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Full Length Research Paper

Classical and Bayesian estimation of parameters on the generalized exponentiated gamma distribution

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In this paper, we offer generalized exponentiated gamma distribution. We consider the maximum likelihood and Bayesian estimators of unknown parameters and propose Markov chain Monte Carlo (MCMC) method to generate samples from the posterior distribution. The mean square error of estimations is computed and comparisons are made using Monte Carlo simulation.

Key words: Maximum likelihood estimator, Bayesian estimator, squared error loss function, generalized exponentiated gamma distribution.

INTRODUCTION

The exponentiated gamma (EG) distribution has been introduced by Gupta et al. (1998) which has a probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the form, respectively;

$$f(x) = \theta x e^{-x}\left(1 - e^{-x}(1+x)\right)^{\theta-1} \quad ; \quad x > 0\,, \theta > 0 \tag{1}$$

$$F(x) = \left(1 - e^{-x}(1+x)\right)^{\theta} \quad ; \quad x > 0\,, \theta > 0 \tag{2}$$

Where θ is the shape parameter. Bayesian and non-Bayesian estimations on the EG distribution was discussed by Shawky and Bakoban (2008). Also order statistics from exponentiated gamma distribution and associated inference was discussed by Shawky and Bakoban (2009). For more information, Basu and Ebrahimi (1991), and Nassar and Eissa (2004) can be consulted.

Ghanizadeh et al. (2011) considered the classical estimation of the EG distribution parameters with presence of k outliers. Nasiri, and Pazira (2010) considered the Bayesian and non-Bayesian estimations of generalized exponential distribution in the presence outlier. In this paper, we consider generalized exponential distribution.

The generalized exponentiated gamma (GEG) distribution has the following p.d.f. and c.d.f., respectively;

$$f(x) = \theta\lambda^2 x e^{-\lambda x}\left(1 - e^{-\lambda x}(1+\lambda x)\right)^{\theta-1} \quad ; \quad x > 0\,, \theta > 0\,, \lambda > 0 \tag{3}$$

$$F(x) = \left(1 - e^{-\lambda x}(1+\lambda x)\right)^{\theta} \quad ; \quad x > 0\,, \theta > 0\,, \lambda > 0 \tag{4}$$

Where λ and θ are scale and shape parameters, respectively. When shape parameter is one f(x) is given by Gupta et al. (1998). Classical and Bayesian estimators are derived for the shape parameter in the case of type – II censored sample.

MAXIMUM LIKELIHOOD ESTIMATION

Suppose a type-II censored sample $\underline{x} = (x_1, x_2, \ldots, x_k)$ where x_i is ith order statistics. This sample are obtained and recorded from $GEG(\lambda,\theta)$ distribution with p.d.f. and c.d.f. given by (3) and (4), respectively. The likelihood function of the observed data is given by,

$$\ell(\underline{x};\lambda,\theta) = \theta^k \lambda^{2k} e^{-T}\left(1 - V^{\theta}\right)^{n-k} \tag{5}$$

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Where

$T = \sum_{i=1}^{k}(x_i - lnx_i - (\theta-1)lnu_i)$, $u_i = 1 - e^{-\lambda x_i}(1+\lambda x_i)$,
$V = 1 - e^{-\lambda x_k}(1+\lambda x_k)$

To estimation of parameter, we consider of $log\ell(\underline{x};\lambda,\theta)$ as,

$$log\ell(\theta) = klog(\theta) + 2klog(\lambda) - T + (n-k)log(1-V^{\theta})$$

$$\frac{\partial log\,\ell(\theta)}{\partial\theta} = \frac{k}{\theta} - \frac{dT}{d\theta} + (n-k)\frac{d(1-V^{\theta})}{1-V^{\theta}}$$

$$= \frac{k}{\theta} + \sum_{i=1}^{k} ln\left(1 - e^{-\lambda x_i}(1+\lambda x_i)\right) - (n-k)\frac{V^{\theta}log(V)}{1-V^{\theta}}$$

Hence,

$$\frac{k}{\theta} + \sum_{i=1}^{k} ln\left(1 - e^{-\lambda x_i}(1+\lambda x_i)\right) - (n-k)\frac{V^{\theta}log(V)}{1-V^{\theta}} = 0 \quad (6)$$

