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Full Length Research Paper

Solving traveling salesman problem by using a fuzzy multi-objective linear programming

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The traveling salesman problem (TSP) is one of the most intensively studied problems in computational mathematics. Information about real life systems is often available in the form of vague descriptions. Hence, fuzzy methods are designed to handle vague terms, and are most suited to finding optimal solutions to problems with vague parameters. This study develops a fuzzy multi-objective linear programming (FMOLP) model with piecewise linear membership function for solving a multi-objective TSP in order to simultaneously minimize the cost, distance and time. The proposed model yields a compromise solution and the decision maker's overall levels of satisfaction with the determined objective values. The primary contribution of this paper is a fuzzy mathematical programming methodology for solving the TSP in uncertain environments. A numerical example is solved to show the effectiveness of the proposed approach. The performance of proposed model with Zimmerman and Hannan's methods is compared. Computational results show that the proposed FMOLP model achieves higher satisfaction degrees.

Key words: Traveling salesman problem, fuzzy multi-objective linear programming, decision maker.

INTRODUCTION

The traveling salesman problem (TSP) is one of the well-studied NP-hard combinatorial optimization problems which determines the closed route of the shortest length or of the minimum cost (or time) passing through a given set of cities where each city is visited exactly once (Majumdar and Bhunia, 2011). In other words, find a minimal Hamiltonian tour in a complete graph of N nodes.

The first instance of the TSP was from Euler in 1759 whose problem was to move a knight to every position on a chess board exactly once (Michalewicz, 1994). The traveling salesman first gained fame in a book written by German salesman BF Voigt in 1832 on how to be a successful traveling salesman (Michalewicz, 1994). He mentions the TSP, although not by that name, by suggesting that to cover as many locations as possible without visiting any location twice is the most important aspect of the scheduling of a tour. The origins of the TSP in mathematics are not really known - all we know for certain is that it happened around 1931 (Bryant, 2000).

In TSP as a multi-objective combinatorial optimization problem, each objective function is represented in a distinct dimension. Of this form, to decide the multi objective TSP in the optimality means to determine the k-dimensional points that pertaining to the space of feasible solutions of the problem and that possess the minimum possible values according to all dimension. The permissible deviation from a specified value of a structural dimension is also considerable because traveling sales man can face a situation in which he is not able to achieve his objectives completely. There must be a set of alternatives from which he can select one that best meets his aspiration level. A conventional programming approach does not deal with this situation however some researchers have specifically treated the multi-objective TSP (Rehmat et al., 2007). Branch and Bound approach was used to solve TSP with two sum criteria (Chaudhuri and De, 2011) An E-constrained based algorithm for Bi-Objective TSP was suggested by Melamed and Sigal (1997) and Rehmat et al. (2007). A

Abbreviations: TSP, Traveling salesman problem; **FMOLP,** fuzzy multi-objective linear programming; **FLP,** fuzzy linear programming; **MOLP,** multi- objective linear programming; **DM,** decision maker; **LP,** linear programming.

Table 1. Some of the recent solution strategies in the literature for TSP.

Author(s)	Year	Solution strategy
Chaudhuri et al.	2011	Fuzzy multi- objective linear programming
Rehmat et al.	2007	Fuzzy multi- objective linear programming
Paquete et al.	2004	Pareto local search method which extended local search algorithm
Angel et al.	2004	Dynamic search algorithm by local search
Yan et al.	2003	Evolutionary algorithm
Jaszkiewicz	2002	Genetic local search

brief solution strategy in the accessible literature is summarized in Table 1.

In most real world problems it is not possible to have all constraints and resources in exact form rather they are in expected or vague form. This leads to use of Fuzzy Logic which enables us to emulate human reasoning process and make decisions based on vague or imprecise data. Fuzzy Programming gives methodology of solving problems in Fuzzy environment (Chaudhuri and De, 2011).

In this work a paradigm deals with vague parameters and achieves certain aspiration level of optimality for multi-objective symmetric TSP by transforming it into a linear program using fuzzy multi- objective linear programming (FMOLP) technique. The route selection of problem is done by exploiting aspiration level parameters.

The aim of this paper is to develop a FMOLP model for solving the multi-objective TSP in the fuzzy environment. We compared in a case study the results obtained by the proposed model, with Zimmerman and Hannan's methods. The structure of the paper is as follows: Subsequently, the problem description of this study is shown, after which the fuzzy multi-objective programming is presented. This is followed by a case study for TSP which is solved. Finally, this paper is concluded.

PROBLEM DESCRIPTION

The proposed TSP attempts simultaneously to minimize the cost, distance and time. Set of indices, parameters and decision variables for the multi-objective linear programming (MOLP) model are defined in the nomenclature (Table 2).

Objective functions

Objective function for minimization of cost:

$$Z_1 : \mathrm{Min} \sum_i^n \sum_j^n C_{ij} X_{ij} \tag{1}$$

Objective function for minimization of distance:

$$Z_2 : \mathrm{Min} \sum_i^n \sum_j^n d_{ij} X_{ij} \tag{2}$$

Objective function for minimization of total time:

$$Z_3 : \mathrm{Min} \sum_i^n \sum_j^n t_{ij} X_{ij} \tag{3}$$

Constraints

$$\sum_i^n X_{ij} = 1 \quad \text{for all} \quad j, \tag{4}$$

$$\sum_i^n X_{ij} = 1 \quad \text{for all} \quad i, \tag{5}$$

A route can not be selected more than once. That is,

$$X_{ij} + X_{ji} \le 1 \quad \text{for} \quad \text{all} \quad i, j \tag{6}$$

And the non-negativity constraints:

$$X_{ij} \ge 0 \tag{7}$$

FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING

Fuzzy multi-objective linear programming, an amalgamation of fuzzy logic and multi-objective linear programming, deals with flexible aspiration levels or goals and fuzzy constraints with acceptable deviations.

