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Full Length Research Paper

Soft matrices

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In this paper, soft matrices are defined based on soft set. Cartesian product of two soft sets is defined here. The very common operations of soft matrices are defined like AND, OR, union, intersection. commutative, associative, distributive, De Morgan's laws and convergent property for soft matrices are investigated.

Key words: Soft set, soft relation, soft matrices, AND, OR operations, complement, union and intersection of soft matrices, commutative, associative, distributive laws, De Morgan's laws, convergence.

INTRODUCTION

We can not successfully use classical method to solve complicated problem in economics, engineering and environment because of various uncertainties typical for those problems. There are theories, viz. theory of probability, theory of fuzzy sets (Zadeh, 1965), theory of intuitionistic fuzzy sets (Atanassov, 1986), theory of vague set (Gau and Buehrer, 1993), theory of interval mathematics (Moore, 1996) and theory of rough sets (Pawlak, 1982) which can be consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. The reason of the difficulties is, possibly, the inadequacy of the parametrization tool of the theories. To overcome these difficulties Molodtsov (1999) introduced the concept of soft set as a new mathematical tool for dealing with uncertainties which is free from the difficulties that have troubled the usual theoretical approaches.

Molodtsov successfully applied the soft theory into several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, theory of probability, theory of measurement and so on.

At present, works on the soft set theory are progressing rapidly. Maji et al. (2002) described the application of soft set theory to a decision making problem. They also studied (2003) several operations on the theory of soft set and fuzzy soft set like as `OR' operation, `AND' operation, `NOT' operation, complement, union, intersection, etc. Ali et al. (2009) define some new operations in soft set and proved the De Morgan's laws in soft set theory. Yang et al. (2009) combine the interval-valued fuzzy set and soft set.

DEFINITION AND PRELIMINARIES

In this section, we recall some basic notion of soft set theory introduced by Molodtsov (1999) and some useful definition from (Maji et al., 2001,2003; Ali et al., 2009). Here we take U to be an initial universal set and E to be a set of parameters and $A, B \subset E$.

Definition 1 (soft set)

A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U .

In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\varepsilon)$, $\varepsilon \in E$, from this family may be considered as the set of ε-approximate elements of the soft set.

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As an illustration, let us consider the following example.

Example 1

A soft set (F, E) describes the attractiveness of the bikes which Mr X is going to buy.

U is the set of bikes under consideration.
E is the set of parameters. Each parameter is a word or a sentence.
$E =$ ($e_1 =$ *stylish;* $e_2 =$ *heavy duty;* $e_3 =$ *light;* $e_4 =$ *steel body;* $e_5 =$ *cheap;* $e_6 =$ *good milage;* $e_7 =$ easily started; $e_8 =$ long driven; $e_9 =$ costly; $e_{10} =$ fiber body)
In this case, to define a soft set means to point out stylish bikes, heavy duty bikes, and so on.

Definition 2 (operation with soft sets)

Suppose a binary operation denoted by $*$, is defined for all subsets of the set U. Let (F, A) and (G, B) be two soft sets over U. Then the operation $*$ for the soft sets is defined in the following way:

$$(F, A) * (G, B) = (H, A \times B)$$

where $H(\alpha, \beta) = F(\alpha) * G(\beta)$, $\alpha \in A, \beta \in B$ and $A \times B$ is the cartesian product of the sets A and B.

Definition 3 (NOT set of a set of parameters)

Let $E = \{e_1, e_2, e_3, \ldots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\rceil E$ and is defined by $\rceil E = \{\neg e_1, \neg e_2, \neg e_3, \ldots, \neg e_n\}$ where $\neg e_i = not\ e_i \quad for\ all\ i$. It may be noted that $\rceil$ and $\neg$ are two different operations.

Definition 4 (complement of a soft set)

The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \rceil A)$ where

$F^c : \rceil A \to P(U)$ is a mapping which is defined by

$$F^c(\alpha) = U - F(\neg \alpha), \quad for\ all\ \alpha \in \rceil A.$$

Definition 5 (relative complement of a soft set)

The relative complement of a soft set (F, A) is denoted by $(F, A)^r$ and is defined by $(F, A)^r = (F^r, A)$ where $F^r : A \to P(U)$ is a mapping given by $F^r(\alpha) = U - F(\alpha), \quad for\ all\ \alpha \in A$.

Definition 6 (NULL soft set)

A soft set (F, A) over U is said to be a NULL soft set denoted by Φ, if $for\ all\ \varepsilon \in A, F(\varepsilon) = \phi$ (null-set).

Definition 7 (relative NULL soft set)

A soft set (F, A) over U is said to be relative NULL soft set with respect to parameter set A denoted by Φ_A if $\varepsilon \in A$, $F(\varepsilon) = \phi$ (null set).

Definition 8 (relative whole soft set)

A soft set (F, A) over U is said to be relative whole soft set (with respect to parameter set A) denoted by U_A, if $for\ all\ \varepsilon \in A, F(\varepsilon) = U$.

Definition 9 (absolute soft set)

The relative whole soft set U_E with respect to the universe set of parameters E is called the absolute soft set over U.

