the larger coefficients and



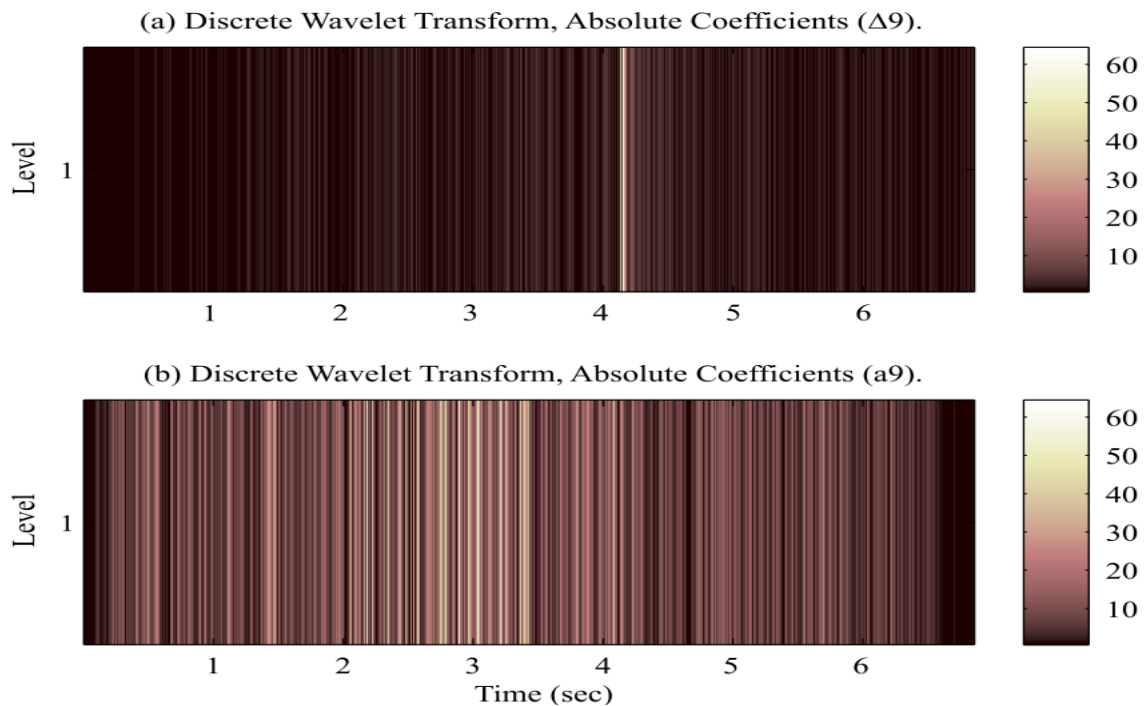
**Figure 3.** Normalized Round Valley acceleration record with respect to its peak ground acceleration (PGA).



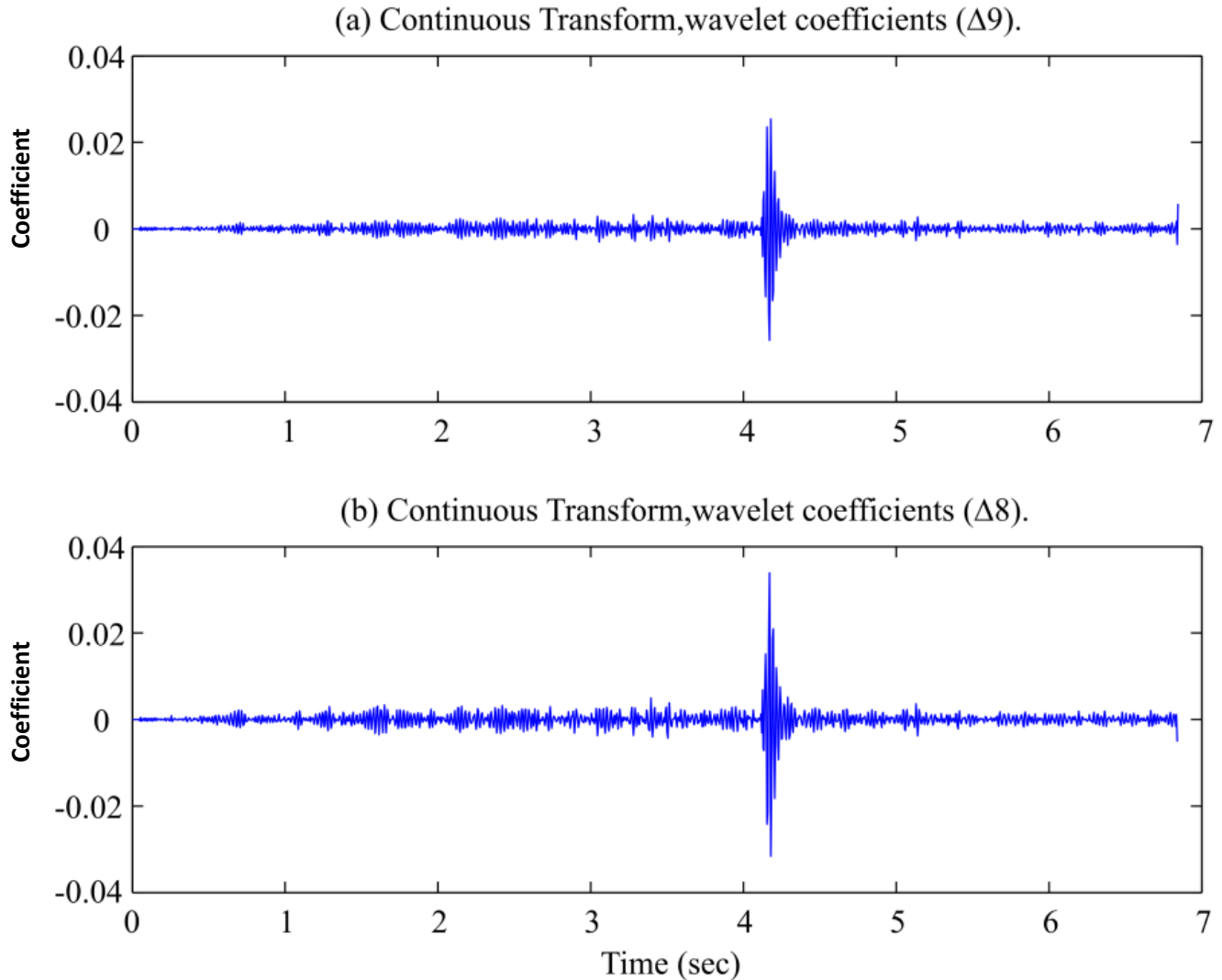
**Figure 4.** Difference between earthquake response and linear response for first and second stories.



**Figure 5.** Comparing details of the first decomposition level, the difference curve ( $\Delta$ -curve) and horizontal acceleration is shown (for node 8). As the figures indicate, the absolute coefficients, at initiation of damage, have suddenly changed at 4.14 s (a), while sudden change is not visible in the wavelet decomposition level of the horizontal acceleration response (b).



**Figure 6.** Comparing details of the first decomposition level, the  $\Delta$ -curve and horizontal acceleration are shown (for node 9). As the figures indicate, the absolute coefficients, at initiation of damage, have suddenly changed at 4.14 s (a), while sudden change is not visible in the wavelet decomposition level of the horizontal acceleration response (b).



**Figure 7.** Wavelet analysis of  $\Delta$ function for different stories, (a) second storey (node 9) and (b) first storey (node 8).

**Table 1.** Wavelet coefficient (Figure 7).

Storey	Time $T=4.14$ s
St2	0.025
St1	0.03

For  $T=4.14$  s:  $WC2 \leq WC1 \rightarrow$  Damage in 2<sup>nd</sup> Storey.

darker colours to the smaller ones. These coefficients are based on a discrete wavelet analysis. It is clearly shown in Figures 5 and 6 that one level of decomposition would suffice.

As shown in Figures 5(a) and 6(a), the time of hinge formations were detected using the wavelet analysis of difference curve functions while using wavelet analysis of the horizontal acceleration response could not reveal the time of damage occurrence (Figures 5(b) and 6(b)). As

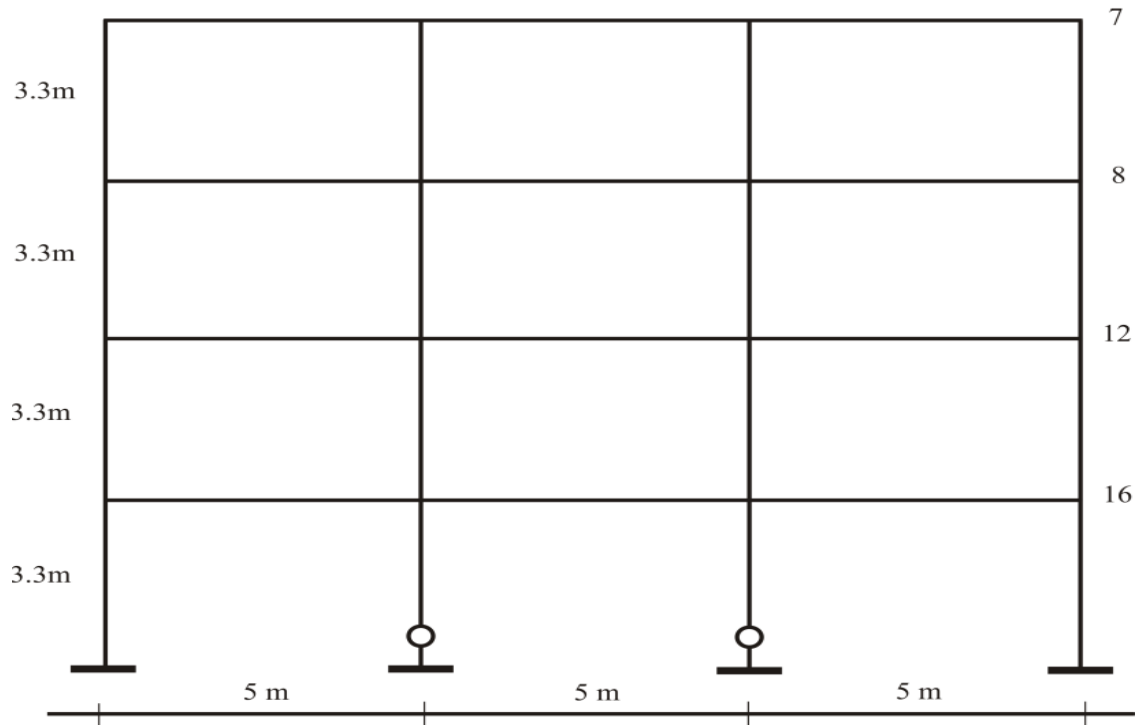
shown in Figure 7, the time of the first hinge formation is at  $T=10.48$  s which has occurred at the second storey (Table 1).

