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Full Length Research Paper

Frenet-Serret motion and ruled surfaces with constant slope

Yusuf YAYLI and Evren ZIPLAR*

Department of Mathematics, Faculty of Science, University of Ankara, Tandoğan, Ankara, Turkey.

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In this paper, we study Frenet-Serret motion and ruled surfaces with constant slope in Euclidean 3-space. By applying Frenet-Serret motion to the points of a cone, we obtain ruled surfaces with constant slope and we investigate these surfaces. We give the definition of a Smarandache curve in Euclidean 3-space. Also, we define a new type of general helix curves on a ruled surface in Euclidean 3-space.

Key words: Frenet-Serret motion, Smarandache curves, constant slope surfaces, ruled surfaces, helix curves.

INTRODUCTION

Frenet-Serret motion has the most important position of the study of kinematics. In particular, the study of one-parameter motions became an interesting topic in kinematics. The motion was investigated by Bottema and Roth (1979) in Euclidean n-space.

Ruled surfaces are one of the most important topics of differential geometry. The surfaces were found by Gaspard Monge, who was a French mathematician and inventor of descriptive geometry. Besides, these surfaces have the most important position of the study of one parameter motions.

In Euclidean 3-space E^3, each regular unit speed curve $\alpha : \mathrm{I} \subset \mathrm{IR} \to E^3$ has the orthonormal frame $\{\vec{t}, \vec{n}, \vec{b}\}$ at all the points of its space. The elements of the frame are called the tangent, the principal normal and the binormal vectors, respectively. Furthermore, the planes spanned by $\{\vec{t}, \vec{n}\}$, $\{\vec{t}, \vec{b}\}$ and $\{\vec{n}, \vec{b}\}$ are called as the osculating plane, the rectifying plane and the normal plane, respectively.

We now recall the definition of general helix in Euclidean 3-space. A curve $\alpha : \mathrm{I} \subset \mathrm{IR} \to E^3$ with unit $\vec{u}$, so that $\langle \vec{t}, \vec{u} \rangle = \cos(\theta)$ is constant along the curve. It has been known that the curve is general helix if and only if $\frac{k_2}{k_1}(s)$ is constant, where k_1 is curvature and k_2 is torsion of α, respectively.

Constant slope surfaces are considerable subject of geometry. For their shapes, we can say that constant slope surfaces are one of the most fascinated surfaces in the Euclidean 3-space.

There are so many types of these surfaces. Surface for which the unit normals make a constant angle with a fixed vector direction is a kind of constant slope surfaces. Munteanu and Nistor (2009) obtained a classification of all these surfaces. Moreover, a ruled surface for which the generating lines make a constant angle with a given plane is another kind of constant slope surfaces. Maleček et al. (2009) investigated these surfaces.

The main purpose of this study is to obtain a ruled surface with a constant slope with respect to the osculating planes (or rectifying planes, normal planes) to a curve α by applying Frenet-Serret motion to the points of a cone.

Also, we give the β-helix curves on a ruled surface. But before this, we mention some basic facts which are useful for the rest of the paper.

*Corresponding author. E-mails: evrenziplar@yahoo.com.

speed is a general helix if there is some constant vector

PRELIMINARIES

Definition 1

One parameter motion of body in Euclidean 3-space is generated by the transformation:

$$H : E^3 \to E^3$$

$$X \to H(\mathrm{X}) = AX + \vec{C} = Y$$

Here, A is a 3×3 orthogonal matrix and C is a displacement vector of the origin. Also A and C are C^∞ functions of a real parameter t, the motion parameter.
In the special Frenet-Serret motion, C represents a space curve α and the matrix A is $[\vec{t} \quad \vec{n} \quad \vec{b}]$, where $\{\vec{t}, \vec{n}, \vec{b}\}$ is the Frenet-Serret vector fields of the curve α. Definition 1 was given by Yayli and Masrouri (2011).

