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Fuzzy multi-objective linear programming for traveling salesman problem

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Traveling Salesman Problem (TSP) is an important problem in Artificial Intelligence and Operations Research domain. The problem has been investigated under different headings and solved with various approaches including soft computing and linear programming. The conventional linear programming deals with crisp parameters. However, information available in real life system is of vague, imprecise and uncertain nature. The impreciseness and uncertainty aspects are handled using Fuzzy Sets to obtain optimal solutions. Multi-Objective Linear Programming effectively deals with flexible aspiration levels or goals. Fuzzy Multi-Objective Linear Programming enhances the effectiveness of solutions with acceptable solutions through fuzzy constraints. In this work, Fuzzy Multi-Objective Linear Programming is used for solving TSP with vague and imprecise parameters. An example of TSP with multiple objectives and imprecise parameters is also discussed.

Key words: Traveling salesman problem, fuzzy multi-objective linear programming, multiple objectives, vague parameters, aspiration levels.

INTRODUCTION

Traveling Salesman Problem (TSP) is well-known NP-hard combinatorial optimization problem. It represents the class of problems in which least-cost sequence is found for visiting set of cities, starting and ending at the same city such that each city is visited exactly once. The desire of decision maker where least time span or distance is significant formulates TSP as Multi-Objective Problem. Considering TSP as Multi-Objective Optimization Problem, each objective function is represented in distinct dimension. To decide Multi-Objective TSP in optimality means determining $k-$dimensional points pertaining to space of feasible solutions of problem and minimum possible values according to all dimensions.

The permissible deviation from specified value of structural dimension is also considerable because salesman can face a situation in which objectives are achieved completely. There are a set of alternatives from which he selects one that best meets his aspiration levels. Conventional programming approach does not deal with

this situation. Branch and Bound approach was used to solve TSP with two sum criteria (Fischer and Richter, 1982). For max ordering TSP, 2-opt and 3-opt heuristics was used (Gupta and Warburton, 1986). Sigal (1994) proposed Decomposition approach for solving TSP with respect to two criteria of route length and bottlenecks where both objectives are obtained from same cost matrix.

Branch and Bound method with multiple labeling schemes was used to keep track of possible Pareto optimal tours (Tung, 1994). An E-constrained based algorithm for Bi-Objective TSP was suggested by Melamed and Sigal (1997). An Approximation algorithm with worst case performance bound was proposed by Ehrgott (2000). Hansen (2000) applied Tabu Search algorithm to Multi-Objective TSP. Borges and Hansen (2000) used weighted sums to study global convexity for Multi-Objective TSP. Jaszkiewicz (2002) proposed Genetic Local Search which combines ideas from Evolutionary Algorithms, Local Search with modifications of aggregation of Objective functions. Two Phase Local Search procedures to tackle Bi-Objective TSP was proposed by Paquete and Stützle (2003). During first

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phase, a good solution to one single objective is found by using an effective single objective algorithm. This solution provides starting point for second phase in which Local Search algorithm is applied to sequence of different aggregations of objectives, where each aggregation converts Bi-Objective problem into single objective one. Yan et al. (2003) used an Evolutionary Algorithm to solve Multi-Objective TSP. Angel et al. (2004) proposed a Dynamic Search algorithm which uses Local Search with an exponential sized neighborhood that can be searched in polynomial time using Dynamic Programming and Rounding technique. Paquete et al. (2004) suggested Pareto Local Search method which extended Local Search algorithm for single objective TSP to Bi-Objective case. This method uses an archive to hold non-dominated solutions found in the search process. Furthermore, in TSP salesman takes decision of selecting an optimal and feasible route between any couple of cities on basis of expected measures. In most real world problems it is not possible to have all constraints and resources in exact form rather they are in expected or vague form.

This leads to use of Fuzzy Logic which enables us to emulate human reasoning process and make decisions based on vague or imprecise data. Fuzzy Programming gives methodology of solving problems in Fuzzy environment. An ideal solution method would solve every TSP problem to optimality, but this is not practical in most large problems. While advances have been made in solving TSP, these advances have been obtained at expense of intractable and complex nature of solutions. It is required to meet aspiration level of decision maker under which current optimal solution remains still optimal and feasible.

In this work a paradigm is developed which deals with vague parameters and achieve certain aspiration level of optimality for Multi-Objective Symmetric TSP by transforming it into a Linear Program using Fuzzy Multi-Objective Linear Programming (FMOLP) technique. The route selection of problem is done by exploiting aspiration level parameters. The decision maker introduces tolerances to accommodate vagueness.

