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Full Length Research Paper

Comparative study of four methods for estimating Weibull parameters for Halabja, Iraq

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The Weibull distribution is the standard function used by the wind energy community to model the wind speed frequency distribution. In this study, four methods are presented for estimating Weibull parameters (Shape and Scale), namely, Maximum likelihood method (MLM), Rank regression method (RRM), Mean-standard deviation method (MSD), and Power density method (PDM). To compare the methods, a period of 4 years (2001 - 2004) of monthly time series data of Halabja city was considered. Two distinct analytical methods are studied to determine the parameter estimation accuracy of these methods; coefficient of determination and root mean square error (RMSE) are used as measurement tools. The Rank regression and MSDs are recommended to estimate the shape parameter; also the Rank regression is recommended for use with our time series wind data to estimate the scale parameter.

Key words: Weibull distribution, parameter estimation, energy pattern factor, accuracy.

INTRODUCTION

Weibull has been recognized as an appropriate model in reliability studies and life testing problems such as time to failure or life length of a component or product. Over the years, estimation of the shape and scale parameters for a Weibull distribution function has been approached through Maximum likelihood method (MLM), linear method, and several versions of regression analysis. In recent years, Weibull distribution has been one of the most commonly used, accepted, recommended distribution to determine wind energy potential and it is also used as a reference distribution for commercial wind energy softwares such as Wind Atlas Analysis and Application Program (WAsP). The two-parameter Weibull distribution function is commonly used to fit the wind speed frequency distribution.

The preferred method of estimating the Weibull parameters was a graphical way using the cumulative wind speed distribution, plotting it on special Weibull graph paper. Estimation of the two-parameter Weibull distribution occurs in many real-life problems. The Weibull distribution is an important model especially for reliability and maintainability analysis.

Weibull distribution can be used to model the wind speed distribution at a particular site and hence, it can help in wind resource assessment of a site. By calculating the two parameters (shape and scale) for Weibull distribution the wind speed frequency curve for a site can be made (Prasad et al., 2009) and the key to perform wind turbine and wind farm energy calculation. Several methods have been proposed to estimate Weibull parameters (Marks, 2005; Rider, 1961; Kao, 1959; Pang et al., 2001; Pandey et al., 2011; Seguro and Lambert, 2000; Stevens and Smulders, 1979; Bhattacharya and Bhattacharjee, 2010). In literature about wind energy, these methods are compared several times and in different ways (Akdag and Ali, 2009; Silva et al., 2004; Yilmaz et al., 2005; Gupta, 1986; Rahman et al., 1994; Lei, 2008; Kantar and Senoglu, 2007), however, results and conclusions of the previous studies are different. Several of fit tests are used in literature. A method for estimating parameters of mixed distributions using sample moments has been outlined by Paul (1961) who considered compound Poisson, binomial, and a special case of the mixed Weibull distribution. A graphical method for estimating the mixed Weibull parameters in life testing of electron tubes is proposed by John (1959). For these reasons, according to the results of the studies, it might be concluded that suitability of the method may vary with the sample data size, sample data distribution, sample data format and goodness of fit test (Akdag and Ali, 2009).

The present work is based on the time series wind data collected over a period of 4 years (2001 - 2004) (hourly). The location concerned in this study named Halabja is situated in east Sulaimani/North Iraq 35° 11' 7" North latitude, 45° 58' 42" East longitude and it is at an elevation of 692 m above sea level. There is no obstacle around wind speed measuring location, the wind data recorded from a mechanical cup type anemometer at height of 2 m above the ground level.

In present study, four methods for estimating the parameters of the Weibull wind speed distribution are presented [MLM, Rank regression method (RRM), Mean-standard deviation method (MSD), and the Power density method (PDM)] by Akdag and Ali (2009). The aim of this work was to select a method that gives more accurate estimation for the Weibull parameters at this location in order to reduce uncertainties related to the wind energy output calculation from any Wind Energy Conversion Systems (WECS).