### Example 2: 4-storey 3-bay steel frame

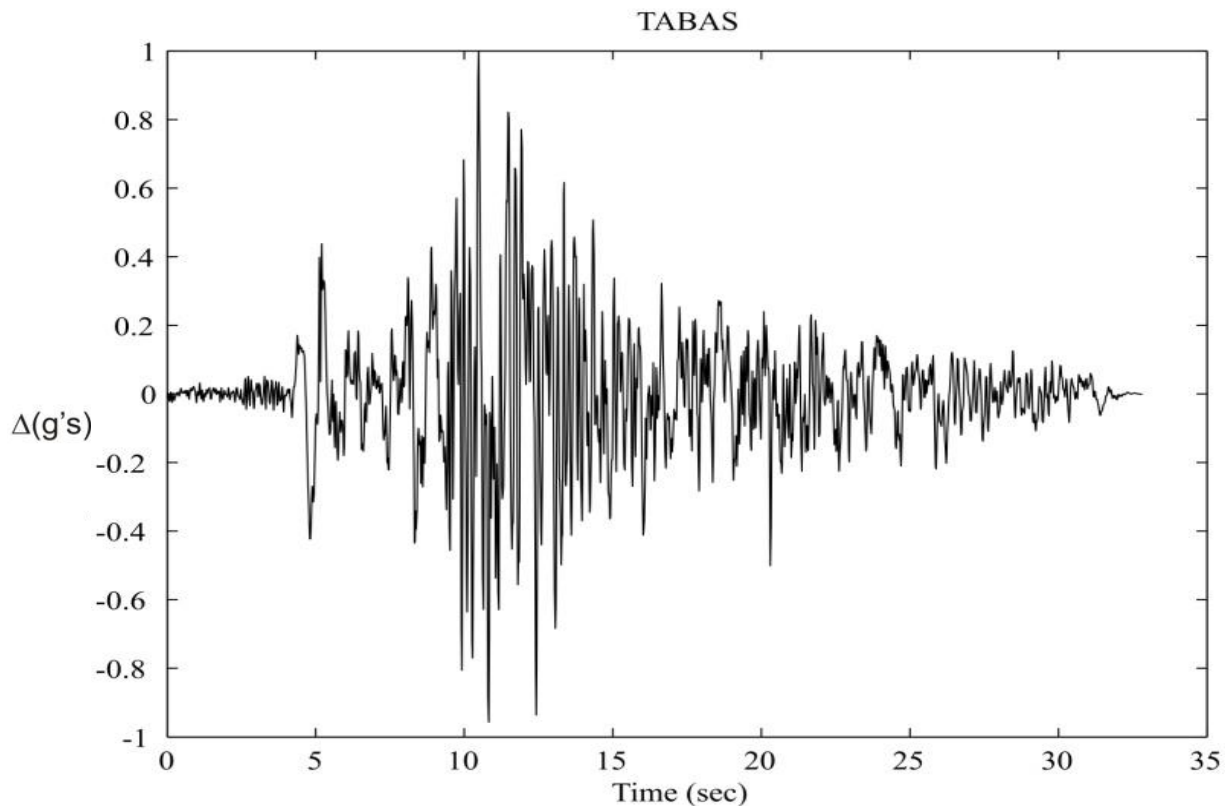
In this example, as shown in Figure 8, a 4-storey 3-bay plane steel frame is subjected to longitudinal component of Tabas excitation scaled to  $PGA=0.2$  g. This normalized earthquake excitation was registered at Tabas, Iran, September 16, 1978 (Figure 9).

A uniform gravity load equal to 2000 kg/m is applied to the spans. Plate girder sections were used for beams and columns (Table 2) with 2400 kg/cm<sup>2</sup> yield strength and ultimate strength of 3500 kg/cm<sup>2</sup>. In this example, as shown in Figure 5, the first hinge developed at  $T_1=13$  s and the second hinge developed at  $T_2=13.2$  s at the first





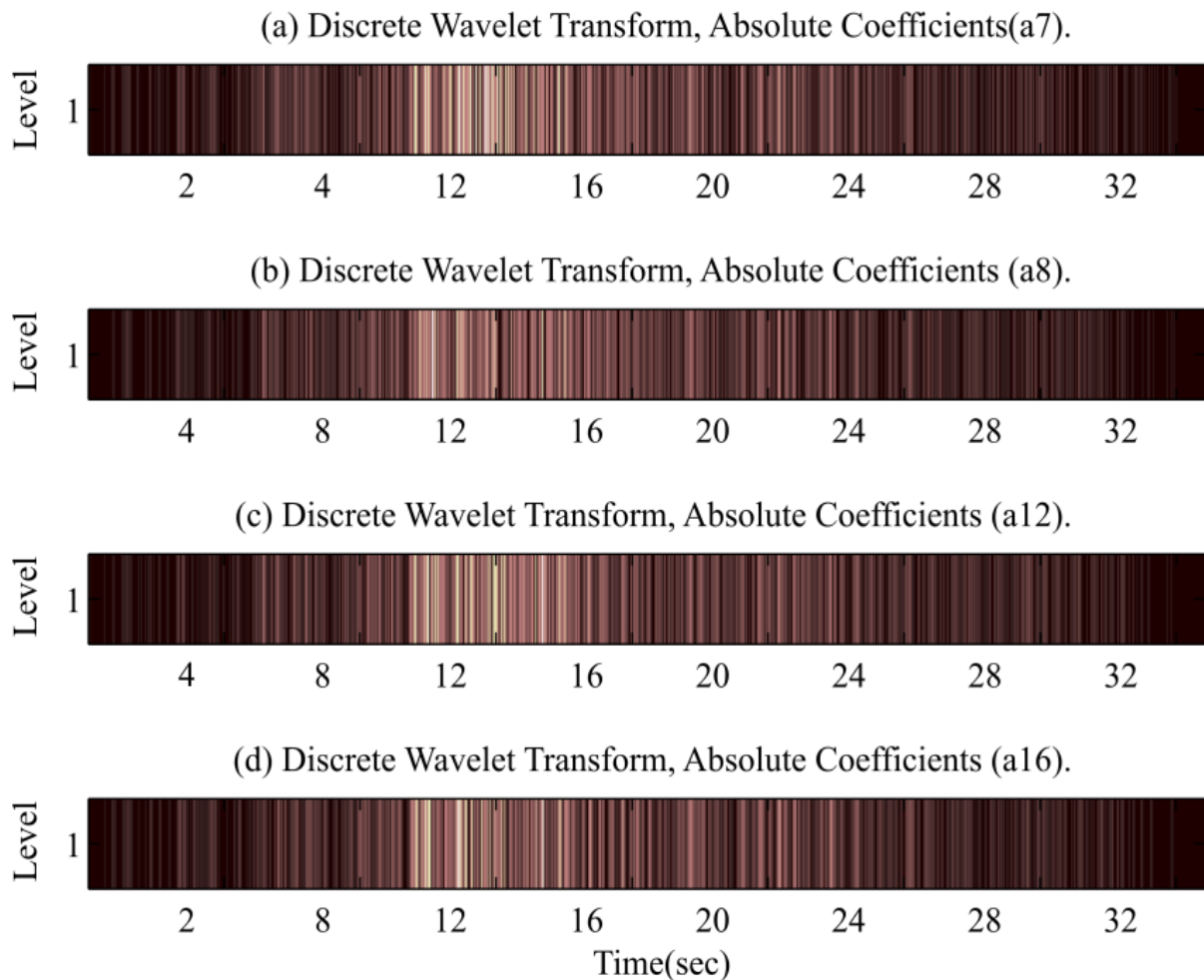
**Figure 8.** Schematic representation of the main damage in 3-bay, 4-story frame and nodal numbers and location of plastic hinge formation.