Zimmermann's method

In 1978, Zimmermann first extended his fuzzy linear programming (FLP) approach to a conventional MOLP problem (Javadia et al., 2008). For each of the objective functions of this problem, assume that the decision maker (DM) has a fuzzy goal such as 'the objective functions should be essentially less than or equal to some value'.

Table 2. Nomenclature.

Sets of indices i, j	Set of cities (i, j= 1, 2, ..., n)
Decision variables X_{ij}	$X_{ij} = \begin{cases} 1 & \text{if city j is visited from city i} \\ 0 & \text{Otherwise} \end{cases}$
Objective functions	
Z_1:	Total cost
Z_2:	Total distance
Z_3:	Total time
Parameters	
C_{ij}:	The cost of traveling from city i to city j
d_{ij}:	The distance from city i to city j
t_{ij}:	The time spent in traveling from city i to city j

Then, the corresponding linear membership function is defined and the minimum operator proposed by Bellman and Zadeh (1970) is applied to combine all the objective functions.

In this method, decision maker introduces tolerances to accommodate vagueness. By adjusting tolerances, range of solutions with different aspiration level are found from which decision maker chooses one that best meets his satisfactory level within given domain. An adopted fuzzy model due to Zimmerman is:

$$\text{Min}CX \leq\sim Z^0$$
S.t
$$Ax \leq\sim b$$

Where $Z^0 = (Z_1^0, Z_2^0, ..., Z_n^0)$ are the goals or aspiration levels and >~ and <~ are the fuzzy inequalities that are fuzzification of ≤ and ≤ respectively. For measurement of satisfaction levels of objectives and constraints Zimmerman suggested the simplest kind of membership function (Chaudhuri and De, 2011):

$$\mu_{1k}(C_k X) = \begin{cases} 0 & C_k X \geq Z_k^0 + t_k \\ 1 - \dfrac{(c_k x - z_k^0)}{t_k} & Z_k^0 \leq C_k X \leq Z_k^0 + t_k \\ 1 & C_k X \leq Z_k^0 \end{cases} \quad (8)$$

$$\begin{aligned} &\text{Max}L \\ &\text{S.t} \\ &L \leq 1 - \frac{(c_k x - z_k^0)}{t_k} \\ &Ax \leq b \end{aligned} \quad (9)$$

Where k=1,…, n and t_k is the admissible violation for the objective z_k, which is decided by the decision maker.

Hannan's method

The crisp MOLP model can be extended to the FMOLP model using the piecewise linear membership function of Hannan (1981) to represent the fuzzy goals of the DM in the MOLP model, together with the fuzzy decision-making of Bellman and Zadeh (1970). The piecewise linear membership functions and the fuzzy decision-making of Bellman and Zadeh converts the problem into one that is suitable for solution by an ordinary LP solver (by introducing the auxiliary variable L, the auxiliary variable L (0 ≤ L ≤ 1) represents overall DM satisfaction with the given objective values). The algorithm follows here (Javadia et al., 2008; Tavakoli-Moghaddam et al., 2010; Liang, 2006).

Step 1: Formulate the original fuzzy MOLP model for the TSP according to Equation 1 to 7.

Step 2: Specify the degree of membership f_j (z_j) for several values of each objective functions z_j, j = 1, 2 and 3 (Table 3).

Step 3: Draw the piecewise linear membership functions (z_j, f_j (z_j)), j = 1, 2 and 3.

Step 4: Formulate the linear equations for each of the piecewise linear membership functions f_j (Z_j) (i = 1, 2 and 3).

Step 4a: Convert the membership functions $f_j(Z_j)$ into the form (Table 3):

$$f_j(Z_j) = \sum_{b=1}^{p_j} \alpha_{jb} \left| Z_{j-Y_{jb}} \right| + \beta_j Z_j + \theta_j, \alpha_{jb} = -\frac{\gamma_{j,b+1} - \gamma_j}{2},$$

$$\beta_j = \frac{\gamma_{j,V_{j+1}} + \gamma_{j1}}{2}, \theta_j = \frac{S_{j,V_{j+1}} + S_{j1}}{2} \quad j = 1,2,3 \quad (10)$$

Table 3. Membership function $f_j(Z_j)$.

Z_1	$> Y_{10}$	Y_{10}	Y_{11}	Y_{12}	…	Y_{1,V_1+1}	$Y_{1,V_1+1} <$
$f(Z_1)$	0	0	q_{11}	q_{12}	…	1	1
Z_2	$>Y_{20}$	Y_{20}	Y_{21}	Y_{22}	…	Y_{2,V_2+1}	$Y_{2,V_2+1} <$
$f(Z_2)$	0	0	q_{21}	q_{22}	…	1	1
Z_3	$>Y_{30}$	Y_{30}	Y_{31}	Y_{32}	…	Y_{3,V_3+1}	$Y_{3,V_3+1} <$
$f(Z_3)$	0	0	q_{31}	q_{32}	…	1	1

($0\ q_{jb}\ 1$, $q_{jb}\ q_{jb+1}$, j=1,2, b=1,2,…,v_j).