Definition 10 (AND operation on two soft sets)

If (F, A) and (G, B) be two soft sets then ``(F, A) AND (G, B)" denoted by $(F, A) \wedge (G, B)$ and is defined by

$(F, A) \wedge (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cap G(\beta) \quad for\ all\ (\alpha, \beta) \in A \times B$.

Definition 11 (OR operation on two soft sets)

If (F, A) and (G, B) be two soft sets then ``(F, A) OR (G, B)" denoted by $(F, A) \vee (G, B)$ is defined by

$(F,A)\vee(G,B)=(O,A\times B)$ where $O(\alpha,\beta)=F(\alpha)\cup G(\beta)$ *for all* $(\alpha,\beta)\in A\times B$.

Definition 12 (union of two soft sets)

Union of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C) where $C=A\cup B$ and *for all* $e\in C$

$$\begin{aligned} H(e) &= F(e), \quad if\ e\in A-B \\ &= G(e), \quad if\ e\in B-A \\ &= F(e)\cup G(e), \quad if\ e\in A\cap B \end{aligned}$$

We write $(F,A)\cup(G,B)=(H,C)$.

Definition 13 (restricted union of two soft set)

Let (F,A) and (G,B) be two soft sets over the same universe U such that $A\cap B\neq\Phi$. The restricted union of (F,A) and (G,B) is denoted by $(F,A)\cup_R(G,B)$, and is defined as $(F,A)\cup_R(G,B)=(H,C)$ where $C=A\cap B$ and *for all* $e\in C, H(e)=F(e)\cup G(e)$.

Definition 14 (extended intersection of two soft sets)

The extended intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C) where $C=A\cup B$ and *for all* $e\in C$

$$H(e)=\begin{cases} F(e), & if\ e\in A-B \\ G(e), & if\ e\in B-A \\ F(e)\cap G(e), & if\ e\in A\cap B \end{cases}$$

We write $(F,A)\cap_E(G,B)=(H,C)$.

Definition 15 (restricted intersection of two soft sets)

Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\Phi$. The restricted intersection of (F,A) and (G,B) is denoted by $(F,A)\cap_R(G,B)$, and is defined as $(F,A)\cap_R(G,B)=(H,C)$ where $C=A\cap B$ and for all $e\in C$, $H(e)=F(e)\cap G(e)$.

Definition 16 (soft relation)

A soft relation may be defined as a soft set over the power set of the cartesian product of two crisp sets. If X and Y are two non-empty crisp sets of some universal set and E is a set of parameters then a soft relation denoted as (R,E) is defined as a mapping from E to $P(X\otimes Y)$.

CARTESIAN PRODUCT AND RELATION BETWEEN TWO SOFT SETS

Using operation with soft sets defined by Molodtsov, the cartesian product and relation between two soft sets are defined below.

Definition 17 (Cartesian product of two soft sets)

Let (F,A) and (G,B) be two soft sets over a common universe U, then the cartesian product of these two soft sets is denoted by $(F,A)\times(G,B)$ and is defined by

$$(F,A)\times(G,B)=(H,A\times B) \quad \text{where}\ H(\alpha,\beta)=F(\alpha)\times G(\beta).$$

As an illustration let us consider the following example.

Example 2

For the problem in Example 1, let us consider two subsets A and B as

$A=$ ($e_1=$ stylish; $e_2=$ heavy duty) $\subset E$ and
$B=$ ($e_5=$ cheap; $e_9=$ costly) $\subset E$.

Then (F,A) describes the ``attractiveness of the bikes" and $(G,B)$ describes the ``cost of the bikes". Let $U=\{b_1,b_2,b_3,b_4,b_5\}$, $F(e_1)=\{b_2,b_4,b_5\}$, $F(e_2)=\{b_1,b_2,b_3\}$, $G(e_5)=\{b_1,b_5\}$ and $G(e_9)=\{b_2,b_4\}$.

Here $A\times B=\{(e_1,e_5),(e_1,e_9),(e_2,e_5),(e_2,e_9)\}$.
Then the cartesian product of (F,A) and (G,B) is $(H,A\times B)=(F,A)\times(G,B)$

where

$$H(e_1,e_5)=\{(b_2,b_1),(b_2,b_5),(b_4,b_1),(b_4,b_5),(b_5,b_1),(b_5,b_5)\}$$
$$H(e_1,e_9)=\{(b_2,b_2),(b_2,b_4),(b_4,b_2),(b_4,b_4),(b_5,b_2),(b_5,b_4)\}$$
$$H(e_2,e_5)=\{(b_1,b_1),(b_1,b_5),(b_2,b_1),(b_2,b_5),(b_3,b_1),(b_3,b_5)\}$$
$$H(e_2,e_9)=\{(b_1,b_2),(b_1,b_4),(b_2,b_2),(b_2,b_4),(b_3,b_2),(b_3,b_4)\}$$

Definition 18 (relation between two soft sets)

Let (F,A) and (G,B) be two soft sets over a common universe U, then the relation R of these two soft sets is defined as $(R,A\times B)\subset(F,A)\times(G,B)$ such that $F(\alpha)\times G(\beta)\in(R,A\times B)$ implies that $F(\alpha)$ is related with $G(\beta)$ where $\alpha\in A$ and $\beta\in B$.