**Figure 9.** Normalized TABAS record with respect to its Peak Ground Acceleration (PGA).

**Table 2.** Dimension of plate girders.

Dimensions	Column (cm)	Beam (cm)
Outside Height	33	30
Top Flange width	20	20
Top Flange Thickness	1.5	1.2
Web Thickness	1	1
Bottom Flange Width	20	20
Bottom Flange Thickness	1.5	1.2

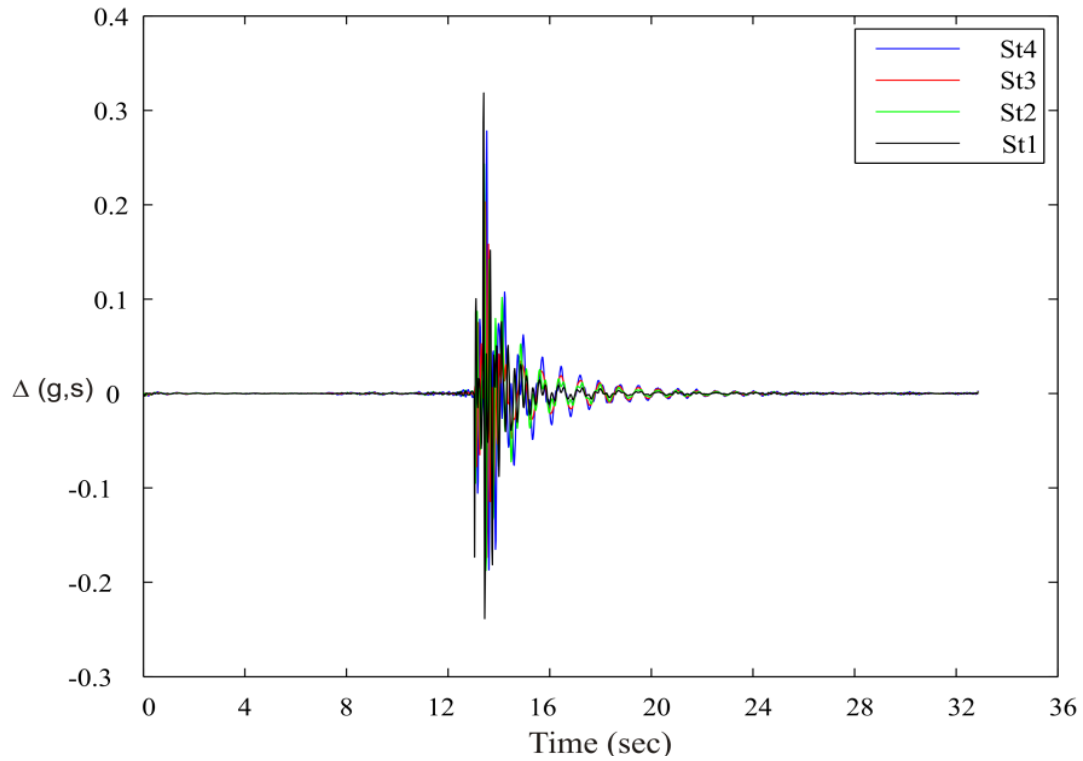


**Figure 10.** Wavelet analysis of horizontal acceleration for different stories (a) fourth storey (node 7) (b) third storey (node 8) (c) second storey (node 12) and (d) first storey (node 16) are shown. As the figures indicate, sudden changes are not visible in the wavelet decomposition level of the horizontal acceleration responses.

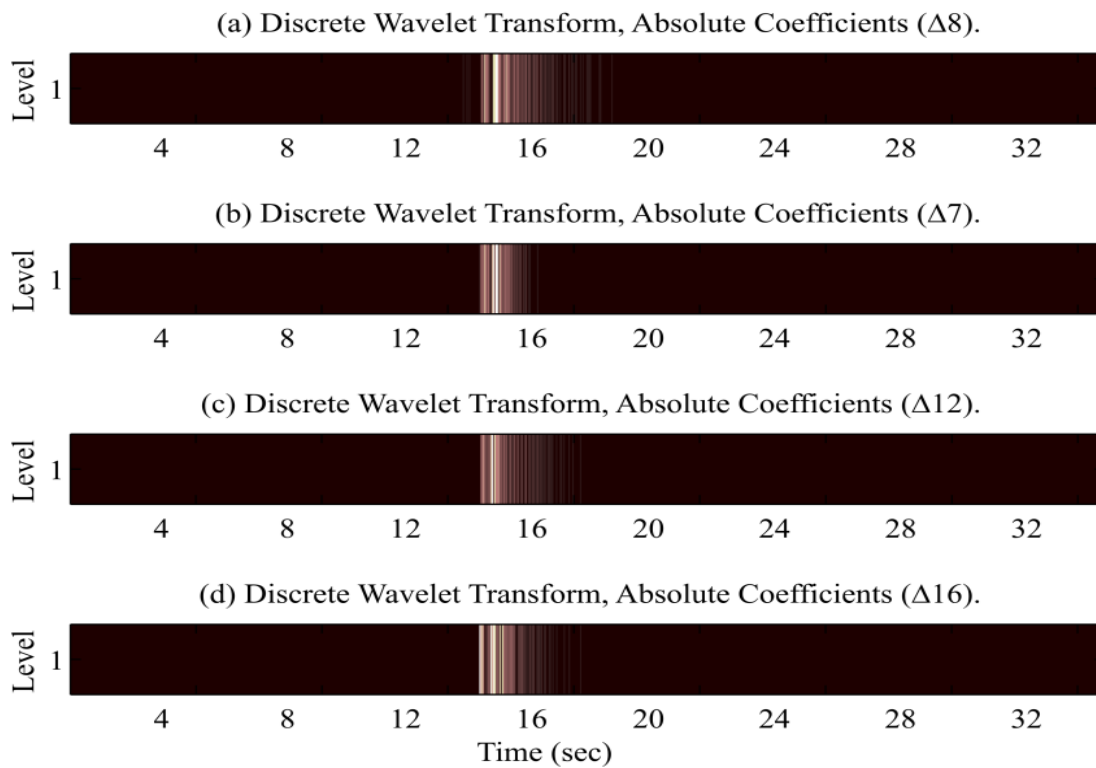
storey. Plastic hinges were obtained from inelastic dynamic analyses performed in OpenSees.

To show the advantage of the difference curve (Figure 11) compared with transitional response in damage detection, results of wavelet transform on difference curve functions and a horizontal response function for the above example is compared. Wavelet coefficients can be

plotted as shown in Figures 10 and 12. In these figures, the colour at each point is associated with the magnitude of the wavelet coefficients. Lighter colours correspond to the larger coefficients and darker colours to the smaller ones. These coefficients are based on a discrete wavelet analysis. As shown in Figure 12, the time of hinge formations were detected using the wavelet analysis of



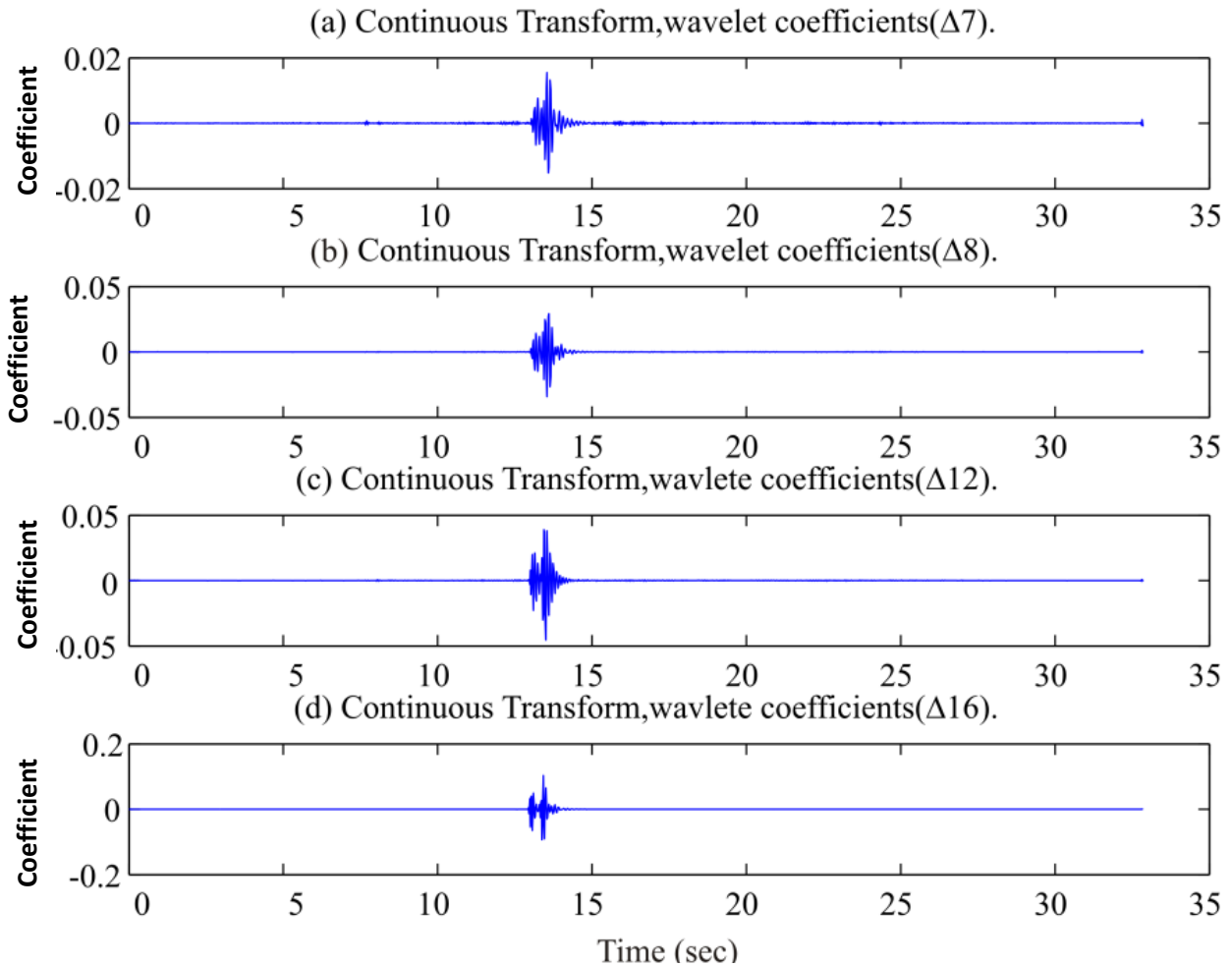
**Figure 11.** Difference between earthquake response and linear response (difference curve).



**Figure 12.** Wavelet analysis of  $\Delta \square$  function for different stories (a) fourth storey (node 7), (b) third storey (node 8), (c) second storey (node 12) and (d) first storey (node 16) are shown. As the figures indicate, the absolute coefficients, at initiation of damage ( $T_1=13$  s and  $T_2=13.2$  s), have suddenly changed.

**Table 3.** Wavelet coefficient (Figure13).

Storey	Time	
	T= 13 s	T= 13.2 s
St4	0.007	0.01
St3	0.01	0.03
St2	0.02	0.04
St1	0.06	0.1



**Figure 13.** Wavelet analysis of  $\Delta$  function for different stories (a) fourth storey (node 7) (b) third storey (node 8) (c) second storey (node 12) (d) first storey (node 16).

difference curve functions while using wavelet analysis of the horizontal acceleration response could not reveal the time of damage occurrence (Figure 10).