By adjusting tolerances, range of solutions with different aspiration level are found from which decision maker chooses one that best meets his satisfactory level within given domain. This paper discuses, the concept of Fuzzy programming, as well as illustrates FMOLP for TSP. A simulation example is given and conclusions are presented.

FUZZY PROGRAMMING

Fuzzy Multi-Objective Linear programming is illustrated to derive algorithm of TSP. It is based on the concept of Fuzzy Logic and Multi-Objective Linear Programming which is discussed in the next two subsections.

Fuzzy membership functions

Fuzzy Logic introduced by Zadeh (1965) is an extension of conventional two state logic which has been used to handle partial truth; that is, truth values between completely true and completely false. This logic underlies modes of reasoning which are approximate in nature. Linguistic term better represents subjective viewpoint of decision makers in more intuitive way and natural language format. The significance of Fuzzy Logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature. Fuzzy Sets use linguistic variables rather than quantitative variables to represent imprecise concepts. A membership function of Fuzzy Set viz., Fuzzy Membership function is mapped on interval $[0,1]$ which is an arbitrary grade of truth. The notation for Fuzzy Membership function $\mu_A(X)$ of set A is

$$\mu_A : X \rightarrow [0,1].$$

Multi-objective linear programming

The first formal representation of Linear Programming Problem and an efficient technique for solving it was developed by Dantzig (1997). The general Linear Programming model for maximization problem proposed by Dantzig is given by,

$$\max Z = \sum_{i=1}^{n} c_i x_i \tag{1}$$

Subject to $\displaystyle\sum_{i=1}^{n} a_{ij} x_{ij} \leq b_i$; $j = 1, ..., m$

$x_i \geq 0$

Where, Z is objective function, x_i are decision variables, m is number of constraints, n is number of decision variables and b_i are given resources. Linear Programming model is solved by different methods such as Graphical method, Simplex method etc.

Linear Programming is limited by the fact that it can deal only with single objective function and does not incorporate soft constraints. Multi-Objective Linear Programming is an extension of Linear Programming. It was introduced by Zeleny (1974). A general Linear Multiple Criteria Decision Making model can be represented as follows:

Find a vector X such that $X^T = [x_1, ...x_n]$ which

maximize k objective functions with n variables and m constraints as:

$$\max \ Z_i = \sum_{j=1}^{n} c_{ij} x_j \ ; \ i = 1,........,k \tag{2}$$

Subject to $\sum_{i=1}^{n} a_{ij} x_i \le b_j \ ; \ j = 1,........,m$

Where, c_{ij}, a_{ij} and b_j are given crisp values. In precise form, multiple objective problems can be represented by following Multi-Objective Linear Programming model:

$$optimize \ Z = CX \tag{3}$$

Subject to $AX \le b$

Where, $Z = [z_1,........,z_n]$ is vector of objectives, C is $K \times N$ matrix of constants and X is $N \times 1$ vector of decision variables, A is $M \times N$ matrix of constants and b is $M \times 1$ vector of constants.

Fuzzy multi-objective linear programming

The concept of decision making in Fuzzy environment involving several objectives was first proposed by Bellman and Zadeh (1970). Zimmerman (1978) applied their approach to vector maximum problem by transforming FMOLP Problem to single objective linear program. Considering the following Multi-Objective Linear Programming model,

$$\max \ Z = CX \tag{4}$$

Subject to $AX \le b$

Adopted Fuzzy model by Zimmerman is given by,

$$\max \ Z^0 \ \widetilde{\le} \ CX \tag{5}$$

Subject to $AX \ \widetilde{\le} \ b$

Where, $Z^0 = [z_1^0,........,z_n^0]$ are goals or aspiration levels; $\widetilde{\ge}$ and $\widetilde{\le}$ are fuzzy inequalities that are fuzzifications of $\ge$ and $\le$ respectively. For measurement of satisfaction levels of objectives and constraints Zimmerman suggested simplest type of Membership function given by,

$$\mu_{1k}(C_k X) = \begin{cases} 0 & if \\ 1 - (z_k^0 - C_k X)/t_k & if \\ 1 & if \end{cases}$$