Definition 2

Let $\mathrm{I} \subset \mathrm{IR}$ be an interval, let $\alpha : \mathrm{I} \to E^3$ be a regular parametrized curve and let $\mathrm{X} : \mathrm{I} \to E^3$ be an arbitrary smooth function with $\vec{\mathrm{X}}(s) \neq \vec{0}$ for all $s \in \mathrm{I}$. Thus, we defined a parametrized surface by:

$$\varphi(s, v) = \vec{\alpha}(s) + v\vec{\mathrm{X}}(s)\,,\ s \in \mathrm{I}\,, v \in \mathrm{IR} \quad (1)$$

This is called a ruled surface with the base curve α and the director curve $\vec{X}(s)$. Definition 2 was given by Sarıoğlugil and Tutar (2007).

Definition 3

If the generated lines (the lines whose direction vectors are $\vec{\mathrm{X}}(s)$) of a surface φ have a constant slope $\tan\theta = \sigma$ (θ is the angle between $\vec{\mathrm{X}}(s)$ and osculating plane at the point $\alpha(s)$, $\theta \in (0, \pi/2)$ and $\sigma \in (0, +\infty)$) with respect to the osculating planes to the curve at every point on the curve α, then φ is called a surface with a constant slope with respect to osculating planes to the curve α in Szarka and Szarková (2009) work.

Definition 4

If the generated lines (the lines whose direction vectors are $\vec{\mathrm{X}}(s)$) of a surface φ have a constant slope $\tan\theta = \sigma$ (θ is the angle between $\vec{\mathrm{X}}(s)$ and rectifying plane at the point $\alpha(s)$, $\theta \in (0, \pi/2)$, $\sigma \in (0, +\infty)$) with respect to the rectifying planes to the curve at every point on the curve α, then φ is called a surface with a constant slope with respect to rectifying planes to the curve α.

Definition 5

If the generated lines (the lines whose direction vectors are $\vec{\mathrm{X}}(s)$) of a surface φ have a constant slope $\tan\theta = \sigma$ (θ is the angle between $\vec{\mathrm{X}}(s)$ and normal plane at the point $\alpha(s)$, $\theta \in (0, \pi/2)$, $\sigma \in (0, +\infty)$) with respect to the normal planes to the curve at every point on the curve α, then φ is called a surface with a constant slope with respect to normal planes to the curve α.

Definition 6

A regular curve in Euclidean 3-space, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve. Definition 6 was given by Ali (2010).

Example 1

Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{\vec{t}, \vec{n}, \vec{b}\}$ be its moving Frenet-Serret frame. Then,

$$\gamma = \frac{1}{\sqrt{2}}\vec{t} + \frac{1}{\sqrt{2}}\vec{n} + \vec{b} \quad (2)$$

is a Smarandache curve in E^3.

Example 2

Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{\vec{t}, \vec{n}, \vec{b}\}$ be its moving Frenet-Serret frame. Then,

$$\gamma = \frac{1}{2}\vec{t} + \frac{\sqrt{3}}{2}\vec{b} \quad (3)$$

is a Smarandache curve in E^3.

MAIN THEOREMS

Theorem 1

If we apply the Frenet-Serret motion to the points of the cone surface:

$$x^2 + y^2 = \frac{1}{\sigma^2} z^2, \tag{4}$$

then we obtain a surface with constant slope σ.

Proof

Let $(v\cos w(s), v\sin w(s), \sigma v)$ be parametric representation of the cone surface

$$x^2 + y^2 = \frac{1}{\sigma^2} z^2.$$

Now, we can apply the Frenet-Serret motion to the points of the cone.