WEIBULL DISTRIBUTION

The Weibull distribution is characterized by two parameters, one is the scale parameter c (m/s) and the other is the shape parameter k (dimensionless). In Weibull distribution, the variations in wind speed are characterized by two functions which are the probability density function (PDF) and the cumulative distribution function (CDF). The PDF, $f_{v,k,c}$ indicates the fraction of time (or probability) for which the wind is at a given speed *V*. It is given by Bhattacharya and Bhattacharjee (2010) and Weisser (2003).

$$f_{(v,k,c)} = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-\left(V/c\right)^k} \tag{1}$$

Where v > 0, and k, c > 0

The CDF of the speed *V* gives the fraction of the time (or probability) that the wind speed is equal or lower than *V*, thus, the cumulative distribution $f_{v,k,c}$ is the integral of the PDF, given by

$$F_{(V,k,c)} = \int_0^\infty f(V)dV = 1 - e^{-\left(V/c\right)^k} \tag{2}$$

The average wind speed can be expressed as:

$$\overline{V} = \int_0^\infty V f(V)dV \tag{3}$$

$$\overline{V} = \int_0^\infty V \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-\left(V/c\right)^k} dV \tag{4}$$

This can be rearranged as:

$$\overline{V} = k\int_0^\infty \left(\frac{v}{c}\right)^k e^{-\left(v/c\right)^k} dV \tag{5}$$

Taken

$$x = \left(\frac{v}{c}\right)^k, \; dV = \frac{c}{k} x^{\left(\frac{1}{k}-1\right)} dx \tag{6}$$

Equation 5 can be simplified as:

$$\overline{V} = c\int_0^\infty e^{-x} x^{1/k} dx \tag{7}$$

This is the form of the standard gamma function, which is given by

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx \tag{8}$$

From Equations 7 and 8, let $n = 1 + \frac{1}{k}$ the average speed can be expressed as:

$$\overline{V} = c\Gamma\left(1 + \frac{1}{k}\right) \tag{9}$$

The standard deviation of wind speed *V* is given by

$$\sigma = \sqrt{\int_0^\infty \left(v - \bar{v}\right)^2 f(v)dv} \tag{10}$$

$$\sigma = \sqrt{\int_0^\infty v^2 f(v)dv - 2\bar{v}\int_0^\infty v f(v)dv + \bar{v}^2}$$

or

$$\sigma = \sqrt{\int_0^\infty v^2 f(v)dv - 2\bar{v}.\bar{v} + \bar{v}^2} \tag{11}$$

Using

$$\int_0^\infty v^2 f(v)dv = \int_0^\infty v^2 \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv = \int_0^\infty c^2 x^{2/k} \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} dv$$

Equating to

$$\int_0^\infty c^2 x^{2/k} e^{(-x)} dx \tag{12}$$

And putting $n = 1 + \frac{2}{k}$, then the following equation can be obtained. Hence, get the standard deviation

$$\sigma = \left[c^2\Gamma\left(1 + \frac{2}{k}\right) - c^2\Gamma^2\left(1 + \frac{1}{k}\right)\right]^{1/2} \tag{13}$$

or

$$\sigma = c\sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma^2\left(1 + \frac{1}{k}\right)} \tag{14}$$

METHODS FOR ESTIMATING WEIBULL PARAMETERS

Maximum likelihood method (MLM)

Maximum likelihood technique, with many required features is the most widely used technique among parameter estimation techniques. The MLM method has many large sample properties that make it attractive for use; it is asymptotically consistent, which means that as the sample size gets larger, the estimate converges to the true values.

Let v_1, v_2, v_3 …………..v_n be a random sample size n drawn from a PDF $f(v,\theta)$ where θ is an unknown parameter. The likelihood function of this random sample is the joint density of the n random variables and is a function of the unknown parameter. Thus, (Yilmaz et al., 2005; Nilsen, 2011),

$$L = \prod_{i=1}^{n} f_{v_i}(v_i, \theta) \tag{15}$$

The maximum likelihood estimator of θ say $\bar{\theta}$ is the value of θ that maximizes L or, equivalent, the logarithm of L. Often but not always, the MLM of θ is a solution of $\frac{d \log L}{d\theta} = 0$.