A comparison of the wavelet coefficients is presented in Figure 13 and Table 3. For different times and stories, damage location can be predicted as:

i. For T=13 s:  $WC_4 < WC_3 < WC_2 < WC_1 \rightarrow$  Damage in 1<sup>st</sup> Storey.

ii. For T=13.2 s:  $WC_4 < WC_3 < WC_2 < WC_1 \rightarrow$  Damage in 1<sup>st</sup> Storey.

### Example 3: 3-Storey steel frame

The second example given here is a 3-storey steel frame subjected to a distributed gravity load and a random excitation on its supports (Tabas earthquake, Figure 9). A

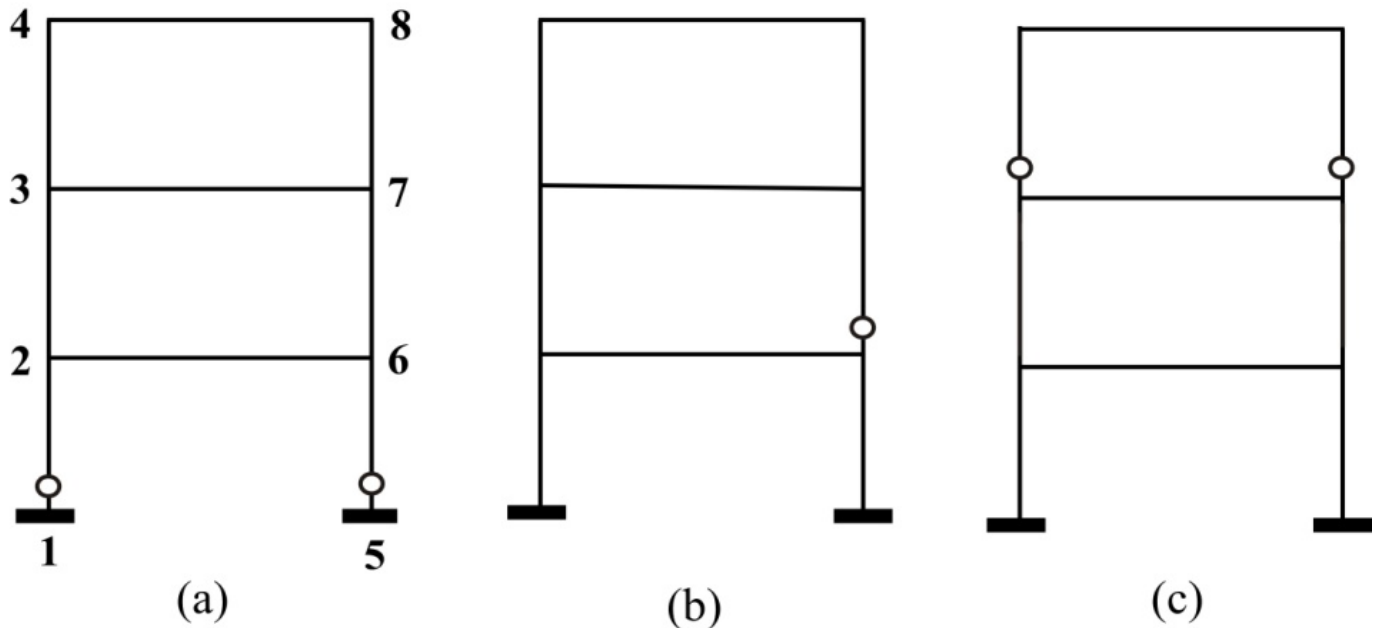


Figure 14. Node numbers and considered damage scenarios (a to c).

Table 4. Wavelet Coefficient (damage case a).

Storey	Time	
	T= 10.04 s	T= 11.76 s
St3	0.00015	0.00137
St2	0.00069	0.00839
St1	0.00609	0.04297

frame with  $L_{\text{column}}=3$  m and  $L_{\text{beam}}=5.0$  m has been designed using AISC-89 ASD (1989) and IPE330 for beams and 360 for columns steel profiles with a  $2400 \text{ Kg/cm}^2$  yield strength and ultimate strength of  $3500 \text{ kg/cm}^2$ . A uniform gravity load equal to  $2000 \text{ kg/m}$  is applied on the beams. This frame has been analysed considering different damage scenarios. These scenarios and node numbers are shown in Figure 14. The general format of the  $\Delta$ -curve obtained here is shown schematically in Figure 15. Each colour is associated with a storey and each one has two parts. One is zero part and the other is the non-zero part.

In the different damage cases presented, the location of damage is determined by comparing the Wavelet coefficients of  $\Delta$ -curve obtained for different stories.

#### Damage case (a)

For this case, as shown in Figure 14(a), the difference between linear and nonlinear time history response ( $\Delta$ -curve) is computed using finite element analysis, as given

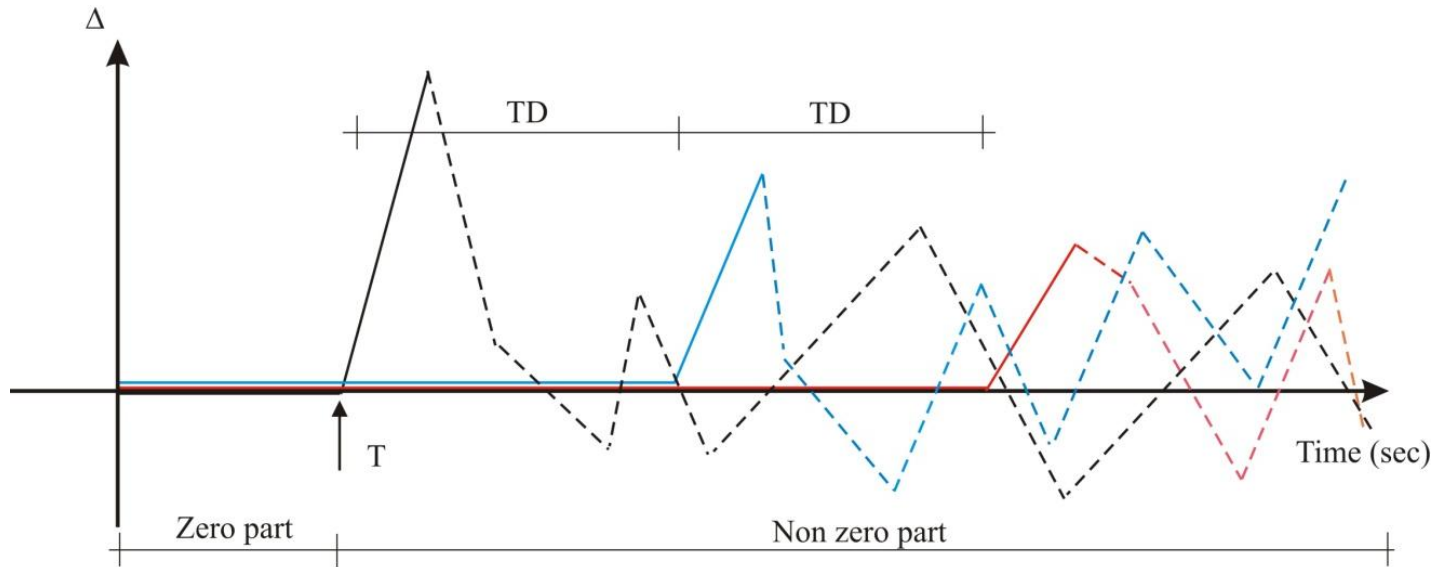
in Figure 16. In this example, the frame is subjected to Tabas excitation scaled to  $\text{PGA}=1.2 \text{ g}$ . There are two visible jumps in the wavelet analysis of  $\Delta$ -curves (Figure 17) and each one demonstrates a hinge formation. Time of first hinge formation is at  $T=10.04 \text{ s}$  and the second one is at  $T=11.76 \text{ s}$ , both of them having occurred at the first storey. Table 4 demonstrates numerical values and a comparison of different Wavelet Coefficients (WC) obtained for the damage case (a).

Comparing the wavelet coefficients is presented in Table 4. For different times and stories, damage location can be predicted as:

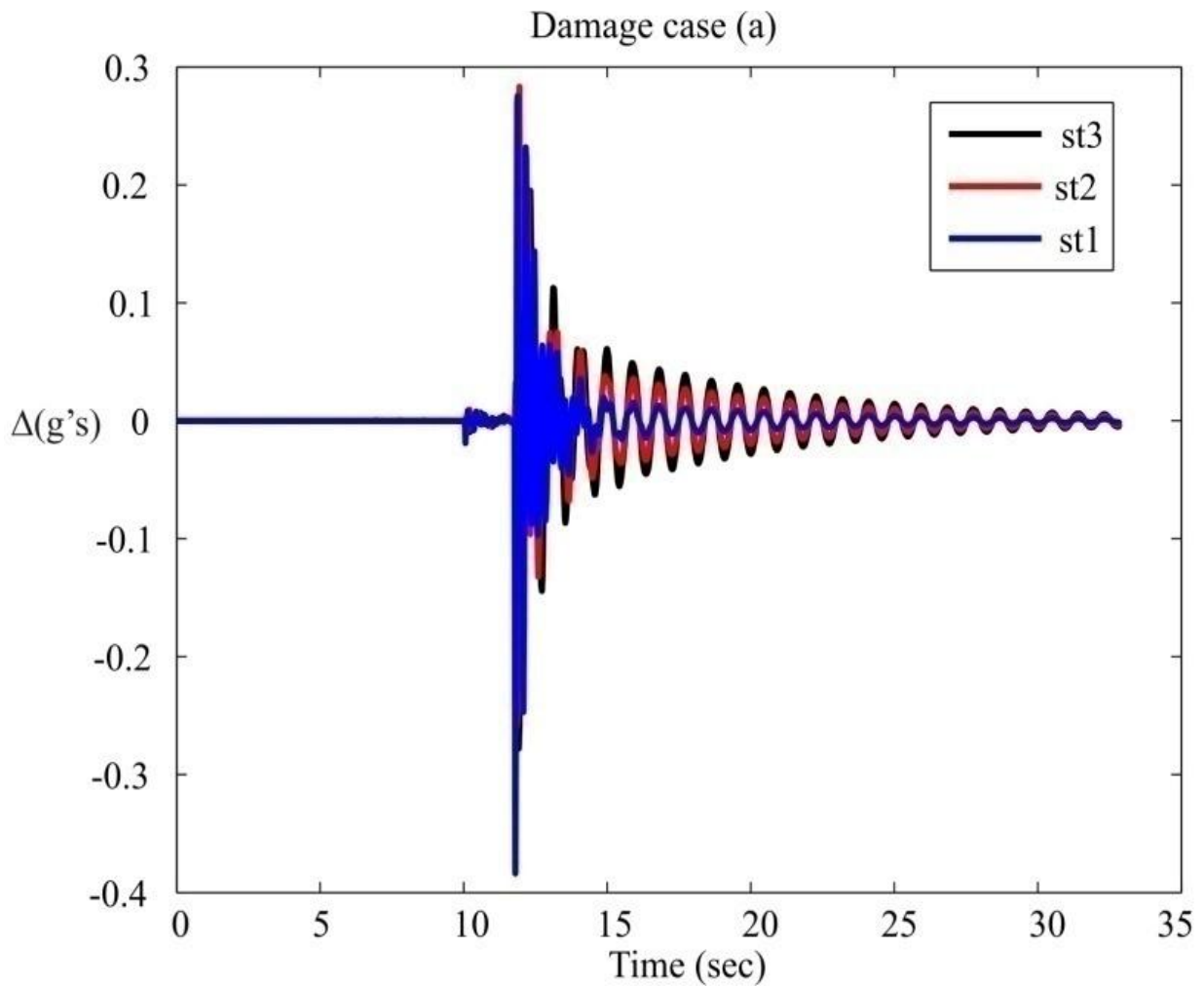
- For  $T=10.04 \text{ s}$ :  $\text{WC}_3 < \text{WC}_2 < \text{WC}_1 \rightarrow$  Damage in 1<sup>st</sup> Storey.
- For  $T=11.76 \text{ s}$ :  $\text{WC}_3 < \text{WC}_2 < \text{WC}_1 \rightarrow$  Damage in 1<sup>st</sup> Storey.

#### Damage case (b)

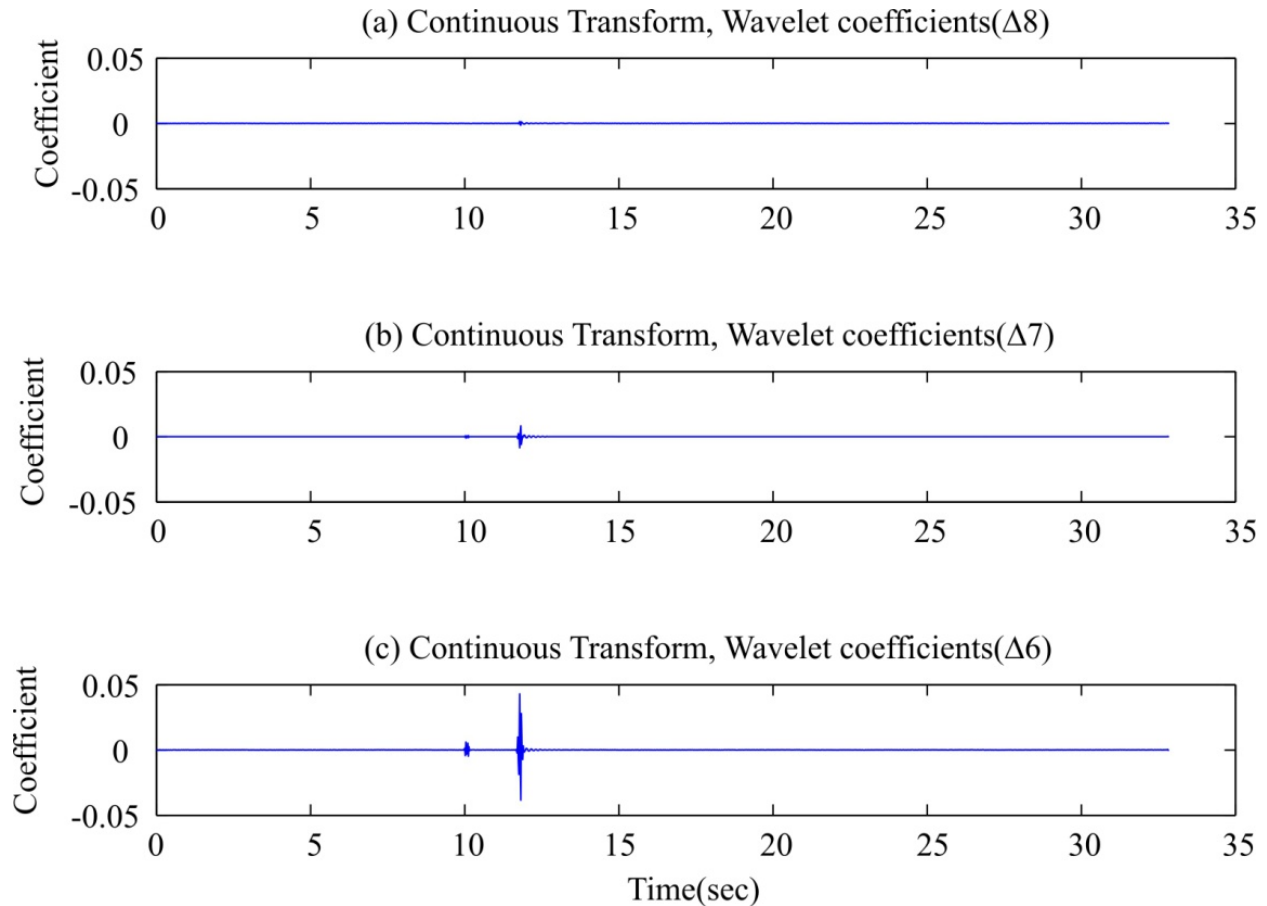
For damage case (b), the difference between linear and



**Figure 15.** Schematic difference between earthquake response and linear response ( $\Delta$ -curve) for 3-storey frames.



**Figure 16.** Difference between earthquake response and linear response (difference curve) damage case (a).



**Figure 17.** Wavelet analysis of  $\Delta \square$  function for different stories (a) third storey (node 8), (b) second storey (node 7) and (c) first storey (node 6).

nonlinear time history response is given in Figure 18. In this example, the frame is subjected to Tabas excitation scaled to  $PGA=0.2$  g. As shown in Figure 19, the time of the first hinge formation is at  $T=10.48$  s which has occurred at the second storey (Table 5).

As it can be seen (Figure 18), the difference between this case and the previous case is that the WC1 and WC2 are about the same. This means that the plastic hinge is formed on the second floor and is closer to the first floor. One can conclude that it has been formed at the lower end of the second floor columns.

Comparing the wavelet coefficients presented in Table 5. For different times and stories, damage location can be predicted as:

i. For  $T=10.48$  s:  $WC_3 < WC_2 \leq WC_1 \rightarrow$  Damage in 2<sup>nd</sup> Storey (closer first storey).

#### Damage case (c)

Figure 20 shows the difference between linear and nonlinear time history response for damage case (c). In

this example, the frame is subjected to Tabas excitation scaled to  $PGA=0.25$  g. It has two visible jumps and so the formation of two hinges can be concluded. As shown in Figure 21, the time of first hinge formation is at  $T=11.34$  s and the second one is at  $T=11.7$  s for which both hinges have occurred at the second storey (Table 6).

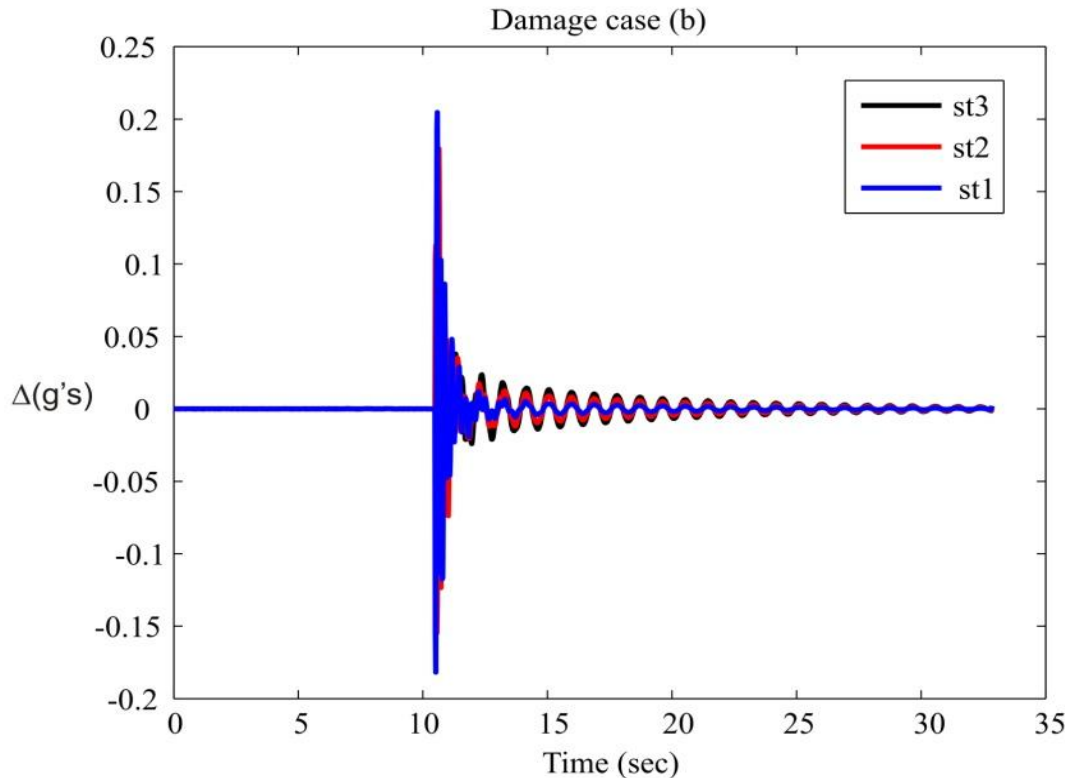
Comparing the wavelet coefficients presented in Table 6. For different times and stories, damage location can be predicted as:

i. For  $T=11.34$  s:  $WC_1 < WC_3 < WC_2 \rightarrow$  Damage in 3<sup>rd</sup> Storey (closer to second storey).