Assume that $f_j(Z_j) = \gamma_{jr} Z_j + S_{jr}$ for each segment $Y_{j,r-1} \leq Z_j \leq Y_{jr}$ where γ_{jr} denotes the slope and S_{jr} is y-intercept of the line segment on $[Y_{j,r-1}, Y_{jr}]$ in the piecewise linear membership function. Hence, we have:

$$f_j(Z_j) = -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)\left|Z_j - Y_{j1}\right| - \left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)\left|Z_j - Y_{j2}\right| - \ldots - \left(\frac{\gamma_{j,V_j+1}-\gamma_j V_j}{2}\right)\left|Z_j - Y_{jP_j}\right| + \left(\frac{\gamma_{ij,}V_{i+1}+\gamma_{j1}}{2}\right)Z_j$$
$$+\frac{S_{j,V_j+1}+S_{j1}}{2}\left(\frac{\gamma_{j,b+1}-\gamma_{jb}}{2}\right) \neq 0 \quad j=1,2,3 \qquad \gamma_{j1} = \left(\frac{q_{j1}-0}{Y_{j1}-Y_{j0}}\right), \quad \gamma_{j2} = \left(\frac{q_{j2}-q_{j1}}{Y_{j2}-Y_{j1}}\right),$$
$$\ldots, \gamma_{j,V_j+1} = \left(\frac{1-q_{jV_j}}{Y_{j,V_j+1}-Y_{jV_j}}\right), b=1,2,\ldots,V_j \qquad (11)$$

V_j is the number of broken points of the jth objective function and $S_{j,V_{i+1}}$ is the y-intercept for the section of the line segment on $[Y_{j,V_j}, Y_{j,V_j+1}]$.

Step 4b: Introduce the following nonnegative deviational variables

$$Z_j + d_{jb}^- - d_{jb}^+ = Y_{jb}, j=1,2,3 b=1,2,\ldots,V_j \qquad (12)$$

Where, d^+_{jb} and d^-_{jb} denote the deviational variables in positive and negative directions at the jth point and Y_{jb} represents the values of the jth objective function at the jth point.

Step 4c: Substituting Equation 12 into Equation 11 yields the following equation:

$$f_j(Z_j) = -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)(d_{j1}^- - d_{j1}^+) - \left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)(d_{j2}^- - d_{j2}^+) - \ldots$$
$$-\left(\frac{\gamma_{j,V_j+1}-\gamma_j V_j}{2}\right)(d_{iV_j}^- - d_{iV_j}^+) + \left(\frac{\gamma_{ij,}V_{i+1}+\gamma_{j1}}{2}\right)Z_j + \frac{S_{j,V_j+1}+S_{j1}}{2}, j=1,2,3 \qquad (13)$$

Step 5: Introduce the auxiliary variable L; the problem can be transformed into the equivalent conventional LP problem. The variable L can be interpreted as representing an overall degree of satisfaction with the DM's multiple fuzzy goals. The FMOLP problem can be solved as a TSP:

$$\text{Max} L$$
$$\text{S.t}$$
$$L \leq -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)(d_{j1}^- - d_{j1}^+) - \left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)(d_{j2}^- - d_{j2}^+) - \ldots$$
$$-\left(\frac{\gamma_{j,V_j+1}-\gamma_j V_j}{2}\right)(d_{iV_j}^- - d_{iV_j}^+)$$
$$+\left(\frac{\gamma_{ij,}V_{i+1}+\gamma_{j1}}{2}\right)Z_j + \frac{S_{j,V_j+1}+S_{j1}}{2} \qquad j=1,2,3$$
$$\text{Constraints } (4) - (7)$$

Step 6: Execute and modify the interactive decision process. If the DM is not satisfied with the initial solution, the model must be changed until a satisfactory solution is found.

Proposed FMOLP

Here an approach to transform the FMOLP model into an equivalent auxiliary crisp mathematical programming model for TSP is defined. The interactive solution procedure of the proposed FMOLP method for solving fuzzy multi-objective TSP includes the following steps:

Step 1: Formulate the original fuzzy MOLP model for the TSP.

Step 2: Specify the degree of membership $f_j\,(z_j)$ for several values of each objective function z_j, j = 1, 2 and 3.
Step 3: Draw the piecewise linear membership functions for each $(z_j, f_j(z_j))$, j = 1, 2 and 3.
Step 4: Formulate the piecewise linear equations for each membership function $f_j(z_j)$, g = 1, 2 and 3.
Step 5: Introducing a two-phase approach (Tavakoli-Moghaddam et al., 2010) for the auxiliary variable L and then the problem can be transformed into the equivalent ordinary LP problem. The variable L can be interpreted to represent an overall degree of the satisfaction with the DM's multiple fuzzy goals.

Phase 1: Using the "max–min" operator proposed by Bellman and Zadeh 1970 and L_0 satisfaction degree:

$$
\begin{aligned}
&\text{MaxL}\\
&\text{S.t}\\
&L \le -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)(d_{j1}^{-}-d_{j1}^{+})-\left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)(d_{j2}^{-}-d_{j2}^{+})-\ldots\\
&-\left(\frac{\gamma_{j,V_j+1}-\gamma_j V_j}{2}\right)(d_{iV_j}^{-}-d_{iV_j}^{+})\\
&+\left(\frac{\gamma_{ij,}V_{i+1}+\gamma_{j1}}{2}\right)Z_j+\frac{S_{j,V_j+1}+S_{j1}}{2} \qquad j=1,2,3\\
&Z_j+d_{jb}^{-}-d_{jb}^{+}=Y_{jb} \qquad j=1,2,3
\end{aligned}
$$