$$C_k X \le z_k^0 - t_k$$
$$z_k^0 - t_k \le C_k X \le z_k^0 \quad k = 1,............,n \tag{6}$$
$$C_k X \ge z_k^0$$

t_k Represent admissible violation for objective z_k which is decided by decision maker. Zimmerman discussed membership function for maximizing objective function. In case of minimizing objective function, Fuzzy Membership function is,

$$\mu_{1k}(C_k X) = \begin{cases} 0 & if \\ 1 - (C_k X - z_k^0)/t_k & if \\ 1 & if \end{cases}$$

$$C_k X \ge z_k^0 + t_k$$
$$z_k^0 \le C_k X \le z_k^0 + t_k \quad k = 1,............,n \tag{7}$$
$$C_k X \le z_k^0$$

Another class of Fuzzy Membership functions suggested by Zimmermann has $\mu_{2i}(a_i X)$ for i^{th} constraint as follows:

$$\mu_{2i}(a_i X) = \begin{cases} 0 & if \ a_i X \ge b_i + d_i \\ 1 - (a_i X - b_i)/d_i & if \ b_i \le a_i X \le b_i + d_i \\ 1 & if \ a_i X \le b_i \end{cases}$$

$$i = 1,......... ..., m \tag{8}$$

d_i is admissible violation for fuzzy resource b_i for i^{th} constraint. These Membership functions express satisfaction of decision maker so they must be maximized. As a result objective function becomes,

$$\max_X (\mu_{11}(C_1 X),..............,\mu_{1k}(C_k X),\mu_{21}(a_1 X),..............\mu_{2m}(a_m X)) \tag{9}$$

According to Fuzzy Sets, membership function of intersection of any two or more sets is minimum Membership function of these sets. By virtue of this objective function becomes:

$$\min(\mu_{11}(C_1 X),............,\mu_{1k}(C_k X),\mu_{21}(a_1 X),............\mu_{2m}(a_m X)) \tag{10}$$

From above representation we have,

$$\max imize\ CX \stackrel{\sim}{\geq} Z^0 \tag{11}$$

Subject to:

$$\stackrel{\sim}{\alpha} \stackrel{\sim}{\leq} 1-(z_k^0 - C_k X)/t_k; k=1,...............,n$$

$$\stackrel{\sim}{\alpha} \stackrel{\sim}{\leq} 1-(a_i X - b_i)/d_i; i=1,................,m$$

$$\stackrel{\sim}{\alpha} \geq 0, X \geq 0$$

Where, $\stackrel{\sim}{\alpha}$ is overall satisfaction level achieved with respect to solution.

FUZZY MULTI-OJECTIVE LINEAR PROGRAMMING APPROACH FOR TSP

The most frequently considered objective of TSP is to determine an optimal order for traveling all cities so that total cost is minimized. Consider the situation when decision maker has to determine optimal solution of TSP with minimized cost, time and overall distance. The individual objective functions can be formed for all objectives of decision maker. Let x_{ij} be the link from city i to j and

$$x_{ij} = \begin{cases} 1, city(i) \rightarrow city(j) \\ 0, otherwise \end{cases} \tag{12}$$

Let c_{ij} be the cost of traveling from city i to j; overall cost of particular route is sum of costs on links comprising the route. Since, decision maker has to minimize overall traveling cost the goal can be set for total estimated cost of entire route for TSP denoted by z_1^0. But there can be situations when estimated cost doesn't meet and so decision maker can set tolerance for estimated cost. Denoting tolerance against this goal as t_1, objective function for minimization of cost is given as follows:

$$z_1 : \min \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \stackrel{\sim}{\leq} z_1^0 \tag{13}$$

Let d_{ij} be the distance from city i to j and z_2^0 be the corresponding aspiration level for objective function for minimization of distance and t_2 be tolerance, then objective function takes following form:

$$z_2 : \min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \stackrel{\sim}{\leq} z_2^0 \tag{14}$$

Let t_{ij} be the time spent in traveling from city i to j and z_3^0 be the corresponding aspiration level for objective function for minimization of total time and t_3 be tolerance. The objective function is written as follows:

$$z_3 : \min \sum_{i=1}^{n} \sum_{j=1}^{n} t_{ij} x_{ij} \stackrel{\sim}{\leq} z_3^0 \tag{15}$$