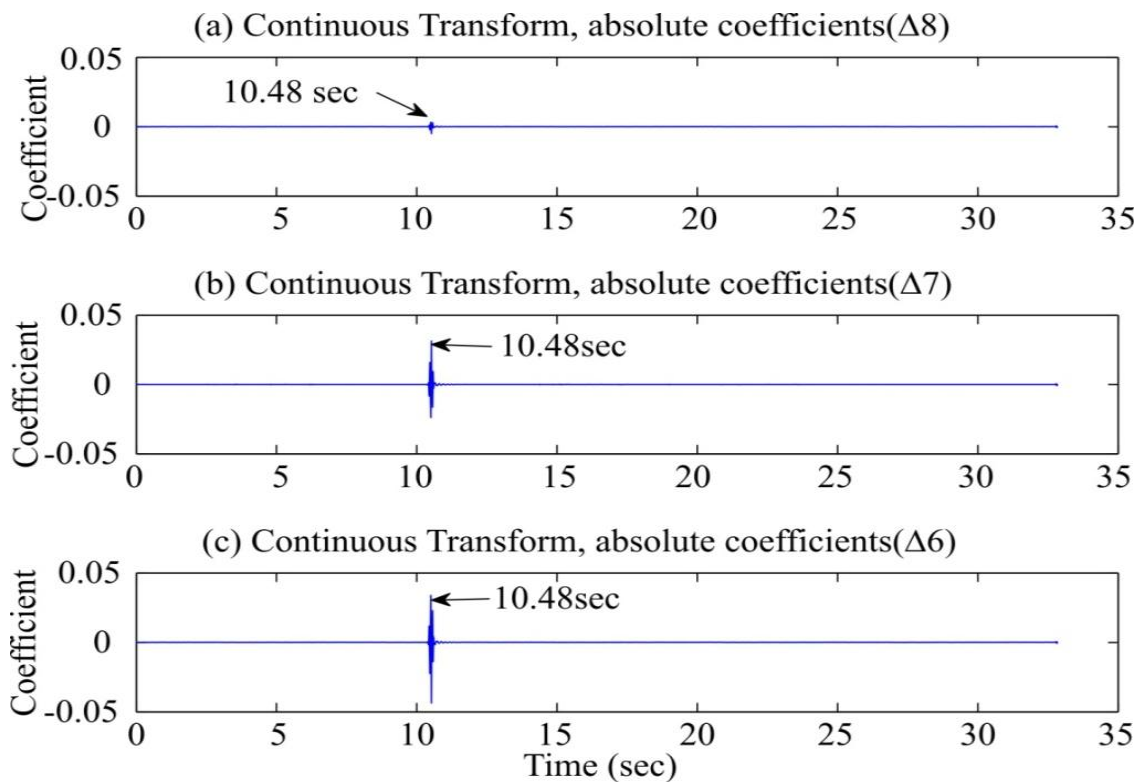
ii. For  $T=11.7$  s:  $WC_1 < WC_3 < WC_2 \rightarrow$  Damage in 3<sup>rd</sup> Storey (closer to second storey).

#### Example 4:3-Storey steel frame (Multiple Damage Location)

In this example, a 3-storey frame under Tabas earthquake is analysed as shown in Figure 22. This frame is subjected to excitation scaled to  $PGA=1.8$  g. It has been assumed that three plastic hinges at different



**Figure 18.** Difference between earthquake response and linear response ( $\Delta$ -curve) damage case (b).

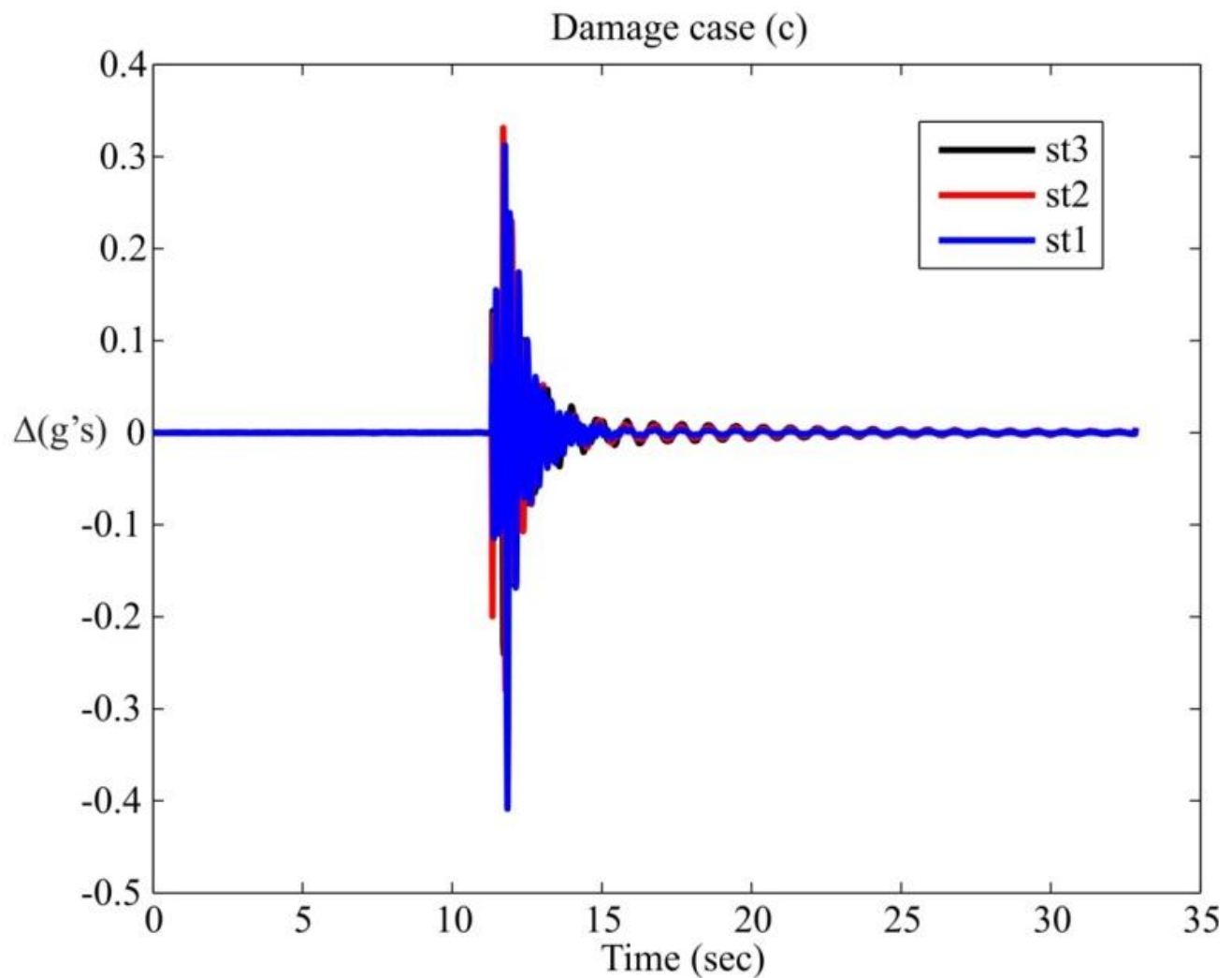


**Figure 19.** Wavelet analysis of  $\Delta$  function for different storeys (a) third storey (node 8) (b) second storey (node 7) (c) first storey (node 6).