For $\lambda = 1$, it is given by Shawky and Bakoban (2008). To estimate of θ we can solve Equation (2.2) by Newton-Raphson method. Hence, solution of the equation is,

$$\theta_{i+1} = \theta_i - \frac{g(\theta_i)}{g'(\theta_i)}$$

Where,

$$g(\theta) = \frac{k}{\theta} + \sum_{i=1}^{k} ln\left(1 - e^{-\lambda x_i}(1+\lambda x_i)\right) - (n-k)\frac{V^{\theta}log(V)}{1-V^{\theta}}$$

$$g'(\theta) = -\frac{k}{\theta^2} - (n-k)\frac{V^{\theta}log(V).(1-V^{\theta}) + V^{2\theta}log(\theta)}{(1-V^{\theta})^2}$$

BAYESIAN ESTIMATION

Suppose that θ has the following gamma prior distribution, when λ is known,

$$\pi(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{\alpha-1}e^{-\beta\theta} \quad ; \quad \theta > 0, \alpha > 0, \beta > 0 \quad (7)$$

Appling Bayesian theorem and using Equation (5) and (7), the posterior density of θ as,

$$\pi(\theta|\underline{x}) = \frac{\ell(\underline{x};\lambda,\theta)\pi(\theta)}{\int_0^{\infty}\ell(\underline{x};\lambda,\theta)\pi(\theta)d\theta}$$

$$= \frac{\theta^{\alpha+k-1}e^{-\beta\theta}(1-V^{\theta})^{n-k}e^{-T}}{\int_0^{\infty}\theta^{\alpha+k-1}e^{-\beta\theta}(1-V^{\theta})^{n-k}e^{-T}d\theta}$$

The Bayes estimate ,$\hat{\theta}_B$, of θ under the squared error loss function given by,

$$\hat{\theta}_B = E(\theta^r) = \frac{I_r}{I_0}$$

Where,

$$I_r = \int_0^{\infty}\theta^{\alpha+k+r-1}e^{-\beta\theta}(1-V^{\theta})^{n-k}e^{-T}d\theta$$

Since $0 < 1 - V^{\theta} < 1$ for $x > 0$ by using the binomial series expression we have,

$$I_r = \sum_{t=0}^{n-k}\binom{n-k}{t}(-1)^t\int_0^{\infty}\theta^{\alpha+k+r-1}e^{-\beta\theta}V^{t\theta}e^{-T}d\theta$$

Let,

$$B = \sum_{i=1}^{k}(x_i - lnx_i + lnu_i)$$

So,

$$T = B - \theta\sum_{i=1}^{k} lnu_i$$

Hence,

$$I_r = \sum_{t=0}^{n-k}\binom{n-k}{t}(-1)^t\int_0^{\infty}\theta^{\alpha+k+r-1}e^{-\beta\theta}V^{t\theta}e^{-\left(B-\theta\sum_{i=1}^{k}lnu_i\right)}d\theta$$

$$= \sum_{t=0}^{n-k}\binom{n-k}{t}(-1)^t e^{-B}\int_0^{\infty}\theta^{\alpha+k+r-1}V^{t\theta}e^{-\theta\left(\beta-\sum_{i=1}^{k}lnu_i\right)}d\theta$$

$$= \sum_{t=0}^{n-k}\binom{n-k}{t}(-1)^t e^{-B}\int_0^{\infty}\theta^{\alpha+k+r-1}e^{-\theta\left(\beta-tlnV-\sum_{i=1}^{k}lnu_i\right)}d\theta$$

$$= \sum_{t=0}^{n-k}\binom{n-k}{t}(-1)^t e^{-B}.\frac{\Gamma(\alpha+k+r)}{A^{\alpha+k+r}}$$

Where,
$A = \beta - tlnV - \sum_{i=1}^{k} lnu_i$

NUMERICAL COMPARISONS

Here we perform some numerical experiments. Our main aim is to compare the MLE and Bayesian estimator and

Table 1. Simulated values of bias and MSE′s of $\hat{\theta}_{ML}$,$\hat{\theta}_B$ and $\lambda = 0.75$.