One important aspect is dependency of objective functions on each other. They are mostly dependent, but determining exact form of dependency is complex process. The proposed framework works in all cases, if there is some feasible solution. These multiple objective functions are represented in vector form comprising multiple objectives with specified goals and tolerances. The membership functions are set for these individual objective functions to check their level of acceptability. A restriction is imposed that every city should be visited from exactly one of its neighboring city and vice versa, that is,

$$\sum_{i=1}^{n} x_{ij} = 1, \forall j \tag{16}$$

$$\sum_{j=1}^{n} x_{ij} = 1, \forall i \tag{17}$$

A route can not be selected more than once, that is, $x_{ij} + x_{ji} \leq 1, \forall i, j$ and non-negativity constraints $x_{ij} \geq 0$. These constraints collectively are expressed in vector form and fuzzy membership functions are defined for all objective functions. Finally, linear model is formulated using FMOLP model using TSP objective functions, constraints and their corresponding Membership functions. The model is solved by mixed Integer Linear Programming.

SIMULATION EXAMPLE

The proposed FMOLP approach for TSP is analyzed with Symmetric TSP, where salesman starts from his home city 0, visits three cities exactly once and comes back to his home city 0 by adopting route with minimum cost, time and distance covered. A map of cities to be visited is shown in Figure 1 and cities listed along with their cost, time and distance matrix in Table 1, where triplet (c, d, t) represents cost, distance and time parameters respectively for corresponding pair of cites.

Let links x_{ij} be decision variable of selection of nk (i, j) from city i to j. The objective functions z_1, z_2,
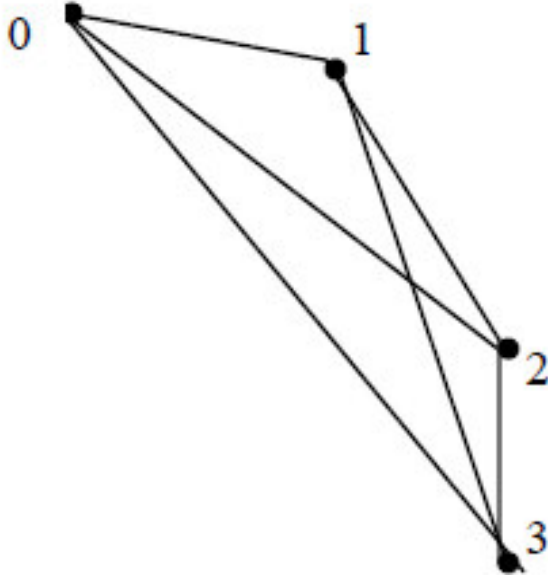
Figure 1. Symmetric traveling salesman problem.

Table 1. The matrix for time, cost and distance for each pair of cities.

City	0	1	2	3
	(c,d,t)	(c,d,t)	(c,d,t)	(c,d,t)
0	(000)	(20,5,4)	(15,5,5)	(11,3,2)
1	(20,5,4)	(000)	(30,5,3)	(10,3,3)
2	(15,5,5)	(30,5,3)	(000)	(20,10,2)
3	(11,3,2)	(10,3,3)	(20,10,2)	(000)

li z_3 are formulated for cost, distance and time, respectively. Their aspirations levels are set as 65, 16 and 11 by solving each objective function subject to given constraints in TSP and their tolerances are decided as 5, 2 and 1. The corresponding objective functions are as follows:

$$\min \begin{array}{l} z_1 = 20x_{01} + 15x_{02} + 11x_{03} + 20x_{10} + 30x_{12} + 10x_{13} + \\ 15x_{20} + 30x_{21} + 20x_{23} + 11x_{30} + 10x_{31} + 20x_{32} \tilde{\leq} 65 \end{array} \tag{18}$$

$$tolerence = t_1 = 5$$

$$\min \begin{array}{l} z_1 = 5x_{01} + 5x_{02} + 3x_{03} + 5x_{10} + 5x_{12} + 3x_{13} + \\ 5x_{20} + 5x_{21} + 10x_{23} + 3x_{30} + 3x_{31} + 10x_{32} \tilde{\leq} 16 \end{array} \tag{19}$$

$$tolerence = t_2 = 2$$

$$\min \begin{array}{l} z_1 = 4x_{01} + 5x_{02} + 2x_{03} + 4x_{10} + 3x_{12} + 3x_{13} + \\ 5x_{20} + 3x_{21} + 2x_{23} + 2x_{30} + 3x_{31} + 2x_{32} \tilde{\leq} 11 \end{array} \tag{20}$$

$$tolerence = t_3 = 1$$

The Fuzzy Membership function for cost, distance and time objective functions are illustrated below which are based on above equations.