**Table 5.** Wavelet Coefficient (damage case b).

Storey	Time T=10.48 s
St3	0.00507
St2	0.03129
St1	0.04366

**Figure 20.** Difference between earthquake response and linear response ( $\Delta$ -curve) damage case (c).

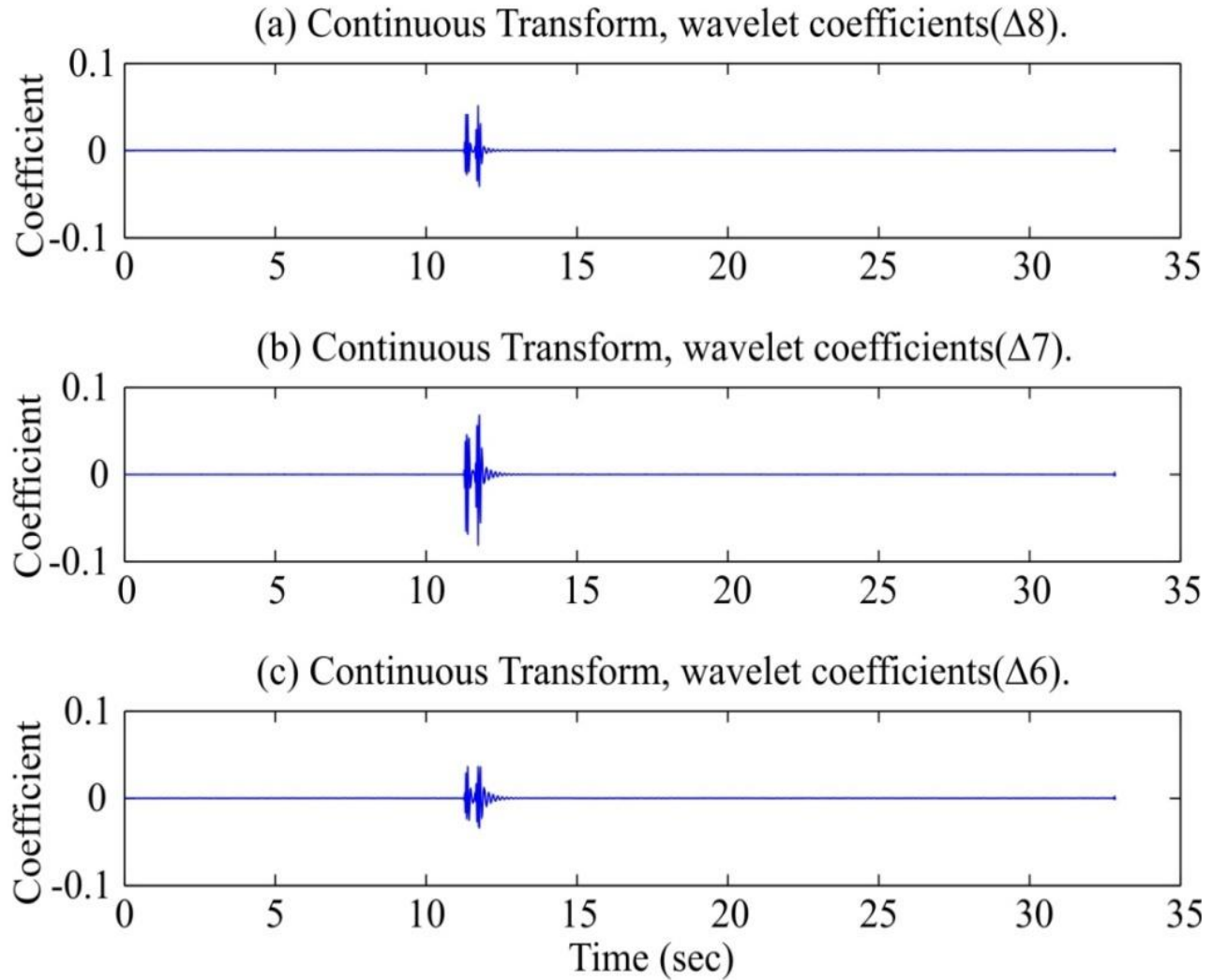
time and locations were formed.

Selection of the proper wavelet function is the first step in the wavelet analysis. The choice depends on the problem at hand and can have a considerable effect on the results. In this case, the morlet wavelet was used.

Absolute coefficients of continuous wavelet transform are shown in Figure 23. In this Figure, the lighter lines represent greater wavelet coefficients, i.e. location of damage is nearer. For example, Figure 23(a) represents

hinges formed at  $T=11$  s being closer to the level of node 8 at the roof (Figure 22). Meanwhile, the same line in Figure 23(c) is darker and it implies that damage is far from node 6 at the first storey. In case of difficulty in differentiation between colours, judgment can be based on values of the coefficients as shown in Figure 24. Table 7 summarises the wavelet coefficients obtained from Figure 24.

A comparison of the wavelet coefficients is presented in



**Figure 21.** Wavelet analysis of  $\Delta$  function for different stories (a) third storey (node 8) (b) second storey (node 7) (c) first storey (node 6).

**Table 6.** Wavelet coefficient (damage case c).

Storey	Time	
	T=11.34 s	T=11.7 s
St3	0.04109	0.05134
St2	0.06498	0.08117
St1	0.02915	0.03627

**Table 7.** For different times and stories, damage location can be predicted as:

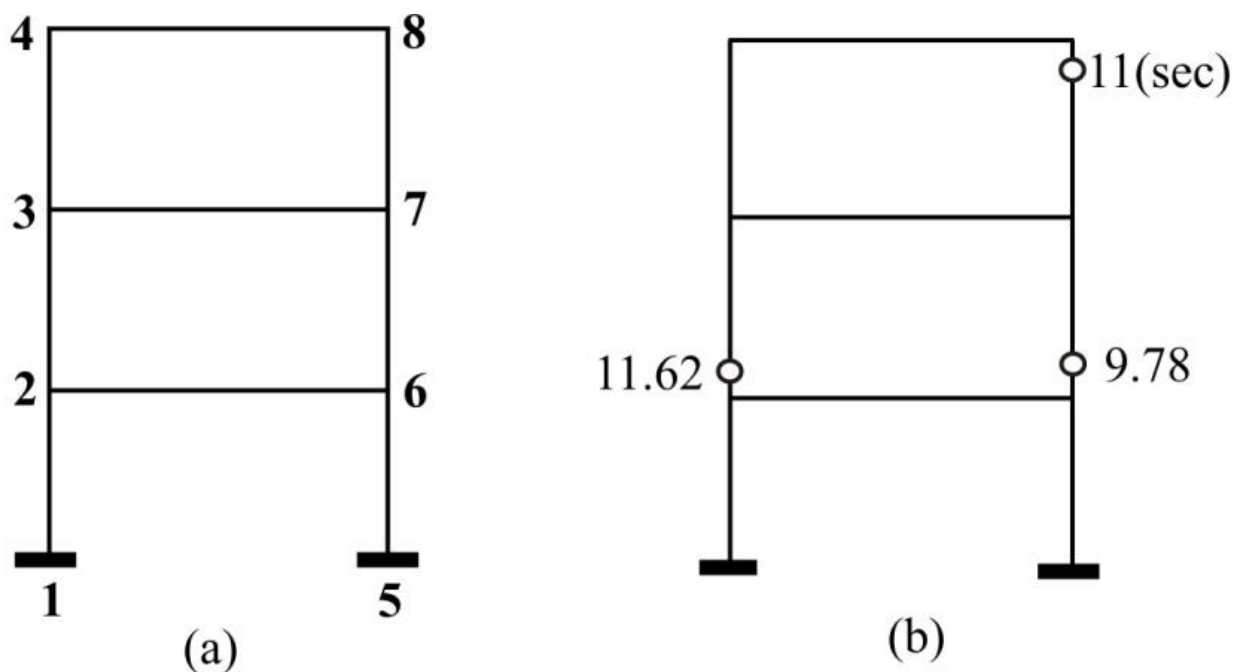
- For  $T=9.78$  s:  $WC_3 < WC_2 \leq WC_1 \rightarrow$  Damage in 2<sup>nd</sup> Storey (closer to first storey)
- For  $T=11.00$  s:  $WC_1 < WC_2 \leq WC_3 \rightarrow$  Damage in 3<sup>rd</sup> Storey (closer to third storey)

- For  $T=11.62$  s:  $WC_3 < WC_2 \leq WC_1 \rightarrow$  Damage in 2<sup>nd</sup> Storey (closer to first storey).

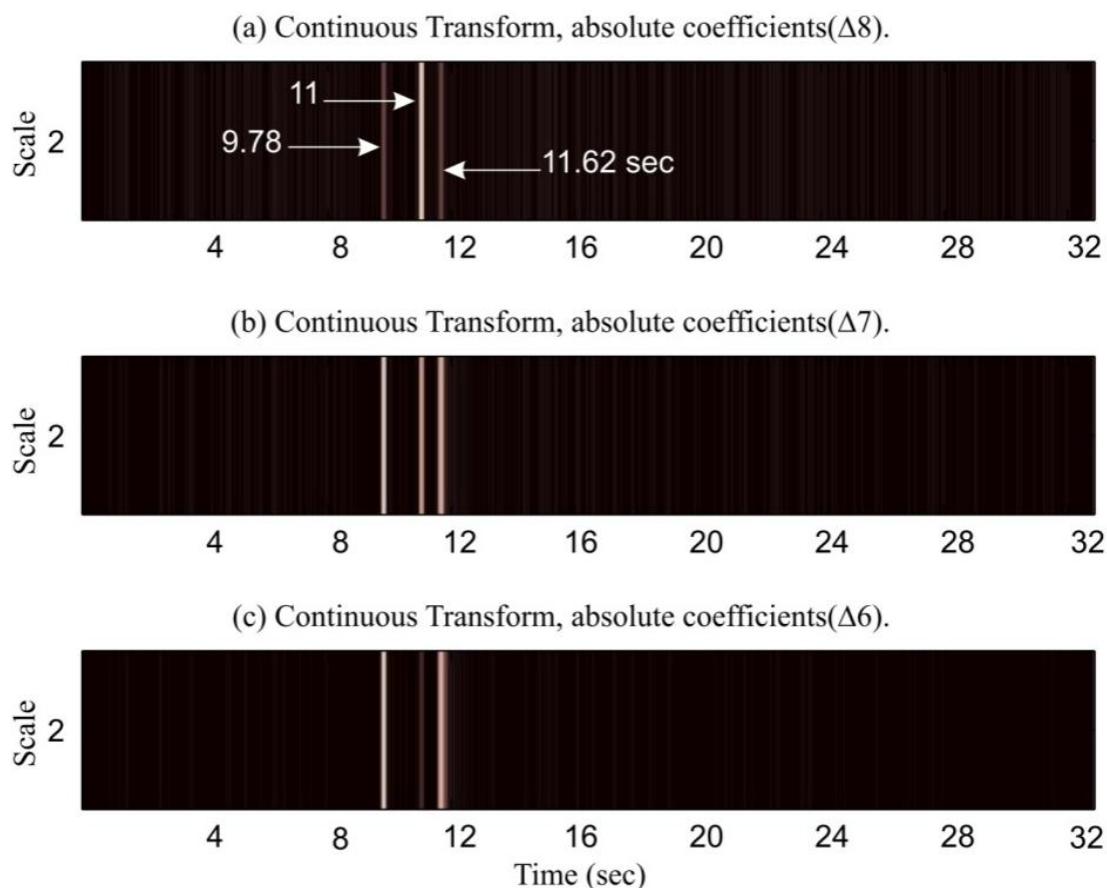
## DISCUSSION

As demonstrated in the previous examples, the proposed method has the ability to determine the time and location of damage regardless of the number of bays, number of stories and type of excitation. Wavelet transform of  $\Delta$ -curve has visible changes in time of hinge formation and this makes it possible to detect the time and location of damage. However, such information can not directly be extracted from the response of the stories.