Example 3

For the problem in Example 2 let (F,A) and (G,B) be two soft set over the common universe U and let

That is,

$$\begin{aligned}(R,A\times B) &= \{\{(b_2,b_1),(b_2,b_5),(b_4,b_1),(b_4,b_5),(b_5,b_1),(b_5,b_5)\},\\ &\quad \{(b_2,b_2),(b_2,b_4),(b_4,b_2),(b_4,b_4),(b_5,b_2),(b_5,b_4)\}\}\\ (R,A\times B) &= \{F(e_1)\times G(e_5),\ F(e_1)\times G(e_9)\}.\end{aligned}$$

Then we say that $F(e_1)$ is related with $G(e_5)$ and $G(e_9)$.

SOFT MATRIX AND ITS OPERATIONS

In soft matrix operations, we define soft matrix. Since matrix is a very important tool in any branches of mathematics so we motivated to study it over soft set.

Definition 19 (soft matrix)

Let (F,A) be a soft set defined on the universe U. Then a soft matrix over (F,A) is denoted by $[M(F,A)]$ is a matrix whose elements are the elements of the soft set (F,A).

$$Mathematically, [M(F,A)]=(m_{ij}) \quad \text{where } m_{ij}=F(\alpha) \text{ for some } \alpha\in A.$$

To illustrate soft matrix the following example is considered.

Example 4

Let us consider $A=E=\{e_1,e_2,e_3,\cdots,e_{10}\}$ and

$U=\{b_1,b_2,b_3,b_4,b_5\}$ where (F, A) ={ F (e_1) = stylish bikes ={b_2, b_4, b_5}, F (e_2) = heavy duty bikes = {b_1, b_2, b_3}, F (e_3) = light bikes = {b_1, b_2}, F (e_4) = steel body bikes = {b_3, b_5}, F (e_5) = cheap bikes = {b_1, b_3, b_5 }, F (e_6) = good milage bikes = {b_2, b_5 }, F (e_7) = easily started bikes = {b_3, b_4}, F (e_8) = long driven bikes = {b_1, b_3, b_4}, F (e_9) = costly bikes = {b_2, b_4}, F (e_{10}) = fiber body bikes = {b_1, b_2, b_4}}. Let

$$[M(F,A)]=\begin{bmatrix} \begin{matrix}\text{heavy duty bikes}\\ \{b_1,b_2,b_3\}\end{matrix} & \begin{matrix}\text{cheap bikes}\\ \{b_1,b_3,b_5\}\end{matrix} & \begin{matrix}\text{stylish bikes}\\ \{b_2,b_4,b_5\}\end{matrix} \\ \begin{matrix}\text{light bikes}\\ \{b_1,b_2\}\end{matrix} & \begin{matrix}\text{good milage bikes}\\ \{b_2,b_5\}\end{matrix} & \begin{matrix}\text{costly bikes}\\ \{b_2,b_4\}\end{matrix} \\ \begin{matrix}\text{cheap bikes}\\ \{b_1,b_3,b_5\}\end{matrix} & \begin{matrix}\text{light bikes}\\ \{b_1,b_2\}\end{matrix} & \begin{matrix}\text{long driven bikes}\\ \{b_1,b_3,b_4\}\end{matrix} \end{bmatrix}$$

Here we see that all the elements of the matrix $[M(F,A)]$ are of the soft set (F,A). Hence the above matrix is a soft matrix.

Definition 20 (AND operation between two soft matrices)

Let $[M(F,A)]=(m_{ij})$ and $[N(F,A)]=(n_{ij})$ are two soft matrices of same order over a common soft set (F,A). Then ``$[M(F,A)]$ AND $[N(F,A)]$" is denoted by $[M(F,A)]\wedge[N(F,A)]$ is a soft matrix $[L(F,A)]=(l_{ij})$ of same order that of $[M(F,A)]$ or $[N(F,A)]$, is define by $l_{ij}=m_{ij}\cap n_{ij}$.

Let us consider the following example.

Example 5

Let

$$[N(F,A)]=\begin{bmatrix} \begin{matrix}\text{steel body bikes}\\ \{b_3,b_5\}\end{matrix} & \begin{matrix}\text{stylish bikes}\\ \{b_2,b_4,b_5\}\end{matrix} & \begin{matrix}\text{easily strated bikes}\\ \{b_3,b_4\}\end{matrix} \\ \begin{matrix}\text{fiber body bikes}\\ \{b_1,b_2,b_4\}\end{matrix} & \begin{matrix}\text{good milage bikes}\\ \{b_2,b_5\}\end{matrix} & \begin{matrix}\text{cheap bikes}\\ \{b_1,b_3,b_5\}\end{matrix} \\ \begin{matrix}\text{heavy duty bikes}\\ \{b_1,b_2,b_3\}\end{matrix} & \begin{matrix}\text{fiber body bikes}\\ \{b_1,b_2,b_4\}\end{matrix} & \begin{matrix}\text{steel body bikes}\\ \{b_3,b_5\}\end{matrix} \end{bmatrix}$$