Now, we apply the MLM to estimate the Weibull parameters, k and c. Consider the Weibull PDF given in Equation 1, then likelihood function will be (Yilmaz et al., 2005; Nilsen, 2011):

$$L(v_1, v_2, \ldots\ldots\ldots v_n, k, c) = \prod_{i=1}^{n} \left(\frac{k}{c}\right)\left(\frac{v_i}{c}\right)^{k-1} e^{-\left(\frac{v_i}{c}\right)^k} \tag{16}$$

On taken the logarithms of Equation 16, differentiating with respect to k and c in turn, and equating to zero, one can obtain the estimating equations

$$\frac{\partial \ln L}{\partial k} = \frac{n}{k} + \sum_{i=1}^{n} \ln v_i - \frac{1}{c}\sum_{i=1}^{n} v_i^k \ln v_i = 0 \tag{17}$$

$$\frac{\partial \ln L}{\partial c} = \frac{-n}{k} + \frac{1}{c^2}\sum_{i=1}^{n} v_i^k = 0 \tag{18}$$

In eliminating c between Equations 17 and 18 and simplifying, one can get

$$\frac{\sum_{i=1}^{n} v_i^k \ln v_i}{\sum_{i=1}^{n} v_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} \ln v_i = 0 \tag{19}$$

This may be solved to get the estimate of k. This can be accomplished by the use of standard iterative procedures (that is, Newton-Raphson method), which can be written in the form

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{20}$$

Where

$$f(k) = \frac{\sum_{i=1}^{n} v_i^k \ln v_i}{\sum_{i=1}^{n} v_i^k} - \frac{1}{k} - \frac{1}{n}\sum_{i=1}^{n} \ln v_i \tag{21}$$

And

$$f'(k) = \sum_{i=1}^{n} v_i^k (\ln v_i)^2 - \frac{1}{k^2}\sum_{i=1}^{n} v_i^k \left(k \ln v_i - 1\right) - \left(\frac{1}{n}\sum_{i=1}^{n} \ln v_i\right)\left(\sum_{i=1}^{n} v_i^k \ln v_i\right) \tag{22}$$

The shape parameter k can be estimated using Equations 21 and 22 with Equation 20 as:

$$k = \left(\frac{\sum_{i=1}^{n} v_i^k \ln v_i}{\sum_{i=1}^{n} v_i^k} - \frac{1}{n}\sum_{i=1}^{n} \ln v_i\right)^{-1} \tag{23}$$

Once k is determined, c can be estimated using Equation 18 as follows:

$$c = \left(\frac{1}{n}\sum_{i=1}^{n} v_i^k\right)^{1/k} \tag{24}$$

Rank regression method (RRM)

The second estimation technique, we shall discuss is known as the least squares method. This is, in essence, a more formalized method of the manual probability plotting technique, in that it provides a mathematical method for fitting a line to plotted failure data points.

It is so commonly applied in engineering and mathematics problems that are often not thought of as an estimation problem. With the help of this method the parameters are estimated with regression line equation by cumulative density function. From Equation 1, the cumulative density function of Weibull distribution function with two parameters can be written as (Justus et al., 1978):

$$F(v_i) = 1 - e^{-\left(\frac{v}{c}\right)^k} \tag{25}$$

This function can be arranged as:

$$\{1 - F(v_i)\}^{-1} = e^{-\left(\frac{v}{c}\right)^k} \tag{26}$$

If we take the natural logarithm of Equation 26

$$-\ln\{1 - F(v_i)\} = \left(\frac{v_i}{c}\right)^k \tag{27}$$

And then retake the natural logarithm of Equation 27, we get the following equation:

$$\ln[-\ln\{1 - F(v_i)\}] = -k \ln c + k \ln v_i \tag{28}$$

Equation 28 represents a direct relationship between ($\ln v_i$) and $-\ln\{1 - F(v_i)\}$ which should be minimized

$$\sum_{i=1}^{n}\{\ln[-\ln(1 - F(v_i)] - \ln[-\ln(1 - E(F(v_i))]\}^2 \tag{29}$$

Parameters of Weibull distribution with two parameters are estimated by minimizing with Equation 29. The two parameters c and k are intersecting by the following equations:

$$k = \frac{n\sum_{i=1}^{n}\ln v_i \ln[-\ln\{1-F(v_i)\}] - \sum_{i=1}^{n}\ln v_i \sum_{i=1}^{n}\ln[-\ln\{1-F(v_i)\}]}{n\sum_{i=1}^{n}\ln v_i^2 - \{\sum_{i=1}^{n}\ln v_i\}^2} \quad (30)$$

$$c = \exp\{\frac{k\sum_{i=1}^{n}\ln v_i - \sum_{i=1}^{n}\ln[-\ln\{1-F(v_i)\}]}{nk}\} \quad (31)$$

From Equations 30 and 31, k and c can be estimated, respectively.