Let α be a regular space curve which is parametrized by the vector function $\alpha = \alpha(s)$, $s \in \mathrm{I}$, the arc length. Then,

$$\Phi(s,v) = \begin{bmatrix} \vec{t} & \vec{n} & \vec{b} \end{bmatrix} \begin{bmatrix} v\cos w(s) \\ v\sin w(s) \\ \sigma \mathrm{v} \end{bmatrix} + \alpha(s) \tag{5}$$

and from Equation 5, we have:

$$\Phi(s,v) = \vec{\alpha}(s) + v\left(\cos w(s)\vec{t} + \sin w(s)\vec{n} + \sigma\vec{b}\right) \tag{6}$$

where $\vec{\mathrm{X}}(s) = \cos w(s)\vec{t} + \sin w(s)\vec{n} + \sigma\vec{b}$.

Generating lines of the surface Φ are given by the points on the curve $\alpha = \alpha(s)$ and they have the constant slope σ with respect to the osculating planes to the curve $\alpha = \alpha(s)$.

Actually, Φ is a constant slope surface with respect to the osculating planes to the curve $\alpha(s)$.

We assume that θ is the angle between $\vec{\mathrm{X}}(s)$ and osculating plane at the point $\alpha(s)$. Then, we can write

$$\tan\theta = \frac{\left\|\sigma\vec{b}\right\|}{\left\|\cos w(s)\vec{t} + \sin w(s)\vec{n}\right\|} = \sigma = const \tag{7}$$

where $\cos w(s)\vec{t} + \sin w(s)\vec{n}$ is orthogonal projection of the vector $\vec{\mathrm{X}}(s)$ on the osculating plane at the point $\alpha(s)$.

This completes the proof.

Theorem 2

If we apply the Frenet-Serret motion to the points of the cone surface:

$$x^2 + z^2 = \frac{1}{\sigma^2} y^2, \tag{8}$$

then we obtain a surface with constant slope σ.

Proof

Let $(v\cos w(s), \sigma v, v\sin w(s))$ be parametric representation of the cone surface $x^2 + z^2 = \frac{1}{\sigma^2} y^2$.

Now, we can apply the Frenet-Serret motion to the points of the cone.

Let α be a regular space curve which is parametrized by the vector function $\alpha = \alpha(s)$, $s \in \mathrm{I}$, the arc length. Then,

$$\phi(s,v) = \begin{bmatrix} \vec{t} & \vec{n} & \vec{b} \end{bmatrix} \begin{bmatrix} v\ \cos w(s) \\ \sigma v \\ v\ \sin w(s) \end{bmatrix} + \alpha(s) \tag{9}$$

and from Equation 9, we have:

$$\phi(s,v) = \vec{\alpha}(s) + v\ \left(\cos w(s)\vec{t} + \sin w(s)\vec{b} + \sigma\vec{n}\right) \tag{10}$$

where $\vec{\mathrm{X}}(s) = \cos w(s)\vec{t} + \sin w(s)\vec{b} + \sigma\vec{n}$.

Generating lines of the surface ϕ are given by the points on the curve $\alpha = \alpha(s)$ and they have the constant slope σ with respect to the rectifying planes to the curve $\alpha = \alpha(s)$. Actually, ϕ is a constant slope surface with respect to the rectifying planes to the curve $\alpha(s)$.

We assume that θ is the angle between $\vec{\mathrm{X}}(s)$ and rectifying plane at the point $\alpha(s)$. Then, we can write:

$$\tan\theta = \frac{\left\|\sigma\vec{n}\right\|}{\left\|\cos w(s)\vec{t} + \sin w(s)\vec{b}\right\|} = \sigma = const \tag{11}$$

where $\cos w(s)\vec{t} + \sin w(s)\vec{b}$ is orthogonal projection of the vector $\vec{X}(s)$ on the rectifying plane at the point $\alpha(s)$.

This completes the proof.

Theorem 3

If we apply the Frenet-Serret motion to the points of the cone surface:

$$y^2 + z^2 = \frac{1}{\sigma^2}x^2, \quad (12)$$

then we obtain a surface with constant slope σ.

Proof

Let $(\sigma v, v\cos w(s), v\sin w(s))$ be parametric representation of the cone surface $y^2 + z^2 = \frac{1}{\sigma^2}x^2$.