Phase 2: We use the result of the presented model to overcome disadvantages of Phase 1. In this phase, the solution is forced to improve, modify, and dominate the one obtained by the "max–min" operator. Also, we add constraints and a new auxiliary objective function to Phase 2 in order to achieve at least the satisfaction degree obtained in Phase 1. Thus, the problem is as follows:

$$
\text{MaxL} = L_0 + \frac{1}{3}\sum_{j=1}^{3}(L_j - L_0) \tag{14}
$$

$$
\begin{aligned}
&\text{S.t}\\
&L_0 \le L_j \le -\left(\frac{\gamma_{j2}-\gamma_{j1}}{2}\right)(d_{j1}^{-}-d_{j1}^{+})-\left(\frac{\gamma_{j3}-\gamma_{j2}}{2}\right)(d_{j2}^{-}-d_{j2}^{+})-\ldots\\
&-\left(\frac{\gamma_{j,V_j+1}-\gamma_j V_j}{2}\right)(d_{iV_j}^{-}-d_{iV_j}^{+})\\
&+\left(\frac{\gamma_{ij,}V_{i+1}+\gamma_{j1}}{2}\right)Z_j+\frac{S_{j,V_j+1}+S_{j1}}{2} \qquad j=1,2,3 \qquad (15)\\
&Z_j+d_{jb}^{-}-d_{jb}^{+}=Y_{jb} \qquad j=1,2,3\\
&\text{Constraints}\,(4)-\ (7)
\end{aligned}
$$

Step 6: Solve the ordinary LP problem and execute the interactive decision process. If the DM is dissatisfied with the initial solutions, the model must be adjusted until a set of satisfactory solutions is derived.

If the solution is L = 1, then each goal is fully satisfied; if it is 0 < L < 1, then all of the goals are satisfied at the level of L, and if it is L = 0, then none of the goals are satisfied (Liang, 2006). Furthermore, the L value may be adjusted to identify a better TSP solution if the DM did not accept the initial overall degree of this satisfaction value. The DM may try to interactively modify the results by adjusting the fuzzy data and related model parameters until a satisfactory solution is obtained. Essentially, the proposed FMOLP method provides a systematic framework that facilitates fuzzy decision-making process, enabling the DM to interactively adjust the search direction during the solution procedure to obtain a DM's preferred satisfactory solution. Figure 1 illustrates the block diagram of the FMOLP model.

CASE STUDY FOR TSP

A traveling salesman has been analyzed with symmetric TSP, who starts from his home city 0; has to visit the three cities exactly once and he is required to come back to his home city 0 by adopting a route with minimum cost, time and distance covered (Chaudhuri and De, 2011). A map of the cities to be visited is shown in Figure 2 and the cities listed along with their cost, time and distance matrix in Table 4, where triple (c,d,t) represents; cost in dollars, distance in kilometers, and time in hours respectively for the corresponding couple of cites.

Zimmerman method

The three objective function z_1, z_2, z_3 are formulated for cost, distance and time respectively. Their Aspiration levels are set as 65, 16 and 11 by solving each objective function subject to the given constraints in the TSP and their corresponding tolerances are decided as 15, 3 and 6.

Objective functions:

$$
\begin{aligned}
&MinZ_1 = 20X_{01}+15X_{02}+11X_{03}+20X_{10}\\
&+30X_{12}+10X_{13}+15X_{20}+30X_{21}+20X_{23}\\
&+11X_{30}+10X_{31}+20X_{32} \lesssim 65, \qquad t_1=15
\end{aligned}
$$

$$
\begin{aligned}
&MinZ_2 = 5X_{01}+5X_{02}+3X_{03}+5X_{10}\\
&+5X_{12}+3X_{13}+5X_{20}+5X_{21}+10X_{23}\\
&+3X_{30}+3X_{31}+10X_{32} \lesssim 16, \qquad t_2=3
\end{aligned}
$$

$$
\begin{aligned}
&MinZ_3 = 4X_{01}+5X_{02}+2X_{03}+4X_{10}\\
&+3X_{12}+3X_{13}+5X_{20}+3X_{21}+2X_{23}\\
&+2X_{30}+3X_{31}+2X_{32} \lesssim 11, \qquad t_3=6
\end{aligned}
$$