Mean-standard deviation method (MSD)

The Weibull factors k and c can also be estimated from the mean and standard deviation σ of wind data, consider the expression for average and standard deviation given in Equations 9 and 14, from these, one has (Fung et al., 2007; Weisser and Foxon, 2003):

$$\left(\frac{\sigma_v}{\bar{v}}\right)^2 = \frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{1}{k}\right)} - 1 \quad (32)$$

Where

$$\bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i \quad (33)$$

And

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(v_i - \bar{v}\right)^2 \quad (34)$$

n is the number of wind observation. Once σ_v and $\bar{v}$ are calculated for a given data set, then k can be determined by solving Equation 32 numerically, once k is determined, c is given by

$$c = \frac{\bar{v}}{\Gamma\left(1+\frac{1}{k}\right)} \quad (35)$$

In a simpler approach, an acceptable approximation for k is (Akhlaque et al., 2006):

$$k = \left(\frac{\sigma_v}{\bar{v}}\right)^{-1.086} \quad (36)$$

Power density method (PDM)

This is a new method suggested by Akdag and Ali (2009). It is used to estimate the two-Weibull parameters, depends on the energy pattern factor method; it is related to the averaged data of wind speed. This method has simpler formulation, easier implementation and also requires less computation. According to the Weibull probability distribution, the mean wind speed of Equation 35 can be written as (Silva et al., 2004; Paula et al., 2012):

$$\bar{v} = c\Gamma\left(1+\frac{1}{k}\right) \quad (37)$$

And hence the cubic mean wind speed is given as:

$$\bar{v}_{cu} = c^3\Gamma\left(1+\frac{3}{k}\right) \quad (38)$$

To determine the energy pattern factor (E_{pf}) one can write Equations 37 and 38 as:

$$E_{pf} = \frac{\overline{v_{cu}^3}}{\bar{v}^3} = \frac{\Gamma\left(1+\frac{3}{k}\right)}{\Gamma^3\left(1+\frac{1}{k}\right)} \quad (39)$$

Equation 39 is known as energy pattern factor (Epf) method. Weibull parameters can be estimated with solving energy pattern factor Equation 39 numerically or approximately by power density technique using the simple formula as follows:

$$k = 1 + \frac{3.69}{(E_{pf})^2} \quad (40)$$

Once k is determined, c can be estimated using Equation 37.

COMPARISON AND ACCURACY OF THE METHODS

Four methods for estimating the parameters of the Weibull wind speed distribution for wind energy analysis for Halabja city are presented. The application of each method is demonstrated using a sample wind speed data set, and a comparison of the accuracy of each method is also performed with the actual time series data for the our case study (Halabja city). In order to compare the methods, monthly mean wind data used for Halabja region is obtained from meteorological automatic station which covers the period of 4 years (2001 - 2004).

Two tests were employed to determine the accuracy of the four methods given in this article, first is the coefficient of determination R2 of Equation 41 used to how well the regression model describes the data, and second is root mean square error (RMSE) of Equation 42.

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(X_i - x_i)^2}{\sum_{i=1}^{N}(X_i - \underline{X})^2} \quad (41)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_i - x_i)^2} \quad (42)$$

Where N is the total number of intervals, Xi the frequencies of observed wind speed data, xi the frequencies distribution value estimated with Weibull distribution, X the mean of Xi values.

RESULTS AND DISCUSSION

Once coefficient of determination and RMSEs are computed the difference methods can be compared in accuracy as shown in Tables 1 and 2. Weibull parameters have been estimated monthly according to the four methods with the actual time series data for all the years (2001 - 2004). Figure 1 shows the histogram of the actual frequency distribution of diurnal wind speed for all these years with the Weibull function for fitting a wind data probability distribution. Figures 2 and 3 show the

Table 1. Monthly estimated Weibull parameters with actual data.