Then, $[M(F,A)] \wedge [N(F,A)]$

$$= \begin{bmatrix} \textit{heavy duty \& steel body bikes } \{b_3\} & \textit{cheap \& stylish bikes } \{b_5\} & \textit{stylish \& easily started bikes } \{b_4\} \\ \textit{light \& fiber body bikes } \{b_1, b_2\} & \textit{good milage bikes } \{b_2, b_5\} & \textit{costly \& cheapbicks } \phi \\ \textit{cheap \& heavy duty bikes } \{b_1, b_3\} & \textit{light \& fiber body bikes } \{b_1, b_2\} & \textit{long driven \& steel body bikes } \{b_3\} \end{bmatrix}$$

The symbol & is used to represents logical

Definition 21 (OR operation between two soft matrices)

Let $[M(F,A)] = (m_{ij})$ and $[N(F,A)] = (n_{ij})$ be two soft matrices of same order over a common soft set (F,A). Then ``$[M(F,A)]$ OR $[N(F,A)]$" is denoted by $[M(F,A)] \vee [N(F,A)]$ is a soft matrix $[L(F,A)] = (l_{ij})$ of same order that of $[M(F,A)]$ or $[N(F,A)]$, is define by $l_{ij} = m_{ij} \cup n_{ij}$.

Example 6

The matrix $[M(F,A)] \vee [N(F,A)]$, for the previous matrices $[M(F,A)] and [N(F,A)]$ is

$$\begin{bmatrix} \textit{heavy duty or steel body bikes } \{b_1, b_2, b_3, b_5\} & \textit{cheap or stylish bikes } \{b_1, b_2, b_3, b_4, b_5\} & \textit{stylish or easily started bikes } \{b_2, b_3, b_4, b_5\} \\ \textit{light or fiber body bikes } \{b_1, b_2, b_4\} & \textit{good milage bikes } \{b_2, b_5\} & \textit{costly or cheap bikes } \{b_1, b_2, b_3, b_4, b_5\} \\ \textit{cheap or heavy duty bikes } \{b_1, b_2, b_3, b_5\} & \textit{light or fiber body bikes } \{b_1, b_2, b_4\} & \textit{long driven or steel body bikes } \{b_1, b_3, b_4, b_5\} \end{bmatrix}$$

Definition 22 (complement of a soft matrix)

Let $[M(F,A)] = (m_{ij})$ be a soft matrix over a soft set (F,A) with respect to a universe U. The complement of the soft matrix $[M(F,A)]$ is denoted by $[M(F,A)]^c$, where $[M(F,A)]^c = (m_{ij}^c)$ is a soft matrix of same order that of $[M(F,A)]$ and is defined by

$$m_{ij}^c = U - F(\alpha), \quad \textit{where } m_{ij} = F(\alpha) \textit{ for some } \alpha \in A.$$

To illustrate complement, let us consider the following example.

Example 7

For previous $[M(F,A)]$

$$[M(F,A)]^c = \begin{bmatrix} \textit{not heavy duty bikes } \{b_4, b_5\} & \textit{not cheap bikes } \{b_2, b_4\} & \textit{not stylish bikes } \{b_1, b_3\} \\ \textit{not light bikes } \{b_3, b_4, b_5\} & \textit{not good milage bikes } \{b_1, b_3, b_4\} & \textit{not costly bikes } \{b_1, b_3, b_5\} \\ \textit{not cheap bikes } \{b_2, b_4\} & \textit{not light bikes } \{b_3, b_4, b_5\} & \textit{not long driven bikes } \{b_2, b_5\} \end{bmatrix}$$

It may be observed that $([M(F,A)]^c)^c = [M(F,A)]$.

Proposition 1

Let $[M(F,A)]$ and $[N(F,A)]$ be two soft matrices of same order over a common soft set (F,A). Then

(i) $[M(F,A)] \wedge [N(F,A)] = [N(F,A)] \wedge [M(F,A)]$

(ii) $[M(F,A)] \vee [N(F,A)] = [N(F,A)] \vee [M(F,A)]$

That is, commutative properties holds for soft matrices.

Proof (i)

Let $[M(F,A)] = (m_{ij})$ and $[N(F,A)] = (n_{ij})$ where $m_{ij} = F(\alpha)$, $n_{ij} = F(\beta)$ for some $\alpha, \beta \in A$.

Now the ij th element of $[M(F,A)] \wedge [N(F,A)]$ is $m_{ij} \cap n_{ij} = n_{ij} \cap m_{ij}$ [By commutative property for crisp sets].

Also the ij th element of $[N(F,A)] \wedge [M(F,A)]$ is $n_{ij} \cap m_{ij}$.

Hence $[M(F,A)] \wedge [N(F,A)] = [N(F,A)] \wedge [M(F,A)]$.

Similarly we can prove the second proposition Ω.