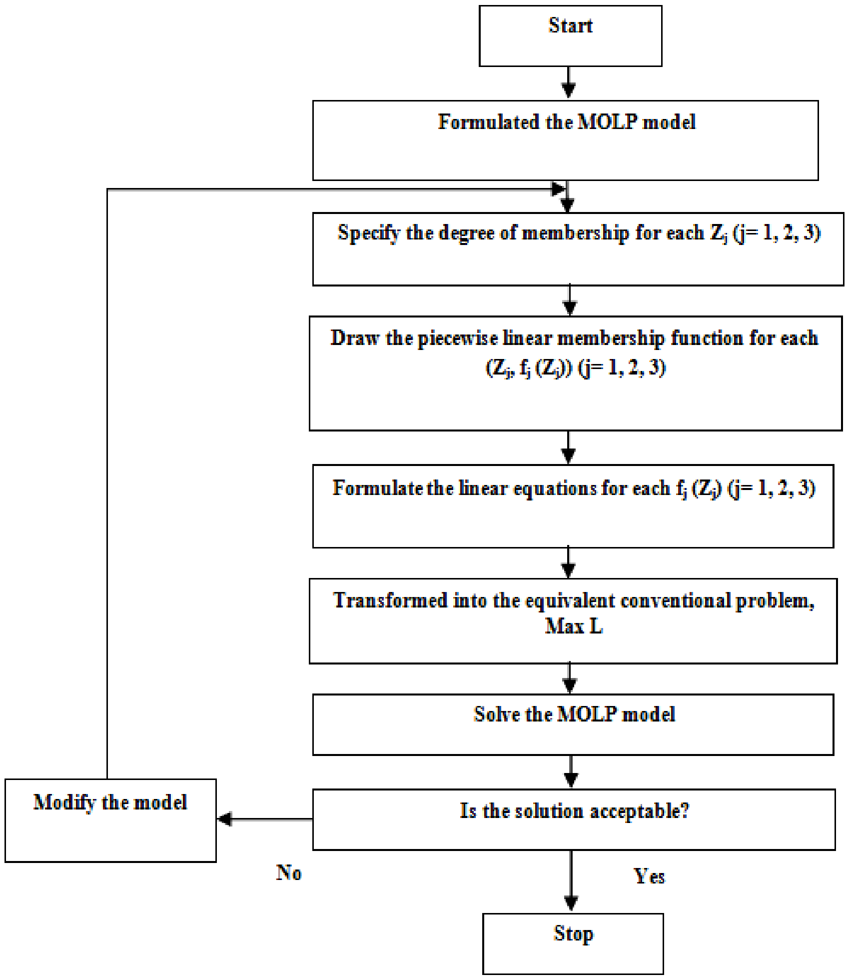

Figure 1. The block diagram of the interactive FMOLP model development (Tavakoli- Moghaddam et al., 2010).

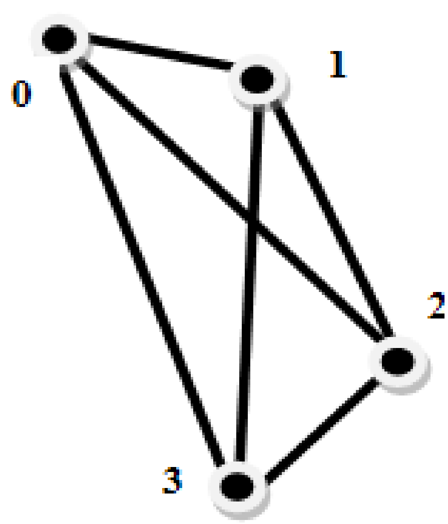

Figure 2. Symmetric traveling salesman problem (Chaudhuri et al., 2011).

Table 4. The matrix for time, cost and distance for each pair of cities (Chaudhuri et al., 2011).

City	(c, d, t) 0	(c, d, t) 1	(c, d, t) 2	(c, d, t) 3
0	(0, 0, 0)	(20, 5, 4)	(15, 5, 5)	(11, 3, 2)
1	(20, 5, 4)	(0, 0, 0)	(30, 5, 3)	(10, 3, 3)
2	(15, 5, 5)	(30, 5, 3)	(0, 0, 0)	(20, 10, 2)
3	(11, 3, 2)	(10, 3, 3)	(20, 10, 2)	(0, 0, 0)

$$\mu(Z_1) = \begin{cases} 0 & , Z_1 \geq 78 \\ 1 - \dfrac{(z_1 - 63)}{15} & , 63 \leq Z_1 \leq 78 \\ 1 & , Z_1 \leq 63 \end{cases},$$

$$\mu(Z_2) = \begin{cases} 0 & , Z_2 \geq 23 \\ 1 - \dfrac{(z_2 - 20)}{3} & , 20 \leq Z_2 \leq 23 \\ 1 & , Z_2 \leq 20 \end{cases},$$

Table 5. Solution of Fuzzy multi-objective linear programming problem (Zimmerman).

Z_1, t_1	Z_2, t_2	Z_3, t_3	L	Rout
65, 15	16, 3	11, 6	0.667	(x_{03}, x_{31}, x_{12}, x_{20})

Table 6. Membership functions.

Z_1	>80	80	75	70	65	<65
$f_1(Z_1)$	0	0	0.5	0.8	1	1
Z_2	>19	19	18	17	16	<16
$f_2(Z_2)$	0	0	0.4	0.7	1	1
Z_3	>17	17	15	13	11	<11
$f_3(Z_3)$	0	0	0.4	0.7	1	1

$$\mu(Z_3)=\begin{cases} 0 & ,Z_3\ge 16 \\ 1-\dfrac{(z_3-10)}{6} & ,10\le Z_3\le 16 \\ 1 & ,Z_3\le 10 \end{cases}$$

A fuzzy multi-objective linear program with max-min approach is given as:

$$\begin{aligned}
&\text{MaxL}\\
&\text{S.t}\\
&L\le 1-\frac{(z_1-63)}{15}\\
&L\le 1-\frac{(z_2-20)}{3}\\
&L\le 1-\frac{(z_3-11)}{6}\\
&\text{Constraints} \quad (4)-(7)\\
&X_{ij}\in\{0,1\}, L\ge 0.
\end{aligned}$$

The Lingo computer software is used to run this ordinary LP model on an Intel® 2.66GHz Processor with 3 GB RAM. As shown in Table 5 when z_1, z_2 and z_3 are considered, an optimal rout with L = 0.667 and Z_1 = 66, Z_2 = 16 and Z_3 = 13 is yielded.

Hannan's method

First, determine the initial solutions for each of the objective functions using the conventional LP model. The results are Z_1 = 65; Z_2 = 16 and Z_3 = 11. Then, formulate the FMOLP model using the initial solutions and the MOLP model presented here. Table 6 gives the piecewise linear membership functions of the proposed model. Figures 3 to 5 illustrate the corresponding shapes of the piecewise linear membership functions. The complete FMOLP model of numerical example is as

$$\begin{aligned}
&\text{MaxL}\\
&\text{S.t}\\
&L\le -0.02(d_{11}^- - d_{11}^+) - 0.01(d_{12}^- - d_{12}^+) - 0.07\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}C_{ij}X_{ij}\right\}+5.8\\
&L\le -0.05(d_{21}^- - d_{21}^+) - 0.35\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}d_{ij}X_{ij}\right\}+6.7\\
&L\le -0.025(d_{31}^- - d_{31}^+) - 0.175\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}t_{ij}X_{ij}\right\}+3.025\\
&\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}C_{ij}X_{ij}\right\}+d_{11}^- - d_{11}^+ = 70\\
&\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}C_{ij}X_{ij}\right\}+d_{12}^- - d_{12}^+ = 75\\
&\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}d_{ij}X_{ij}\right\}+d_{21}^- - d_{21}^+ = 18\\
&\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}t_{ij}X_{ij}\right\}+d_{31}^- - d_{31}^+ = 15\\
&\text{Constraint}(4)-(7)\\
&L, d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+, d_{21}^-, d_{21}^+, d_{31}^-, d_{31}^+ \ge 0.
\end{aligned}$$