Now, we can apply the Frenet-Serret motion to the points of the cone.

Let α be a regular space curve which is parametrized by the vector function $\alpha = \alpha(s)$, $s \in \mathrm{I}$, the arc length. Then,

$$\gamma(s,v) = \begin{bmatrix} \vec{t} & \vec{n} & \vec{b} \end{bmatrix} \begin{bmatrix} \sigma v \\ v\cos w(s) \\ v\sin w(s) \end{bmatrix} + \alpha(s) \quad (13)$$

and from Equation 13, we have:

$$\gamma(s,v) = \vec{\alpha}(s) + v\left(\cos w(s)\vec{n} + \sin w(s)\vec{b} + \sigma\vec{t}\right) \quad (14)$$

where $\vec{X}(s) = \cos w(s)\vec{n} + \sin w(s)\vec{b} + \sigma\vec{t}$.

Generating lines of the surface γ are given by the points on the curve $\alpha = \alpha(s)$ and they have the constant slope σ with respect to the normal planes to the curve $\alpha = \alpha(s)$. Actually, γ is a constant slope surface with respect to the normal planes to the curve $\alpha(s)$.

We assume that θ is the angle between $\vec{X}(s)$ and normal plane at the point $\alpha(s)$. Then, we can write:

$$\tan\theta = \frac{\left\|\sigma\vec{t}\right\|}{\left\|\cos w(s)\vec{n} + \sin w(s)\vec{b}\right\|} = \sigma = cst. \quad (15)$$

where $\cos w(s)\vec{n} + \sin w(s)\vec{b}$ is orthogonal projection of the vector $\vec{X}(s)$ on the normal plane at the point $\alpha(s)$.

This completes the proof.

Example 3

Let the curve $\alpha(s)$ be a cylindrical helix parametrized by the vector function:

$$\vec{\alpha}(s) = \left(4\cos\frac{s}{5}, 4\sin\frac{s}{5}, \frac{3s}{5}\right), \quad s \in [0, 15\pi]$$

The Frenet-Serret frame is given by the vector functions:

$$\vec{t}(s) = \left(-\frac{4}{5}\sin\frac{s}{5}, \frac{4}{5}\cos\frac{s}{5}, \frac{3}{5}\right)$$

$$\vec{n}(s) = \left(-\cos\frac{s}{5}, -\sin\frac{s}{5}, 0\right)$$

$$\vec{b}(s) = \left(\frac{3}{5}\sin\frac{s}{5}, -\frac{3}{5}\cos\frac{s}{5}, \frac{4}{5}\right)$$

Direction vectors of generating lines of the surface $\phi(s,v) = \vec{\alpha}(s) + v\left(\cos w(s)\vec{t} + \sin w(s)\vec{b} + \sigma\vec{n}\right)$ (constant slope surface with respect to the rectifying planes to the curve $\alpha(s)$) are given by the vector function:

$$\vec{X}(s) = \left(-\frac{4}{5}\cos w(s)\sin\frac{s}{5} + \frac{3}{5}\sin w(s)\sin\frac{s}{5} - \right.$$

$$\sigma\cos\frac{s}{5},$$

$$\frac{4}{5}\cos w(s)\cos\frac{s}{5} - \frac{3}{5}\sin w(s)\cos\frac{s}{5} - \sigma\sin\frac{s}{5},$$

$$\left.\frac{3}{5}\cos w(s) + \frac{4}{5}\sin w(s)\right)$$

The surface $\phi(s,v)$ has the parametric representation:

$$y = 4\sin\frac{s}{5} + v\left(\frac{4}{5}\cos w(s)\cos\frac{s}{5} - \frac{3}{5}\sin w(s)\cos\frac{s}{5}\right.$$

$$\left.-\sigma\sin\frac{s}{5}\right)$$

$$z = \frac{3s}{5} + v\left(\frac{3}{5}\cos w(s) + \frac{4}{5}\sin w(s)\right), \quad s \in [0, 15\pi], \; v \in \mathrm{IR}$$