Month	Mean V (m/s)	Actual Data		ML method		RR method		MSD method		PD method	
		K	$C_{(m/s)}$	K	$C_{(m/s)}$	K	$C_{(m/s)}$	K	$C_{(m/s)}$	K	$C_{(m/s)}$
Jan	1.49	2.3551	1.6115	1.6072	1.6868	2.9191	1.5669	1.3648	1.6291	1.9102	1.6798
Feb	1.86	1.8691	2.0018	1.4568	2.0841	2.2292	1.9420	1.2725	2.0082	1.8971	1.8688
Mar	1.87	2.3688	2.0723	2.0169	2.1260	2.6248	2.0487	2.0558	2.1113	2.8403	2.0985
April	1.97	2.5514	2.1995	2.2075	2.2420	3.0264	2.1463	2.1368	2.2244	2.9989	2.2053
May	2.16	2.6301	2.4000	2.2757	2.4431	3.1939	2.3338	2.3462	2.4370	3.0965	2.4144
Jun	2.45	3.4864	2.7189	3.1190	2.7445	4.4182	2.6468	3.2168	2.7343	3.6739	2.7146
July	2.43	4.0289	2.6602	3.2868	2.7031	4.6771	2.6293	3.7930	2.6868	3.7933	2.6868
Aug	2.35	3.3551	2.6115	3.1590	2.6240	4.4598	2.5417	3.5062	2.6090	3.7152	2.6038
Sep	2.01	3.0376	2.2200	2.5361	2.2657	3.4402	2.1931	2.6195	2.2622	3.3178	2.2398
Oct	1.75	2.4456	1.9556	2.1106	1.9840	3.1626	1.8745	2.0821	1.9758	2.9119	1.9614
Nov	1.79	2.2320	1.9837	1.9703	2.0281	2.5358	1.9473	1.8807	2.0169	2.7973	2.0087
Dec	1.86	2.1564	2.0682	1.8891	2.1184	2.4237	2.0338	1.7736	2.0908	2.7021	2.0915
Mean	1.99	2.709	2.2086	2.3029	2.2542	3.2592	2.1587	2.3373	2.2322	2.9712	2.2145

Table 2. Statistical analysis for all the methods with actual data.

Method	Variance	Standard deviation	Coefficient of variation	Coefficient of determination		Root mean square error	
				K	C (m/s)	K	C (m/s)
Actual data	0.5977	0.7731	0.4094				
ML	0.8459	0.9197	0.4605	0.6290	0.4270	0.3235	0.4464
RR	0.4263	0.6529	0.3374	0.6401	0.4335	0.6328	0.1157
MSD	0.8081	0.8989	0.4544	0.6201	0.4232	0.2534	0.3702
PD	0.5251	0.7246	0.3666	0.5307	0.4088	0.3448	0.3089

estimated parameters c and k, respectively versus the months of years, the similarity can be seen among the methods with the true data almost for all the months for parameter c, while for parameter k, the divergence of the methods with the actual data obtained due to the difference in the estimated values, these also have appear in the RMSE results as shown in Table 2. The rank regression and mean standard deviation methods give satisfactory results for the shape parameter estimation, while rank regression method give satisfactory result for the scale parameter estimation. Graphically, Figure 4 shows the annual mean wind data of the probability density function for Weibull distributions using the estimated Weibull parameters by all methods have been compared with the annual mean wind true time series data, as a result the power density method is the most fitted method to estimate the Weibull parameters in our study case. We can also see that all methods are similar enough to show that each method would be sufficient for determining our parameter estimates.

Conclusion

According to the results, it might be concluded that suitability of these methods may vary with the sample such as data size, sample data distribution (months), sample data format, and of fit tests. When wind data is available in time series format, according to the R2 and RMSE tests both the RRM and MSD, respectively are the recommended methods for estimating the shape parameter, while for the scale parameter and for the both tests, the RRM is recommended method to estimate. Graphically, the curves of the methods show that the best way to estimate the two Weibull parameters is the PDM. This fact is also been supported by means of the RMSE and R2 statistical tests (Table 2). From this comparative study, it is observed that the values of RMSE and R2 have magnitudes that are almost similar for all the methods.