n	$\lambda = 0.75$		
	k	$\hat{\theta}_{ML}$	$\hat{\theta}_B$
20	15	0.41587 2.36491	1.90387 2.24980
	20	1.32686 2.00649	0.84301 1.98034
30	25	-0.87106 1.40267	-0.67105 1.02246
	30	0.90247 1.04912	-0.70681 0.99721
40	35	-1.09843 0.73264	-0.86435 0.23697
	40	-0.61870 0.64801	0.81375 0.19745
50	45	0.73915 0.60015	1.40256 0.04364
	50	0.13820 0.24015	-0.80359 0.01697

The first entry is the simulated bias.

their results work for sample sizes n=20, 30, 40 and 50 relative to scale parameter λ=0.75, 1 and 2 respectively. We chose prior parameters as α =2 and β =3 for Bayesian estimation. The biases and mean square errors (MSE's) are computed for different sample size *n* and censored sizes: k=15, 25, 35 and 45. The computations are achieved under complete and censored samples.

CONCLUSIONS

In this paper we have introduced a new distribution of the exponentiated gamma distribution and we called that generalized exponentiated gamma distribution. Also we consider classical and Bayesian estimations of the shape parameter of the generalized exponentiated gamma distribution. We assume the gamma prior on the unknown parameter and provided the Bayes estimator under the assumption of quadratic loss function. The estimations conducted on the basis of complete and type-II censored samples.

Our observations about results are stated in the following points:

1. Table 1 and Figure 1, show that the Bayes estimates under the squared error loss function have the smallest estimated MSE's as compared with the MLE. This is true for both complete and censored samples. It is immediate to note that MSE's decrease as sample size increases.
2. Table 2 and Figure 2 show that the Bayes estimates under the squared error loss function have the smallest estimated MSE's as compared with the MLE when sample size is small but when sample size increase the MLE's have the smallest estimated MSE's as compared with the Bayes estimates. This is true for both complete and censored samples.
3. Table 3 and Figure 3 shows that the MLE's have the smallest estimated MSE's as compared with the Bayes estimates. This is true for both complete and censored samples. It is immediate to note that MSE's decrease as sample size increases for Bayes estimates.

From the previous results and observations, we suggest the use of small value of the parameter l by Bayes approach under squared error loss function for estimating shape parameter of GEG distribution, while the MLE's are better than Bayes estimates when l increase.