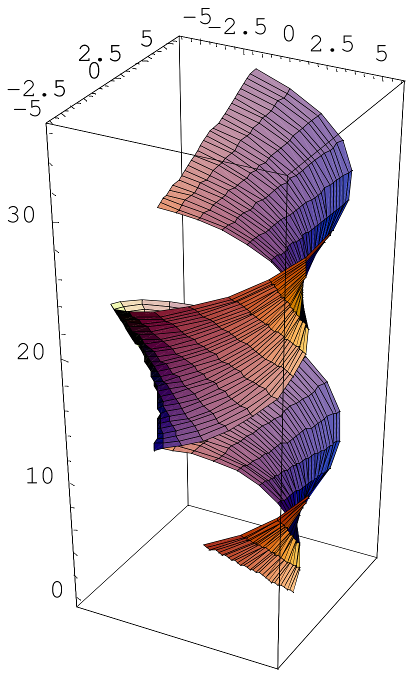

Figure 1. The constant slope surface $\Phi(s,v)$.

The patch of the surface $\Phi(s,v)$ received by choosing $w(s)=\dfrac{\pi}{2}$, $\sigma=\dfrac{1}{4}$ and $v\in[0,3\pi]$ is as shown in Figure 1.

CONSTANT SLOPE RULED SURFACES WHOSE DIRECTOR CURVES ARE SMARANDACHE CURVES

Note that the constant slope surfaces $\Phi(s,v)$, $\phi(s,v)$ and $\gamma(s,v)$ need not to be developable in general.

Here, we assume that the director curves $\vec{X}(s)$ are Smarandache curves. That is, we consider that $\cos w(s)=x_1=const.$ and $\sin w(s)=x_2=const.$

Theorem 4

The surface $\Phi'(s,v)=\vec{\alpha}(s)+v\left(x_1\vec{t}+x_2\vec{n}+\sigma\vec{b}\right)$ is developable if and only if $\alpha(s)$ is a general helix so that:

$$\frac{k_1}{k_2}=\frac{1-x_1^2+\sigma^2}{x_1\sigma} \quad (16)$$

is constant where k_1 is a curvature and k_2 is a torsion of α, respectively.

Proof

We know that a ruled surface is developable iff $\det(\vec{t},\vec{X},\vec{X}')=0$.

So, we will compute $\det(\vec{t},\vec{X},\vec{X}')$:

$$\vec{t}=\alpha'$$

$$\vec{X}=x_1\vec{t}+x_2\vec{n}+\sigma\vec{b}$$

$$\vec{X}'=(-x_2k_1)\vec{t}+(x_1k_1-\sigma k_2)\vec{n}+(x_2k_2)\vec{b}$$

and

$$\det(\vec{t},\vec{X},\vec{X}')=x_2^2k_2-(x_1k_1-\sigma k_2)\sigma=0 \quad (17)$$

From Equation 17, we have:

$$\frac{k_1}{k_2}=\frac{x_2^2+\sigma^2}{x_1\sigma} \quad (18)$$

Since $x_1^2+x_2^2=1$, we can write:

$$\frac{k_1}{k_2}=\frac{1-x_1^2+\sigma^2}{x_1\sigma} \quad (19)$$

This completes the proof.

Theorem 5

The surface $\phi'(s,v)=\vec{\alpha}(s)+v\left(x_1\vec{t}+x_2\vec{b}+\sigma\vec{n}\right)$ is developable if and only if $\alpha(s)$ is a general helix, so that:

$$\frac{k_1}{k_2}=\frac{1-x_1^2+\sigma^2}{x_1x_2} \quad (20)$$

is constant (k_1 is the curvature and k_2 is the torsion of α, respectively).