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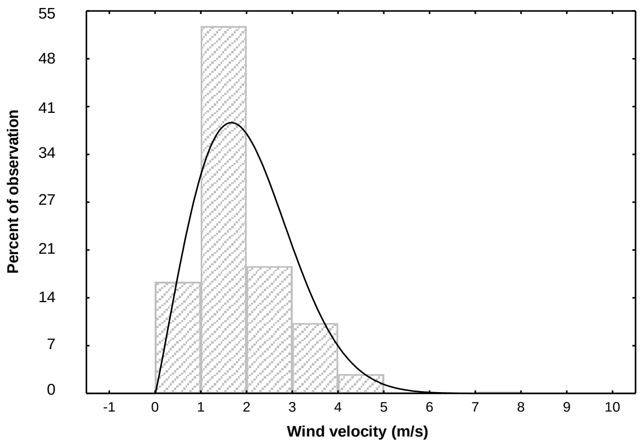

Figure 1. Show the histogram of the time series distribution of the actual wind data.

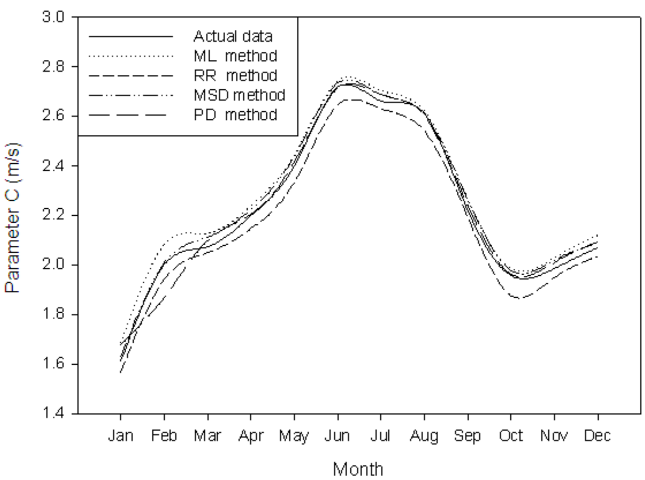

Figure 2. Estimated Weibull parameter C (m/s) versus the months.

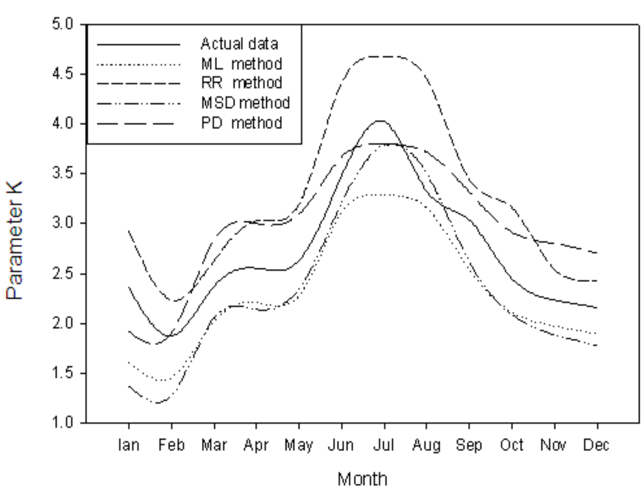

Figure 3. Estimated Weibull parameter K versus the months.

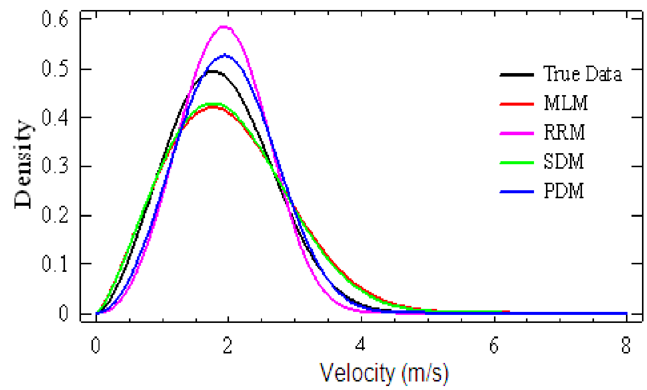

Figure 4. The Weibull probability density function using all methods with actual wind data.

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