Proposition 2

Let $[L(F,A)]$, $[M(F,A)]$ and $[N(F,A)]$ be three soft matrices of same order over a common soft set (F,A). *Then*

(i) $([L(F,A)] \wedge [M(F,A)]) \wedge [N(F,A)] = [L(F,A)] \wedge ([M(F,A)] \wedge [N(F,A)])$

(ii) $([L(F,A)] \vee [M(F,A)]) \vee [N(F,A)] = [L(F,A)] \vee ([M(F,A)] \vee [N(F,A)])$

That is, associative properties holds for soft matrices.

Proof (ii)

Let $[L(F,A)]=(l_{ij})$, $[M(F,A)]=(m_{ij})$ and $[N(F,A)]=(n_{ij})$.
Then the ij th element of $([L(F,A)]\vee[M(F,A)])\vee[N(F,A)]$ is $(l_{ij}\cup m_{ij})\cup n_{ij}$. Similarly the ij th element of $[L(F,A)]\vee([M(F,A)]\vee[N(F,A)])$ is $l_{ij}\cup(m_{ij}\cup n_{ij})$.
Also we know that for crisp sets l_{ij}, m_{ij} and n_{ij}, $(l_{ij}\cup m_{ij})\cup n_{ij}=l_{ij}\cup(m_{ij}\cup n_{ij})$ [By associative property].
Hence
$([L(F,A)]\vee[M(F,A)])\vee[N(F,A)]=[L(F,A)]\vee([M(F,A)]\vee[N(F,A)])$.
Similarly we can prove the first proposition.

Proposition 3

Let $[L(F,A)]$, $[M(F,A)]$ and $[N(F,A)]$ be three soft matrices of same order over a common soft set (F,A). Then

$$(i)\quad [L(F,A)]\wedge([M(F,A)]\vee[N(F,A)])=([L(F,A)]\wedge[M(F,A)])\vee([L(F,A)]\wedge[N(F,A)])$$
$$(ii)\quad [L(F,A)]\vee([M(F,A)]\wedge[N(F,A)])=([L(F,A)]\vee[M(F,A)])\wedge([L(F,A)]\vee[N(F,A)])$$

That is, distributive properties holds for soft matrices.

Proof (i)

Let $[L(F,A)]=(l_{ij})$, $[M(F,A)]=(m_{ij})$ and $[N(F,A)]=(n_{ij})$.
Then the ij th element of $[L(F,A)]\wedge([M(F,A)]\vee[N(F,A)])$ is

$l_{ij}\cap(m_{ij}\cup n_{ij})=(l_{ij}\cap m_{ij})\cup(l_{ij}\cap n_{ij})$ [*By distributive property of crisp sets*]

Also the ij th element of $([L(F,A)]\wedge[M(F,A)])\vee([L(F,A)]\wedge[N(F,A)])$ is $(l_{ij}\cap m_{ij})\cup(l_{ij}\cap n_{ij})$
Hence

$[L(F,A)]\wedge([M(F,A)]\vee[N(F,A)])=([L(F,A)]\wedge[M(F,A)])\vee([L(F,A)]\wedge[N(F,A)])$

The proof of the second proposition is similar.

Theorem 1

The following De Morgan's laws are valid for soft matrices

$$(i)\quad ([M(F,A)]\vee[N(F,A)])^c=[M(F,A)]^c\wedge[N(F,A)]^c$$
$$(ii)\quad ([M(F,A)]\wedge[N(F,A)])^c=[M(F,A)]^c\vee[N(F,A)]^c$$

Proof (i)

Let $[M(F,A)]=(m_{ij})$ and $[N(F,A)]=(n_{ij})$, then $m_{ij}=F(\alpha)$, $n_{ij}=F(\beta)$ for some $\alpha,\beta\in A$. Now the ij th element of $[M(F,A)]\vee[N(F,A)]$ is $a_{ij}=m_{ij}\cup n_{ij}=F(\alpha)\cup F(\beta)$.

Therefore ij th element of $([M(F,A)]\vee[N(F,A)])^c$ is

$$\begin{aligned} a_{ij}^c &= U-a_{ij} \\ &= U-F(\alpha)\cup F(\beta) \\ &= \{U-F(\alpha)\}\cap\{U-F(\beta)\} \\ &= m_{ij}^c\cap n_{ij}^c. \end{aligned}$$

which is the ij th element of $[M(F,A)]^c\wedge[N(F,A)]^c$.

Hence

$([M(F,A)]\vee[N(F,A)])^c=[M(F,A)]^c\wedge[N(F,A)]^c$.

(ii) Here the ij th element of $[M(F,A)]\wedge[N(F,A)]$ is

$b_{ij}=m_{ij}\cap n_{ij}=F(\alpha)\cap F(\beta)$.

Therefore ij th element of $([M(F,A)]\wedge[N(F,A)])^c$ is

$$\begin{aligned} b_{ij}^c &= U-b_{ij} \\ &= U-F(\alpha)\cap F(\beta) \\ &= \{U-F(\alpha)\}\cup\{U-F(\beta)\} \\ &= m_{ij}^c\cup n_{ij}^c. \end{aligned}$$

this is the ij th element of $[M(F,A)]^c\vee[N(F,A)]^c$.