Proof

We will compute $\det(\vec{t},\vec{X},\vec{X}')$:

$$\vec{t}=\alpha'$$

$$\vec{X} = x_1\vec{t} + x_2\vec{b} + \sigma\vec{n}$$

$$\vec{X}' = (-\sigma k_1)\vec{t} + (\sigma k_2)\vec{b} + (x_1k_1 - x_2k_2)\vec{n}$$

and

$$\det(\vec{t}, \vec{X}, \vec{X}') = (x_1x_2)k_1 - x_2^2k_2 - \sigma^2k_2 = 0 \quad (21)$$

From Equation 21, we have:

$$\frac{k_1}{k_2} = \frac{x_2^2 + \sigma^2}{x_1x_2} \quad (22)$$

Since $x_1^2 + x_2^2 = 1$, we can write:

$$\frac{k_1}{k_2} = \frac{1 - x_1^2 + \sigma^2}{x_1x_2} \quad (23)$$

This completes the proof.

Theorem 6

The surface $\gamma'(s,v) = \vec{\alpha}(s) + v\left(x_1\vec{n} + x_2\vec{b} + \sigma\vec{t}\right)$ is developable if and only if $\alpha(s)$ is a general helix so that:

$$\frac{k_1}{k_2} = \frac{1}{x_2\sigma} \quad (24)$$

is constant (k_1 is curvature and k_2 is torsion of α, respectively).

Proof

We will compute $\det(\vec{t}, \vec{X}, \vec{X}')$:

$$\vec{t} = \alpha'$$

$$\vec{X} = x_1\vec{n} + x_2\vec{b} + \sigma\vec{t}$$

$$\vec{X}' = (\sigma k_1 - x_2k_2)\vec{n} + (x_1k_2)\vec{b} + (-x_1k_1)\vec{t}$$

and

$$\det(\vec{t}, \vec{X}, \vec{X}') = x_1^2k_2 - x_2(\sigma k_1 - x_2k_2) = 0 \quad (25)$$

From Equation 25, we have:

$$\frac{k_1}{k_2} = \frac{x_1^2 + x_2^2}{x_2\sigma} \quad (26)$$

Since $x_1^2 + x_2^2 = 1$, we can write:

$$\frac{k_1}{k_2} = \frac{1}{x_2\sigma} \quad (27)$$

This completes the proof.

Corollary 1

We assume that the director curves of Φ' are in the normal planes and the base curve of Φ' is not a line. Then Φ' is developable if and only if α is a planer curve.

Corollary 2

We assume that the director curves of ϕ' are in the normal planes or the osculating planes and the base curve of ϕ' is not a line. Then ϕ' is developable if and only if α is a planer curve.

Corollary 3

We assume that the director curves of γ' are in the osculating planes and the base curve of γ' is not a line. Then γ' is developable if and only if α is a planer curve.

β-HELIX CURVES ON A RULED SURFACE IN EUCLIDEAN 3-SPACE

Definition 7

Let $\varphi(s,v) = \vec{\alpha}(s) + v\vec{X}(s)$ be a ruled surface in E^3 and let η be a curve on the surface φ. If the angle between the unit director vector $\vec{X}(s)$ and the unit tangent vector $\vec{t}$ of η at every point of the curve η is constant, then, the curve η is called as β_t-helix on the surface φ.

Definition 8

Let $\varphi(s,v) = \vec{\alpha}(s) + v\vec{X}(s)$ be a ruled surface in E^3

and let η be a curve on the surface φ. If the angle between the unit director vector $\vec{X}(s)$ and the unit normal vector $\vec{n}$ of η at every point of the curve η is constant, then the curve η is called β_n-helix on the surface φ.

Definition 9

Let $\varphi(s,v)=\vec{\alpha}(s)+v\vec{X}(s)$ be a ruled surface in E^3 and let η be a curve on the surface φ. If the angle between the unit director vector $\vec{X}(s)$ and the unit binormal vector $\vec{b}$ of η at every point of the curve η is constant, then the curve η is called β_b-helix on the surface φ.