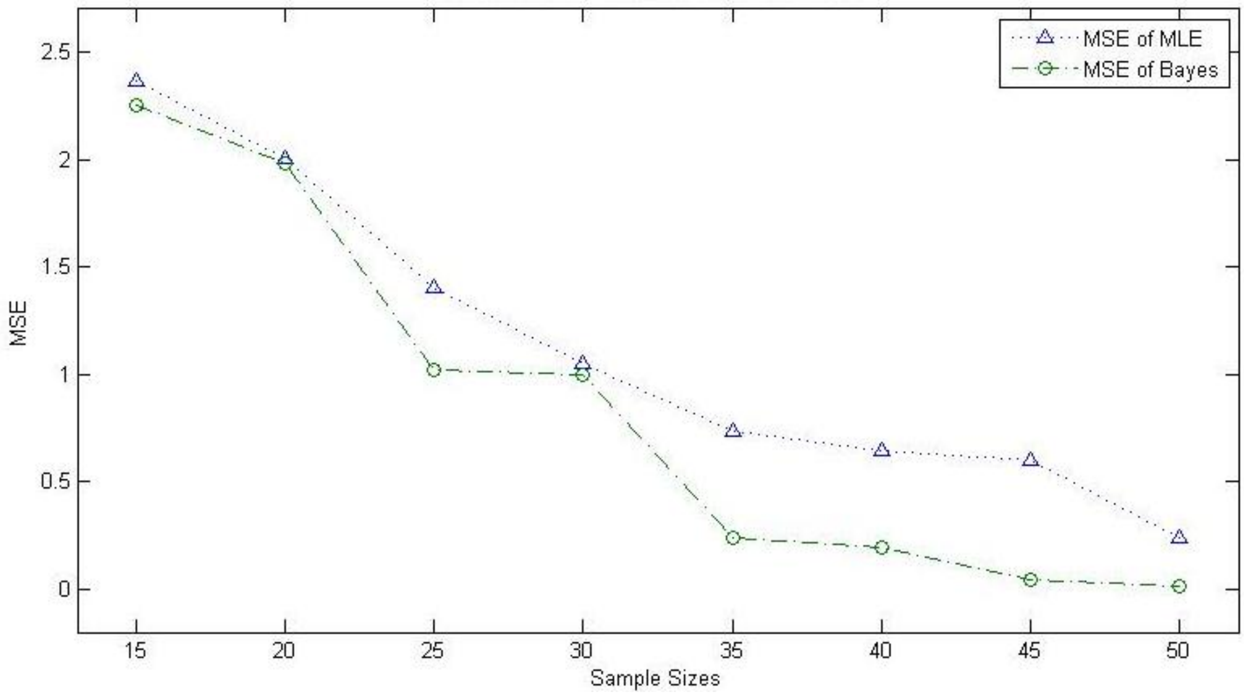

Figure 1. MSE of the estimator of ɵ for λ = 0.75.

Table 2. Simulated values of bias and MSE's of $\hat{\theta}_{ML}$,$\hat{\theta}_B$ and $\lambda = 1$.

n	$\lambda = 1$		
	k	$\hat{\theta}_{ML}$	$\hat{\theta}_B$
20	15	-0.71540 2.00036	2.03046 1.83264
	20	2.36187 1.97621	1.34682 1.67921
30	25	1.36484 1.00346	0.97813 0.97685
	30	-0.15785 0.97875	-1.32645 0.84316
40	35	-1.64975 0.34982	-1.09785 0.76134
	40	0.06974 0.19032	0.94652 0.46971
50	45	-0.97452 0.03648	-0.64985 0.16970
	50	0.91254 0.01360	1.32645 0.09340

The first entry is the simulated bias.

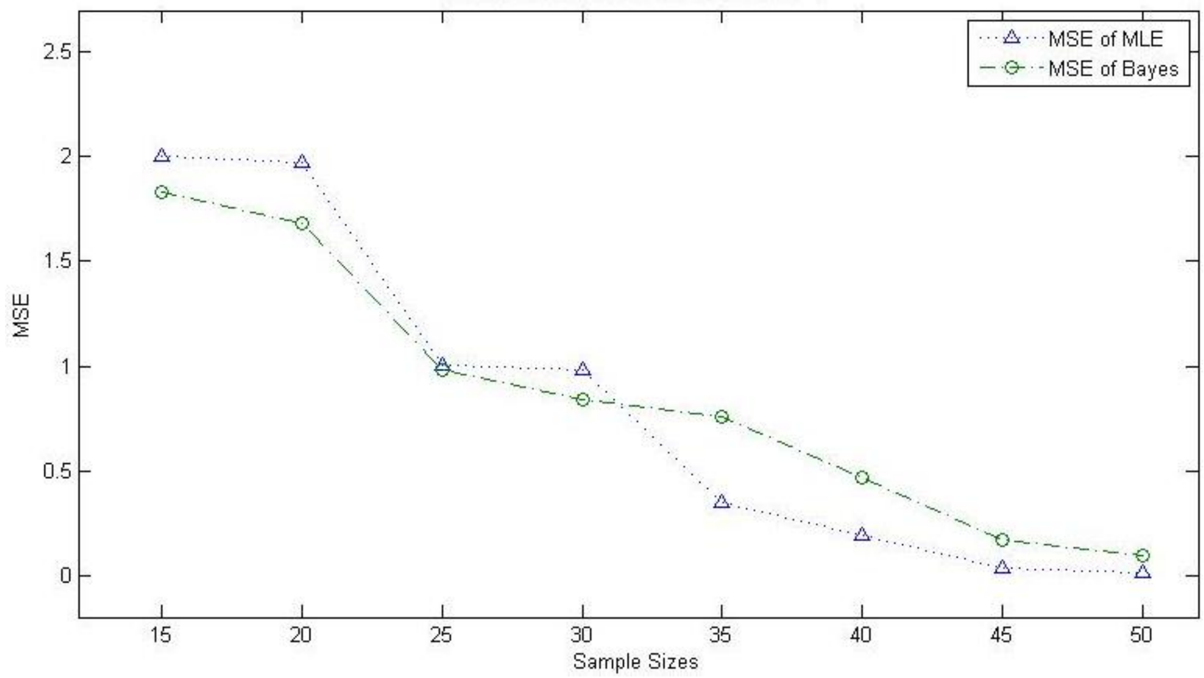

Figure 2. MSE of the estimator of ɵ for λ = 1.