The Lingo computer software is used to run this ordinary LP model on an Intel® 2.66GHz Processor with 3 GB RAM. Results for the example are as follows: Z_1 = 66, Z_2 = 16 and Z_3 = 13. Also, the overall degree of satisfaction with the DM's multiple fuzzy goals is 0.7. The best rout of cities is X_{30}– X_{13}– X_{21}– X_{02}.

Proposed FMOLP

The complete FMOLP model of the numerical example is as given:

$$\begin{aligned}
&\text{MaxL} = L_0+\frac{1}{3}[(L_1-L_0)+(L_2-L_0)+(L_3-L_0)]\\
&\text{S.t}\\
&L_0\le L_1\le -0.02(d_{11}^- - d_{11}^+) - 0.01(d_{12}^- - d_{12}^+) - 0.07\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}C_{ij}X_{ij}\right\}+5.8\\
&L_0\le L_2\le -0.05(d_{21}^- - d_{21}^+) - 0.35\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}d_{ij}X_{ij}\right\}+6.7\\
&L_0\le L_3\le -0.025(d_{31}^- - d_{31}^+) - 0.175\times\left\{\sum_{i=0}^{3}\sum_{j=0}^{3}t_{ij}X_{ij}\right\}+3.025
\end{aligned}$$

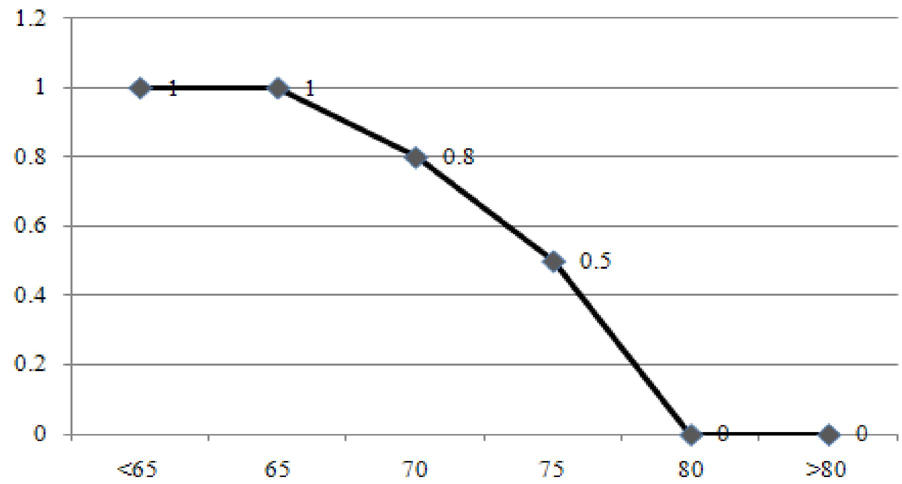

Figure 3. Shape of membership function (Z_1, f_1 (Z_1).

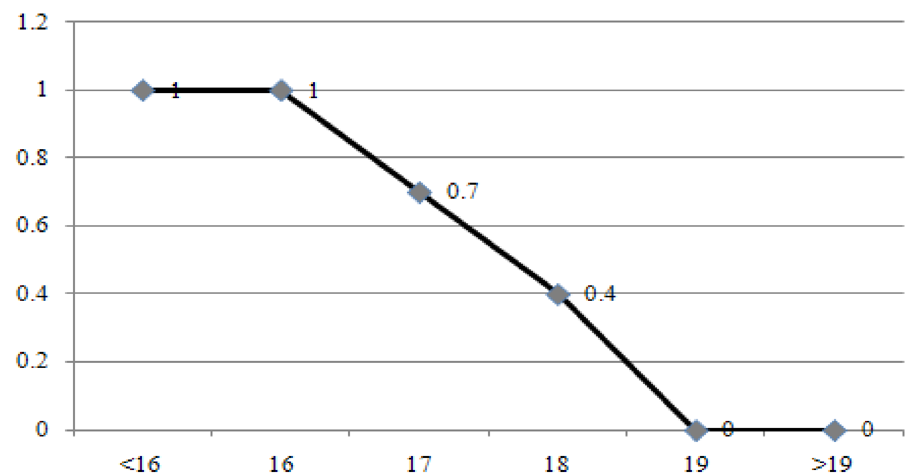

Figure 4. Shape of membership function (Z_2, f_2 (Z_2).