Hence

$$([M(F,A)] \wedge [N(F,A)])^c = [M(F,A)]^c \vee [N(F,A)]^c$$

UNION AND INTERSECTION OF SOFT MATRICES

In this section we define the union and intersection of two soft matrices.

Definition 23 (union of two soft matrices)

Let $[M(F,A)] = (m_{ij})$ *and* $[N(F,A)] = (n_{ij})$ *be two soft matrices of any order over a common soft set* (F,A)*, then* $m_{ij} = F(\alpha)$ *and* $n_{ij} = F(\beta)$ *for some* $\alpha, \beta \in A$*. The union of* $[M(F,A)]$ *and* $[N(F,A)]$ *is denoted by* $[M(F,A)] \cup [N(F,A)] = [L(F,A)]$*, where* $[L(F,A)] = (l_{ij})$ *is a soft matrix whose number of rows is equal to the number of rows of* $[M(F,A)]$ *and number of columns is equal to the number of columns of* $[N(F,A)]$ *and is defined by*

$l_{ij} = \bigcup_{\alpha} F(\alpha)$, where α is the common parameter of the ith row of $[M(F,A)]$ and jth column of $[N(F,A)]$.

As an illustration we consider the following example.

Example 8

Let $B = \{e_1, e_2, e_3, e_4\} \in E$ *and*

$(G,B) = \{G(e_1) = stylish\ bikes = \{b_2, b_4, b_5\}, G(e_2) = heavy\ duty\ bikes = \{b_1, b_2\}, G(e_3) = light\ bikes = \{b_1, b_2, b_3\}, G(e_4) = steel\ body\ bikes = \{b_3, b_5\}\}$.

Let us consider two matrices $[P(G,B)]$ and $[Q(G,B)]$ as:

$$[P(G,B)] = \begin{bmatrix} \begin{matrix} stylish\ bikes \\ \{b_2,b_4,b_5\} \end{matrix} & \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} heavy\ duty\ bikes \\ \{b_1,b_2\} \end{matrix} \\ \begin{matrix} steel\ body\ bikes \\ \{b_3,b_5\} \end{matrix} & \begin{matrix} heavy\ duty\ bikes \\ \{b_1,b_2\} \end{matrix} & \begin{matrix} stylish\ bikes \\ \{b_2,b_4,b_5\} \end{matrix} \\ \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} steel\ body\ bikes \\ \{b_3,b_5\} \end{matrix} & \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} \end{bmatrix}$$

$$[Q(G,B)] = \begin{bmatrix} \begin{matrix} steel\ body\ bikes \\ \{b_3,b_5\} \end{matrix} & \begin{matrix} heavy\ duty\ bikes \\ \{b_1,b_2\} \end{matrix} & \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} \\ \begin{matrix} stylish\ bikes \\ \{b_2,b_4,b_5\} \end{matrix} & \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} steel\ body\ bikes \\ \{b_3,b_5\} \end{matrix} \\ \begin{matrix} heavy\ duty\ bikes \\ \{b_1,b_2\} \end{matrix} & \begin{matrix} stylish\ bikes \\ \{b_2,b_4,b_5\} \end{matrix} & \begin{matrix} heavy\ duty\ bikes \\ \{b_1,b_2\} \end{matrix} \end{bmatrix}$$

Then P(G,B)] ∪ [Q(G,B).

$$= \begin{bmatrix} \begin{matrix} stylish\ or\ heavy\ duty\ bikes \\ \{b_1,b_2,b_4,b_5\} \end{matrix} & \begin{matrix} stylish\ or\ light\ or\ heavy\ duty \\ bikes\ \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} light\ or\ heavy\ duty\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} \\ \begin{matrix} steel\ body\ or\ stylish\ or\ heavy \\ duty\ bikes\ \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} stylish\ or\ heavy\ duty\ bikes \\ \{b_1,b_2,b_4,b_5\} \end{matrix} & \begin{matrix} steel\ body\ or\ heavy\ duty \\ bikes\ \{b_1,b_2,b_3,b_5\} \end{matrix} \\ \begin{matrix} steel\ body\ bikes \\ \{b_3,b_5\} \end{matrix} & \begin{matrix} light\ bikes \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} light\ or\ steel\ body\ bikes \\ \{b_1,b_2,b_3,b_5\} \end{matrix} \end{bmatrix}$$

If there exists no such common parameter then the element is denoted by θ, called the null element.

Definition 24 (intersection of two soft matrices)

Let $[M(F,A)] = (m_{ij})$ and $[N(F,A)] = (n_{ij})$ be two soft matrices of any order over a common soft set (F,A), then $m_{ij} = F(\alpha)$ and $n_{ij} = F(\beta)$ for some $\alpha, \beta \in A$. The intersection of $[M(F,A)]$ and $[N(F,A)]$ is denoted by $[M(F,A)] \cap [N(F,A)] = [L(F,A)]$, where $[L(F,A)] = (l_{ij})$ is a soft matrix whose number of rows is equal to the number of rows of $[M(F,A)]$ and number of columns is equal to the number of columns of $[N(F,A)]$ and is defined by

$l_{ij} = \bigcap_{\alpha} F(\alpha)$, where α is any parameter of the ith row of $[M(F,A)]$ or jth column of $[N(F,A)]$.