$$\mu(z_1) = \begin{cases} 0 & if \ z_1 \geq 70 \\ 1 - (z_1 - 65)/5 & if \ 65 \leq z_1 \leq 70 \\ 1 & if \ z_1 \leq 65 \end{cases} \tag{21}$$

$$\mu(z_2) = \begin{cases} 0 & if \ z_2 \geq 18 \\ 1 - (z_2 - 16)/2 & if \ 16 \leq z_2 \leq 18 \\ 1 & if \ z_2 \leq 16 \end{cases} \tag{22}$$

$$\mu(z_3) = \begin{cases} 0 & if \ z_3 \geq 12 \\ 1 - (z_3 - 11)/1 & if \ 11 \leq z_3 \leq 12 \\ 1 & if \ z_3 \leq 11 \end{cases} \tag{23}$$

The FMOLP with max-min approach is given as follows:

$$\max imize \ CX \tilde{\geq} Z^0 \tag{24}$$

Subject to:

$$\tilde{\alpha} \tilde{\leq} 1 - (z_1 - 65)/5$$

$$\tilde{\alpha} \tilde{\leq} 1 - (z_2 - 16)/2$$

$$\tilde{\alpha} \tilde{\leq} 1 - (z_3 - 11)/1$$

$$x_{01} + x_{02} + x_{03} = 1 \tag{25}$$

$$x_{10} + x_{12} + x_{13} = 1 \tag{26}$$

$$x_{20} + x_{21} + x_{23} = 1 \tag{27}$$

$$x_{30} + x_{31} + x_{32} = 1 \tag{28}$$

$$x_{10} + x_{20} + x_{30} = 1 \tag{29}$$

$$x_{01} + x_{21} + x_{31} = 1 \tag{30}$$

$$x_{02} + x_{12} + x_{32} = 1 \tag{31}$$

Table 2. Solution of Fuzzy multi-objective linear programming problem.

Solution	z_1, t_1	z_2, t_2	z_3, t_3	α	Route
1	65,5	16,2	-	0.80	(x_{03}, x_{31}, x_{20})
2	65,5	16,2	11,1	-	No feasible solution
2	65,5	16,2	11,4	0.55	$(x_{03}, x_{31}, x_{12}, x_{20})$
3	65,5	16,2	11,5	0.62	$(x_{03}, x_{31}, x_{12}, x_{20})$

$$x_{03} + x_{13} + x_{23} = 1 \tag{32}$$

$$x_{01} + x_{10} \leq 1 \tag{33}$$

$$x_{02} + x_{20} \leq 1 \tag{34}$$

$$x_{03} + x_{30} \leq 1 \tag{35}$$

$$x_{12} + x_{21} \leq 1 \tag{36}$$

$$x_{13} + x_{31} \leq 1 \tag{37}$$

$$x_{23} + x_{32} \leq 1 \tag{38}$$

$$\tilde{\alpha} \geq 0, x_{ij} \geq 0$$

The above Fuzzy Linear Program and its variants are solved using MATLAB. As given in Table 2 only z_1 and z_2 are considered and z_3 is omitted; an optimal route with $\alpha = 0.8$ is obtained. When z_3 is also considered, solution becomes infeasible on these tolerances. Again by relaxing tolerance in z_3 to 4, solution becomes feasible. In this case, the optimal path is achieved with $\alpha = 0.55$. By increasing tolerance in z_3 from 4 to 5, an optimal solution with $\alpha = 0.62$ is obtained. These results show that by adjusting tolerance an optimal solution to Multi-Criteria TSP can be determined.

CONCLUSION

In this work, Symmetric TSP is analyzed as Fuzzy problem with vague and imprecise decision parameters. For Multi-Objective TSP in uncertain environment, route selection is done by exploiting these parameters. The tolerances are introduced by decision maker to accommodate this vagueness. By adjusting these tolerances, a range of solutions with different aspiration levels are obtained from which decision maker chooses one that best meets his satisfactory level within given tolerances. FMOLP technique achieves $k-$dimensional points according to decision maker's aspiration level in Multi dimensional solution space. There is a definite potential for further work on development of methods to solve TSP problems with vague description of resources using other techniques like Rough Sets. For efficient results, some heuristics are required such as relative dependencies among objective function.

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