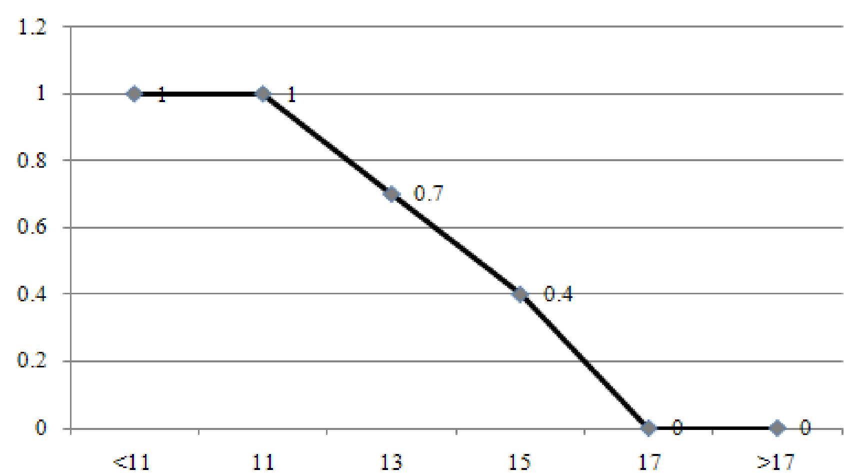

Figure 5. Shape of membership function (Z_3, f_3 (Z_3).

Table 7. Comparison results for the example (Zimmerman's method).

No.	Z_1, t_1	Z_2, t_2	Z_3, t_3	L	Route
1	65, 15	16, 3	-	0.93	(x_{30}, x_{13}, x_{21}, x_{02})
2	-	16, 3	11, 6	0.66	(x_{03}, x_{31}, x_{12}, x_{20})
3	65, 15	-	11, 6	0.66	(x_{03}, x_{31}, x_{12}, x_{20})
4	65, 6	16, 3	11, 6	0.66	(x_{30}, x_{13}, x_{21}, x_{02})
5	65, 15	16, 6	11, 6	0.66	(x_{03}, x_{31}, x_{12}, x_{20})
6	65, 15	16, 3	11, 3	0.33	(x_{03}, x_{31}, x_{12}, x_{20})
7	65, 15	16, 3	11, 9	0.77	(x_{30}, x_{13}, x_{21}, x_{02})

$$\left\{\sum_{i=0}^{3}\sum_{j=0}^{3} C_{ij}X_{ij}\right\} + d_{11}^{-} - d_{11}^{+} = 70$$

$$\left\{\sum_{i=0}^{3}\sum_{j=0}^{3} C_{ij}X_{ij}\right\} + d_{12}^{-} - d_{12}^{+} = 75$$

$$\left\{\sum_{i=0}^{3}\sum_{j=0}^{3} d_{ij}X_{ij}\right\} + d_{21}^{-} - d_{21}^{+} = 18$$

$$\left\{\sum_{i=0}^{3}\sum_{j=0}^{3} t_{ij}X_{ij}\right\} + d_{31}^{-} - d_{31}^{+} = 15$$

$$\text{Constraint } (4)-(7)$$

$$L, d_{11}^{-}, d_{11}^{+}, d_{12}^{-}, d_{12}^{+}, d_{21}^{-}, d_{21}^{+}, d_{31}^{-}, d_{31}^{+} \geq 0.$$

The Lingo computer software is used to run this ordinary LP model on an Intel® 2.66GHz Processor with 3 GB RAM. Results for the example are as follows: Z_1 = 66, Z_2 = 16 and Z_3 = 13. Also, the overall degree of satisfaction with the DM's multiple fuzzy goals is 0.9033. The best rout of cities is X_{30}– X_{13}– X_{21}– X_{02}. The proposed model provides the most flexible decision-making and adjustment processes. For instance, if the DM does not accept the initial overall degree of satisfaction of 0.9033 as in the numerical example, then the DM may try to adjust this L_0 value by taking account of relevant information to seek a set of rational output solutions for TSP decision-making.

MODEL IMPLEMENTATION AND RESULTS ANALYSIS

Implementation

Here the actual implementation of the FMOLP model by manipulating different alternatives based on the preceding numerical example is discussed.

Table 8. Membership function of Scenario 1.

Z_1	>80	80	75	70	65	<65
f_1 (Z_1)	0	0	0.5	0.8	1	1
Z_2	>19	19	18	17	16	<16
f_2 (Z_2)	0	0	0.4	0.7	1	1

Table 9. Membership function of Scenario 2.

Z_1	>80	80	75	70	65	<65
f_1 (Z_1)	0	0	0.5	0.8	1	1
Z_3	>17	17	15	13	11	<11
f_3 (Z_3)	0	0	0.4	0.7	1	1

Zimmerman

As given in Table 7 only Z_1 and Z_2 are considered and Z_3 is omitted; an optimal route with L = 0.93 is obtained. While if Z_1 is omitted and Z_2 and Z_3 are considered and Z_3; an optimal route with L = 0.66 is obtained. In case of Z_1 and Z_3 are consider an optimal route with L= 0.66 is obtained too. Again by relaxing tolerance in Z_3 to 3, the optimal path is achieved with L = 0.33. By increasing tolerance in Z_3 from 3 to 9, an optimal solution with L = 0.77 is obtained. These results show that by adjusting tolerance an optimal solution to Multi-Criteria TSP can be determined.

Proposed FMOLP

The implementation is adapted to the five following scenarios.

(i) Scenario 1: Removing Z_3 (total time), consider only Z_1 (total costs) and Z_2 (total distance) simultaneously. Table 8 presents the membership function of Scenario 1.
(ii) Scenario 2: Removing Z_2, consider only Z_1 and Z_3 simultaneously. Table 9 presents the membership function of Scenario 2.
(iii) Scenario 3: Setting (Z_2, f_2 (Z_2)) and (Z_3, f_3 (Z_3)) to their original values in the numerical example, vary only (Z_1, f_1 (Z_1)). Table 10 presents the data for the implementation of Scenario 3.
(iv) Scenario 4: Setting (Z_2, f_2 (Z_2)) and (Z_1, f_1 (Z_1)) to their original values in the numerical example, vary only (Z_3, f_3 (Z_3)). Table 11 presents the data for the implementation of Scenario 4.