Example 9

For the previous matrices $[P(G,B)]$ and $[Q(G,B)]$, the intersection is given by P(G,B) ∩ Q(G,B)

$$= \begin{bmatrix} stylish\&light\&steelbody\ \&heavydutybikes\ \phi & stylish\&light\&heavyduty\ bikes\{b_2\} & stylish\&light\&steelbody\ \&heavydutybikes\ \phi \\ steelbody\&stylish\&heavy\ dutybikes\ \phi & steelbody\&stylish\&light\ \&heavydutybikes\ \phi & steelbody\&stylish\&light\ \&heavydutybikes\ \phi \\ light\&stylish\&steelbody\ \&heavydutybikes\ \phi & light\&stylish\&steelbody\ \&heavydutybikes\ \phi & light\&steelbody\&heavy\ dutybikes\ \phi \end{bmatrix}$$

Proposition 4

Let $[L(F,A)]$, $[M(F,A)]$ and $[N(F,A)]$ be three soft matrices over a common soft set (F,A), then

$(i)\quad ([L(F,A)]\cup[M(F,A)])\cup[N(F,A)] = [L(F,A)]\cup([M(F,A)]\cup[N(F,A)])$

$(ii)\quad ([L(F,A)]\cap[M(F,A)])\cap[N(F,A)] = [L(F,A)]\cap([M(F,A)]\cap[N(F,A)])$

That is, associative properties holds for soft matrices with respect to union and intersection.

Proof (i)

Let $[L(F,A)]\cup[M(F,A)] = (a_{ij})$ then,

$a_{ij} = \bigcup_{\alpha} F(\alpha)$, where α is the common parameter of the ith row of $[L(F,A)]$ and jth column of $[M(F,A)]$.

Also let $([L(F,A)]\cup[M(F,A)])\cup[N(F,A)] = (b_{ij})$ then,

$b_{ij} = \bigcup_{\alpha} F(\alpha)$, where α is the common parameter of the ith row of $[L(F,A)]\cup[M(F,A)]$ and jth column of $[N(F,A)]$.

Also it can easily be shown that the common parameters of ith row of $[L(F,A)]\cup[M(F,A)]$ are the parameters of ith row of $[L(F,A)]$. Thus,

$b_{ij} = \bigcup_{\alpha} F(\alpha)$, where α is the common parameter of the ith row of $[L(F,A)]$ and jth column of $[N(F,A)]$.

Again let $[M(F,A)]\cup[N(F,A)] = (c_{ij})$ then,

$c_{ij} = \bigcup_{\beta} F(\beta)$, where β is the common parameter of the ith row of $[M(F,A)]$ and jth column of $[N(F,A)]$.

Therefore, if

$[L(F,A)]\cup([M(F,A)]\cup[N(F,A)]) = (d_{ij})$ then,

$d_{ij} = \bigcup_{\beta} F(\beta)$, where β is the common parameter of the ith row of $[L(F,A)]$ and jth column of $[M(F,A)]\cup[N(F,A)]$. i.e., β is the common parameter of the ith row of $[L(F,A)]$ and jth column of $[N(F,A)]$.

[$\because$ The common parameters of jth column of $[M(F,A)]\cup[N(F,A)]$ are the parameters of jth column of $[N(F,A)]$.]

Thus $b_{ij} = d_{ij}$.

That is, $([L(F,A)]\cup[M(F,A)])\cup[N(F,A)] = [L(F,A)]\cup([M(F,A)]\cup[N(F,A)])$.

Similarly, we can prove the second proposition.

Remark 1

From the proof of the above theorem we conclude that if $[L(F,A)]$, $[M(F,A)]$ and $[N(F,A)]$ be three soft matrices over a common soft set (F,A), then,

$$(i)\quad ([L(F,A)]\cup[M(F,A)])\cup[N(F,A)] = [L(F,A)]\cup[N(F,A)] = [L(F,A)]\cup([M(F,A)]\cup[N(F,A)])$$

$$(ii)\quad ([L(F,A)]\cap[M(F,A)])\cap[N(F,A)] = [L(F,A)]\cap[N(F,A)] = [L(F,A)]\cap([M(F,A)]\cap[N(F,A)]).$$

CONVERGENCE OF SOFT MATRIX

In this section we introduce the concept of convergence and power of convergence of a soft matrix.