Table 3. Simulated values of bias and MSE's of $\hat{\theta}_{ML}$,$\hat{\theta}_{B}$ and $\lambda = 2$.

n	k	$\lambda = 2$	
		$\hat{\theta}_{ML}$	$\hat{\theta}_{B}$
20	15	3.46985 3.65842	3.49875 5.68712
	20	2.64814 3.00645	2.64658 5.00925
30	25	-2.34685 2.06975	3.64584 4.16584
	30	1.97824 1.43268	-2.34652 3.74692
40	35	0.12212 1.00254	0.34658 3.65824
	40	-1.46857 0.64985	-2.34658 2.60154
50	45	-3.64851 0.35422	-4.35658 2.03498
	50	1.06497 0.19875	3.06452 1.63408

The first entry is the simulated bias.

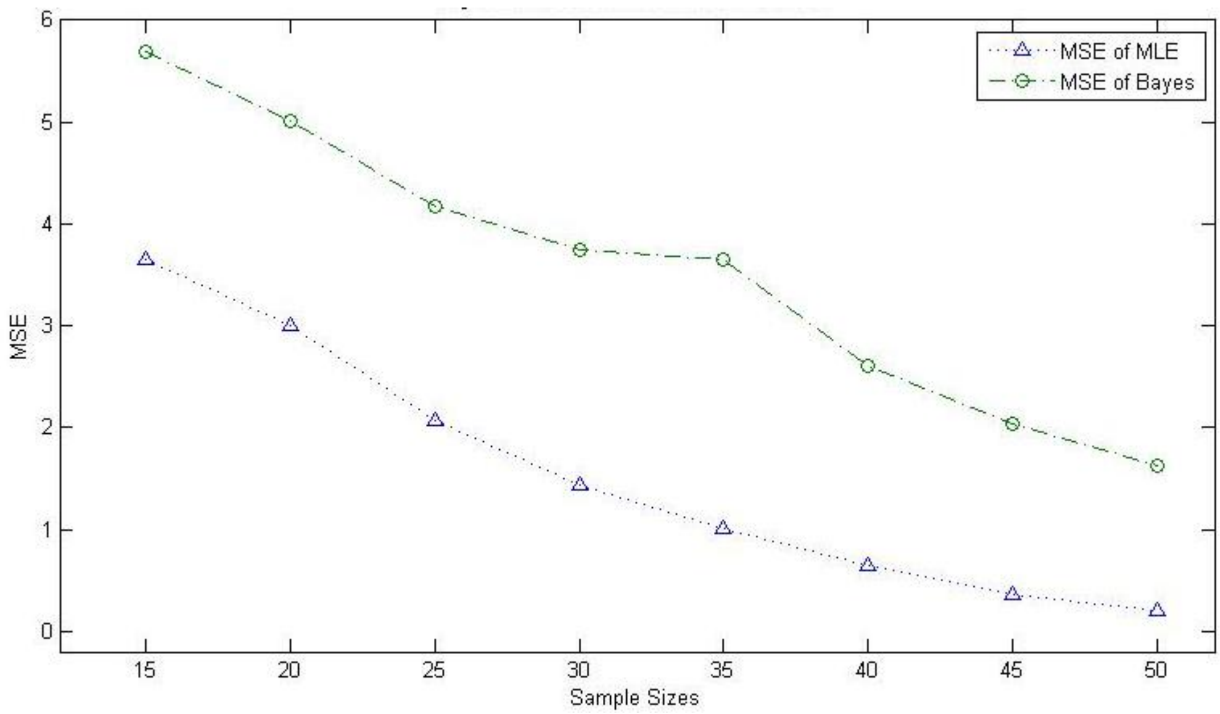

Figure 3. MSE of the estimator of ɵ for λ = 2.

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