Table 10. Membership function of Scenario 3.

Z_2		>19	19	18	17	16	<16
$f_2 (Z_2)$		0	0	0.4	0.7	1	1
Z_3		>17	17	15	13	11	<11
$f_3 (Z_3)$		0	0	0.4	0.7	1	1
Z_1	Run 1	>70	70	65	60	55	<55
		0	0	0.5	0.8	1	1
	Run 2	>75	75	70	65	60	<60
		0	0	0.5	0.8	1	1
	Run 3	>80	80	75	70	65	<65
		0	0	0.5	0.8	1	1
	Run 4	>85	85	80	75	70	<70
		0	0	0.5	0.8	1	1
	Run 5	>90	90	85	80	75	<75
		0	0	0.5	0.8	1	1

Table 11. Membership function of Scenario 4.

Z_1	$f_1 (Z_1)$	Z_2	$f_2 (Z_2)$	Vary (Z_3, $f_3 (Z_3)$)	
				Z_3	$f_3 (Z_3)$
>80	0	>19	0	>19	0
80	0	19	0	19	0
75	0.5	18	0.4	17	0.4
70	0.8	17	0.7	15	0.7
65	1	16	1	13	1
<65	1	<16	1	<13	1

Objective function: 1.0 > 0.9033. Status: the overall degree of satisfaction is accepted.

Table 12. Membership function of Scenario 5.

Z_1	$f_1 (Z_1)$	Z_3	$f_3 (Z_3)$	Vary(Z_2, $f_2 (Z_2)$)	
				Z_2	$f_2 (Z_2)$
>80	0	>17	0	>20	0
80	0	17	0	20	0
75	0.5	15	0.4	19	0.4
70	0.8	13	0.7	18	0.7
65	1	11	1	17	1
<65	1	<11	1	<17	1

Objective function: 1.0 > 0.947. Status: the overall degree of satisfaction is accepted.

(v) Scenario 5: Setting (Z_1, $f_1 (Z_1)$) and (Z_3, $f_3 (Z_3)$) to their original values in the numerical example, vary only (Z_2, $f_2 (Z_2)$). Table 12 presents the data for the implementation of Scenario 5.

Analysis of results

Table 13 summarizes the results of implementing the previous five scenarios. Several significant management implications that emerged when practically applying the proposed model are as follows:

1. Comparing Scenarios 1 and 2 with the numerical example (Run 3 in Scenario 3), reveals the trade-offs and conflicts among dependent objective functions.
2. The results of Scenario 3-5 show that the specific degree of membership for each of the objective functions strong affects the overall level of satisfaction and output solutions for each decision variables. This fact has two significant implications. First, the most important task of DM is to specify the rational degree of membership for each objective function; second, the DM may flexibly revise the range of value of the degree of membership to yield satisfactory solutions.
3. The proposed FMOLP method is based on Hannan's fuzzy programming method, which assumes that the minimum operator is the proper representation of the human DM who aggregates fuzzy sets using logical 'and' operations. It follows that maximization of two or more membership functions is best accomplished by maximizing the minimum membership degree (Liang, 2006).
4. Table 14 compares the results obtained by Zimmerman's and Hannan's approach with the proposed FMOLP method for given example. Minimizing the total cost (Z_1) yields an optimal value of 66. Minimizing the total distance (Z_2) yields an optimal value of 16. Minimizing the total time (Z_3) yields an optimal value of 13. In addition, the overall degree of the DM satisfaction is 0.9033 for the proposed FMOLP model. This table indicate that the results by using the proposed FMOLP method under an acceptable degree of the DM satisfaction in a fuzzy environment.

Conclusion

The focus of this paper is the analysis of the symmetric TSP as a Fuzzy problem. In the paper, a new objective function applied to solve the Multi-Objective TSP. The proposed FMOLP model is constructed using the piecewise linear membership function of Hannan (1981) to represent the fuzzy goals of the DM in the MOLP model, together with the minimum operator of the fuzzy decision-making of Bellman and Zadeh (1970). To verify the model, a case study solved and compared with existing methods (Zimmerman and Hannan's approach).

The result shows that proposed FMOLP is an effective and flexible Optimization method for TSPs and obtains a higher overall degree of DM satisfaction.

Table 13. Results of implementing five scenarios.

Scenario	Objective	Run 1	Run 2	Run 3	Run 4	Run 5
Scenario 1	L	1	-	-	-	-
	Z_1	66	-	-	-	-
	Z_2	16	-	-	-	-
	Z_3	13	-	-	-	-
Scenario 2	L	0.855	-	-	-	-
	Z_1	66	-	-	-	-
	Z_2	16	-	-	-	-
	Z_3	13	-	-	-	-
Scenario 3	L	0.77	0.8366	0.9033	0.97	1
	Z_1	66	66	66	66	66
	Z_2	16	16	16	16	16
	Z_3	13	13	13	13	13
Scenario 4	L	1	-	-	-	-
	Z_1	66	-	-	-	-
	Z_2	16	-	-	-	-
	Z_3	13	-	-	-	-
Scenario 5	L	1	-	-	-	-
	Z_1	66	-	-	-	-
	Z_2	16	-	-	-	-
	Z_3	13	-	-	-	-

Table 14. Comparison results for the example.

Comparison	Zimmerman	Hannan	Proposed FMOLP
Objective function	Max L	Max L	Max L_0
L	0.66	0.7	0.9033
Z_1	66	66	66
Z_2	16	16	16
Z_3	13	13	33
Optimal rout	(x_{03}, x_{31}, x_{12}, x_{20})	(x_{30}, x_{13}, x_{21}, x_{02})	(x_{30}, x_{13}, x_{21}, x_{02})

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