A sequence of matrices $A_1, A_2, A_3, \cdots, A_n, A_{n+1}, \cdots$ That is, $\{A_n\}$ is said to be converge to a finite matrix A (if exist) if

$$\lim_{n\to\infty} A_n = A.$$

Definition 25 (power of convergence of a soft matrix)

A number p is said to be the power of convergence of a soft matrix $[M(F,A)]$ with respect to a binary composition $*$ if

$$[M(F,A)]^{p+n} = [M(F,A)]^{p+n-1} = [M(F,A)]^{p+n-2} = \cdots = [M(F,A)]^{p+1} = [M(F,A)]^{p}.$$

where $n \in N$ and

$$[M(F,A)]^2 = [M(F,A)] * [M(F,A)]$$

$$[M(F,A)]^3 = [M(F,A)] * [M(F,A)] * [M(F,A)]$$

and so on.

Theorem 2

The power of convergence with respect to union and intersection of a soft matrix is 2.

Proof

Let $[M(F,A)] = (a_{ij})$ be a soft matrix of any order. Then $a_{ij} = F(\alpha)$ for some $\alpha \in A$.

Also let $[M(F,A)] \cup [M(F,A)] = (b_{ij})$, then $b_{ij} = \bigcup_{\alpha} F(\alpha)$, where α be the common parameter of i th row and j th column of $[M(F,A)]$.

$\therefore ([M(F,A)] \cup [M(F,A)]) \cup [M(F,A)] = (b_{ij}) \cup (a_{ij}) = (c_{ij})$ (say). Then

$$\begin{aligned} c_{ij} &= \bigcup_{\beta} F(\beta), \text{ where } \beta \text{ be the common parameter of } i\text{th row of } [M(F,A)] \cup [M(F,A)] \\ &\quad \text{and } j\text{th column of } [M(F,A)] \\ &= \bigcup_{\beta} F(\beta), \text{ where } \beta \text{ be the common parameter of } i\text{th row of } [M(F,A)] \text{ and } j\text{th} \\ &\quad \text{column of } [M(F,A)] \\ &= \bigcup_{\beta} F(\beta), \text{ where } \beta \text{ be the common parameter of } i\text{th row and } j\text{th column of} \\ &\quad [M(F,A)] \\ &= b_{ij}. \end{aligned}$$

Hence $([M(F,A)] \cup [M(F,A)]) \cup [M(F,A)] = [M(F,A)] \cup [M(F,A)]$. Similarly, we can prove that $([M(F,A)] \cap [M(F,A)]) \cap [M(F,A)] = [M(F,A)] \cap [M(F,A)]$. Hence the theorem.

As an illustration we consider the following example.

Example 10

For the previous matrices $[P(G,B)]$ we have $[P(G,B)] \cup [P(G,B)]$

$$= \begin{bmatrix} \begin{matrix} \textit{stylish or light bikes} \\ \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} \textit{light or heavy duty bikes} \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} \textit{stylish or light or heavy} \\ \textit{duty bikes } \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} \\ \begin{matrix} \textit{stylish or steel body bikes} \\ \{b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} \textit{steel body or heavy duty bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{heavy duty or stylish bikes} \\ \{b_1,b_2,b_4,b_5\} \end{matrix} \\ \begin{matrix} \textit{steel body or light bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{light or steel body bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{light bikes} \\ \{b_1,b_2,b_3\} \end{matrix} \end{bmatrix}$$

and $[P(G,B)] \cup [P(G,B)] \cup [P(G,B)]$

$$= \begin{bmatrix} \begin{matrix} \textit{stylish or light bikes} \\ \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} \textit{light or heavy duty bikes} \\ \{b_1,b_2,b_3\} \end{matrix} & \begin{matrix} \textit{stylish or light or heavy} \\ \textit{duty bikes } \{b_1,b_2,b_3,b_4,b_5\} \end{matrix} \\ \begin{matrix} \textit{stylish or steel body bikes} \\ \{b_2,b_3,b_4,b_5\} \end{matrix} & \begin{matrix} \textit{steel body or heavy duty bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{heavy duty or stylish bikes} \\ \{b_1,b_2,b_4,b_5\} \end{matrix} \\ \begin{matrix} \textit{steel body or light bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{light or steel body bikes} \\ \{b_1,b_2,b_3,b_5\} \end{matrix} & \begin{matrix} \textit{light bikes} \\ \{b_1,b_2,b_3\} \end{matrix} \end{bmatrix}$$

Thus

$$[P(G,B)] \cup [P(G,B)] \cup [P(G,B)] = [P(G,B)] \cup [P(G,B)]$$

Theorem 3

If all the elements of a soft matrix are distinct then the power of convergence is 1, with respect to union and intersection.

Proof

Let us consider a soft matrix $[M(F,A)] = (a_{ij})$ where each a_{ij} are distinct and $a_{ij} = F(\alpha)$ for some $\alpha \in A$. Then,

$[M(F,A)] \cup [M(F,A)] = \bigcup_{\alpha} F(\alpha)$, where α be the common parameter of i th row and j th column of $[M(F,A)]$.

or, $[M(F,A)] \cup [M(F,A)] = (a_{ij})$

Since all a_{ij} are distinct, the common element is the ij th element of the matrix. That is, $[M(F,A)] \cup [M(F,A)] = [M(F,A)]$.

Similarly, we can prove the above theorem for intersection.

Remark 2

The power of convergence of a soft matrix with respect to union and intersection is at most 2.

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