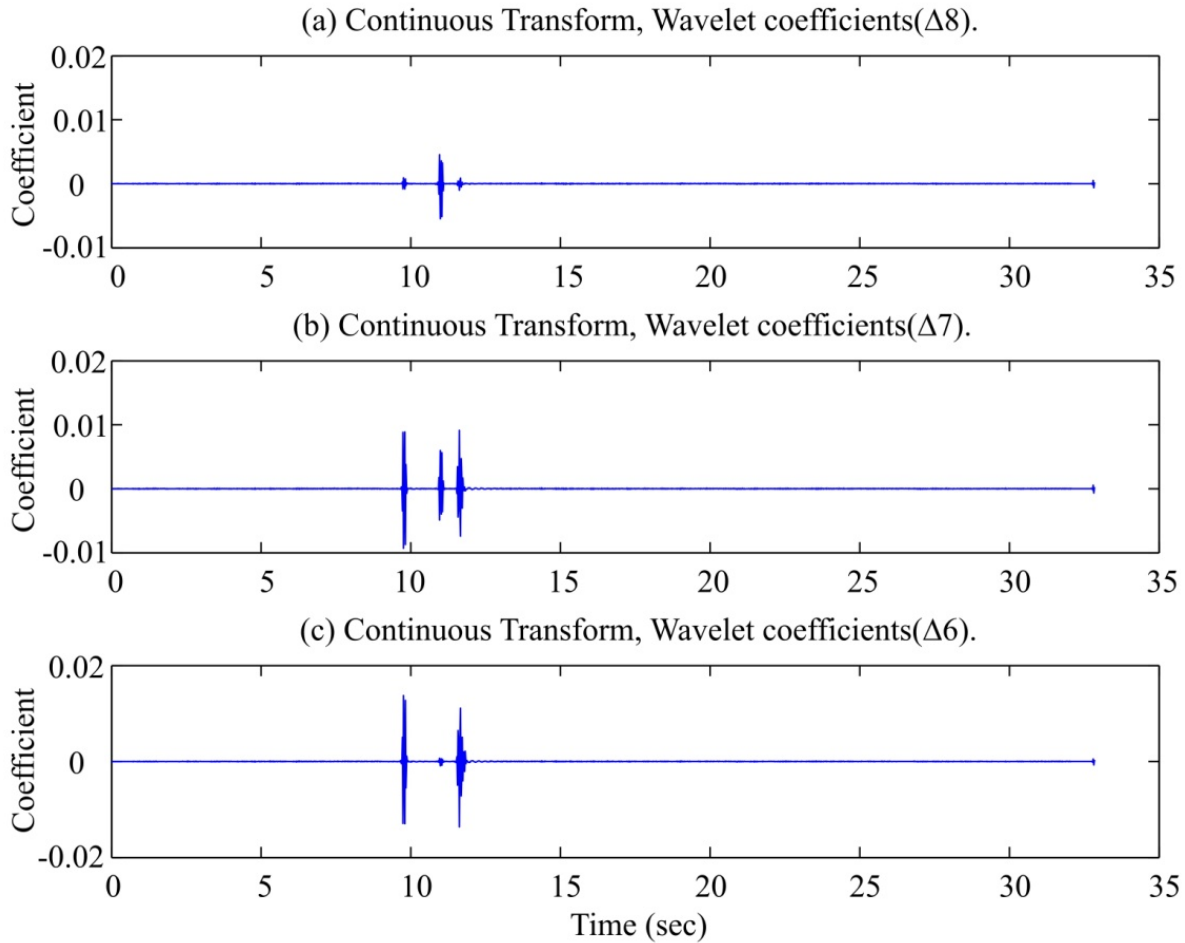
Even in cases where the boundary between the two parts of the difference function is not detectable, the wavelet can effectively makes it clear. In addition, the absolute value of wavelet coefficients is an appropriate criterion for determining the location of damage.



**Figure 22.** (a) Considered frame and node numbers (b) Time and location of plastic hinge formation.



**Figure 23.** Wavelet scalogram of  $\Delta \square$  function for different stories (a) third storey (node 8) (b) second storey (node 7) (c) first storey (node 6).



**Figure 24.** Wavelet analysis of  $\Delta \square$  function for different stories (a) third storey (node 8), (b) second storey (node 7) and (c) first storey (node 6).

**Table 7.** Comparison of wavelet coefficient for 3 storey steel frame.

Storey	Time		
	T=9.78 s	T=11 s	T=11.62 s
St3	0.00093	0.00546	0.00086
St2	0.00880	0.00507	0.00914
St1	0.01370	0.00068	0.01110

Moreover, inclusion of other structural parameters such as damping, type of structure (steel, concrete) and non-structural components may affect the difference curve. These issues have not been separately investigated here but this methodology has the ability to include their effects as well.

## Conclusion

System identification has several methods and the

processing of the output signal is one of the effective methods and is based on changes in system properties. These changes are sometimes visible in measuring space or transformed space. The proposed method uses selective parameters of damage and can appropriately determine the time and location of damage due to its sensitivity. By calculating the difference of linear and nonlinear response of a structure, a function can be found which contains useful information regarding the time and location of damage formation ( $\Delta$ -curve). Wavelet analysis of  $\Delta$ -curve is an effective method to

reveal the time and location of damage.

## REFERENCES

- AISC (1989). Specification for Structural Steel Buildings-Allowable Stress Design and Plastic Design. Am. Institute of Steel Construction. Chicago.
- Bisht SS (2005). Methods for Structural Health Monitoring and Damage Detection of Civil and Mechanical Systems. Thesis submitted to the faculty of the Virginia Polytech. Institute and State University in partial fulfilment of the requirements for the degree of Master of Science in Engineering Science and Mechanics. Blacksburg, Virginia.
- Campbell SD, Richard RM, Partridge JE (2008). Steel Moment Frame Damage Predictions Using Low-Cycle Fatigue. The 14th World Conference on Earthquake Engineering. Beijing, China.
- Farrar CR, Worden K, Todd MD, Park G, Nichols J, Adams DE, Bement MT, Farinholt KM (2007). Nonlinear System Identification for Damage Detection. Los Alamos National Lab., LA-14353.
- Federal Emergency Management Agency (FEMA)-356 (2000). Prestandard and commentary for the seismic rehabilitation of buildings. Washington DC.
- Hui Cao, Hong Yang, Friswell MI, Shaoliang Bai (2004). The Analysis of earthquake waves based on nonlinear response of RC structures. Proceedings of ESDA04 7th Biennial conference on engineering system design and analysis. Manchester, United Kingdom, pp. 19-22.
- Jankovic S, Stojadinovic B (2008). Determining Inter-storey Drift Capacity of R/C Frame Building. 14th World Conference on Earthquake Engineering. Beijing, China.
- Lynn PA, Fuerst W, Thomas B (1997). Introductory Digital Signal Processing with Computer Applications. John Wiley.
- Mallat S, Hwang W L (1992). Singularity detection and processing with wavelets. IEEE Trans. Info. Theory, 38(2): 617-643.
- McKenna F, Fenves GL, Scott MH (2000). An object-oriented software for earthquake engineering simulation. Univ. of California, Berkeley, California, <http://opensees.berkeley.edu/>.
- Misiti M, Misiti Y, Oppenheim G, Poggi J (2007). Wavelets and Their Application. ISTE Ltd. UK.
- Ovanesova AV, Suarez LE (2004). Applications of Wavelet Transforms to Damage Detection in Frame Structures. Eng. Struct., 26: 39-49. DOI:10.1016/j.engstruct.2003.08.009.
- Powell GH, Allahabadi R (1987). Seismic Damage Prediction by Deterministic Methods: Concepts and Procedures. Earthquake Eng. and Structural Dynamics. 16: 719-734. DOI: 10.1002/eqe.4290160507.
- Prieto GA, Lawrence JF, Chung AI, Kohler MD (2010). Impulse response of civil structures from ambient noise analysis. Bull. Seismol. Soc. Am. 100: 2322-2328. DOI:10.1785/0120090285.
- Raghu Prasad B K, Lakshmanan N, Muthumani K, Gopalakrishnan N (2006). Enhancement of Damage Indicators in Wavelet and Curvature Analysis. Sadhana, 31: 463-486.
- Raufi F, Bahar O (2010). Damage detection in 3-storey moment frame building by wavelet analysis. 14th European conference on Earthquake Eng. Ohrid, Macedonia.
- Safak E, Hudnut K (2006). Real-Time Structural Monitoring and Damage Detection by Acceleration and GPS Sensors. 8th US National Conference on Earthquake Eng. San Francisco, California.
- Shanshan W, Qingwen R, Pizhong Q (2006). Structural Damage Detection Using Local Damage Factor. DOI: 10.1177/1077546306068286.
- Snieder R, Safak E (2006). Extracting the building response using seismic interferometry. Theory and application to the Millikan library in Pasadena. California Bull. Seismol. Soc. Am., 96: 586-598.
- Todorovska MI, Trifunac M D (2008). Earthquake damage detection in structures and early warning. 14th World Conference on Earthquake Eng. Beijing, China.
- Todorovska MI, Trifunac MD (2009). Earthquake damage detection in the Imperial County Services Building II. *Analysis of novelties via wavelets*. Struct. Control Health Monit. 17:895-917. DOI: 10.1002/stc.350.
- Todorovska MI, Trifunac MD (2005). Structural Health Monitoring by Detection of Abrupt Changes in Response Using Wavelets: Application to a 6-storey RC Building Damaged by an Earthquake Proc. 37th Joint Panel Meeting on Wind and Seismic Effects. Japan, U.S.-Japan Natural Resources Program (UJNR). p. 20.
- Zabel V (2005). A Wavelet-based Approach for Damage Detection on Civil Engineering Structures. Proceedings of the 23rd International Modal Analysis Conference (IMAC XXIII). Orlando, Florida, USA. Paper no. 178.