Example 4

We consider the cone surface

$$\begin{aligned}\varphi(s,v)&=\left((1+v)\cos(s),(1+v)\sin(s),-\sqrt{2}v\right)\\&=\left(\cos(s),\sin(s),0\right)+v\left(\cos(s),\sin(s),-\sqrt{2}\right)\end{aligned}$$

For $v=e^s-1$, $\eta(s)=\left(e^s\cos(s),e^s\sin(s),(1-e^s)\sqrt{2}\right)$ is a curve on the surface φ, and the curve η is a β_t-helix on the surface φ. Actually,

$$<\frac{\eta'(s)}{\|\eta'(s)\|},\frac{\vec{X}(s)}{\|\vec{X}(s)\|}>=\frac{\sqrt{3}}{2}=const,$$

where

$$\eta'(s)=(e^s(\cos(s)-\sin(s)),e^s(\cos(s)+\sin(s)),\\ -\sqrt{2}e^s),$$

$$\vec{X}(s)=\left(\cos(s),\sin(s),-\sqrt{2}\right),$$

$$\|\eta'(s)\|=2e^s$$

and

$$\|\vec{X}(s)\|=\sqrt{3}.$$

Theorem 7

The base curve α of the surface $\Phi(s,v)=\vec{\alpha}(s)+v\left(\cos w(s)\vec{t}+\sin w(s)\vec{n}+\sigma\vec{b}\right)$ is a β_b-helix on the surface Φ.

Proof

In fact,

$$<\vec{b},\frac{\vec{X}(s)}{\|\vec{X}(s)\|}>=\frac{\sigma}{(1+\sigma^2)^{1/2}}=const, \quad (27)$$

where $\vec{b}$ is the unit binormal vector field of α, $\vec{X}(s)=\cos w(s)\vec{t}+\sin w(s)\vec{n}+\sigma\vec{b}$ and $\|\vec{X}(s)\|=\left(1+\sigma^2\right)^{1/2}$.

This completes the proof.

Theorem 8

The base curve α of the surface $\phi(s,v)=\vec{\alpha}(s)+v\left(\cos w(s)\vec{t}+\sin w(s)\vec{b}+\sigma\vec{n}\right)$ is a β_n-helix on the surface ϕ.

Proof

In fact,

$$<\vec{n},\frac{\vec{X}(s)}{\|\vec{X}(s)\|}>=\frac{\sigma}{(1+\sigma^2)^{1/2}}=const, \quad (28)$$

where $\vec{n}$ is the unit normal vector field of α, $\vec{X}(s)=\cos w(s)\vec{t}+\sin w(s)\vec{b}+\sigma\vec{n}$ and $\|\vec{X}(s)\|=\left(1+\sigma^2\right)^{1/2}$.

This completes the proof.

Theorem 9

The base curve α of the surface $\gamma(s,v)=\vec{\alpha}(s)+v\left(\cos w(s)\vec{n}+\sin w(s)\vec{b}+\sigma\vec{t}\right)$ is a β_t-helix on the surface γ.

Proof

In fact,

$$< \vec{t}, \frac{\vec{X}(s)}{\left\|\vec{X}(s)\right\|} >= \frac{\sigma}{(1+\sigma^2)^{1/2}} = const, \quad (29)$$

where $\vec{t}$ is the unit tangent vector field of α, $\vec{\mathrm{X}}(s) = \cos w(s)\vec{n} + \sin w(s)\vec{b} + \sigma\vec{t}$ and $\left\|\vec{\mathrm{X}}(s)\right\| = \left(1+\sigma^2\right)^{1/2}$.

This completes the proof.

Conclusion

In this study, the relationship between the Frenet motion and the ruled surfaces with constant slope was given.
A new type of general helix curves, which is called as β-helix, was defined. Also, it was given that the base curves of the constant slope surfaces Φ, φ and γ are β-helix curves.

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