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Some properties of the sequence space $\widehat{BV}_\theta(M,p,q,s)$

Mahmut Işik[1], Yavuz Altin[2*] and Mikail Et[2]

*Correspondence:
yaltin23@yahoo.com
[2]Department of Mathematics, Firat University, Elazığ, 23119, Turkey
Full list of author information is available at the end of the article

Abstract
In this paper we define the sequence space $\widehat{BV}_\theta(M,p,q,s)$ on a seminormed complex linear space by using an Orlicz function. We give various properties and some inclusion relations on this space.

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1 Introduction

Let ℓ_∞ and c denote the Banach spaces of real bounded and convergent sequences $x = (x_n)$ normed by $\|x\| = \sup_n |x_n|$, respectively.

Let σ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^k(n) = \sigma(\sigma^{k-1}(n))$, $k = 1, 2, \dots$. A continuous linear functional φ on ℓ_∞ is said to be an invariant mean or a σ-mean if and only if

(i) $\varphi(x) \geq 0$ when the sequence $x = (x_n)$ has $x_n \geq 0$ for all n,

(ii) $\varphi(e) = 1$, where $e = (1, 1, 1, \dots)$ and

(iii) $\varphi(\{x_{\sigma(n)}\}) = \varphi(\{x_n\})$ for all $x \in \ell_\infty$.

If σ is the translation mapping $n \to n+1$, a σ-mean is often called a Banach limit [1], and V_σ, the set of σ-convergent sequences, that is, the set of bounded sequences all of whose invariant means are equal, is the set $\hat{f}$ of almost convergent sequences [2].

If $x = (x_n)$, set $Tx = (Tx_n) = (x_{\sigma(n)})$. It can be shown (see Schaefer [3]) that

$$V_\sigma = \Big\{ x = (x_n) : \lim_k t_{kn}(x) = Le \text{ uniformly in } n, L = \sigma - \lim x \Big\}, \tag{1.1}$$

where

$$t_{kn}(x) = \frac{1}{k+1} \sum_{j=0}^{k} T^j x_n.$$

The special case of (1.1), in which $\sigma(n) = n + 1$, was given by Lorentz [2].

Subsequently invariant means were studied by Ahmad and Mursaleen [4], Mursaleen [5], Raimi [6] and many others.

We may remark here that the concept $\widehat{BV}$ of almost bounded variation was introduced and investigated by Nanda and Nayak [7] as follows:

$$\widehat{BV} = \left\{ x : \sum_m \left| t_{mn}(x) \right| \text{ converges uniformly in } n \right\},$$

where

$$t_{mn}(x) = \frac{1}{m(m+1)} \sum_{v=1}^{m} v(x_{n+v} - x_{n+v-1}).$$

By a lacunary sequence $\theta = (k_r)_{r=0,1,2,\ldots}^{\infty}$, where $k_0 = 0$, we shall mean an increasing sequence of non-negative integers with $k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$, and we let $h_r = k_r - k_{r-1}$. The ratio $\frac{k_r}{k_{r-1}}$ will usually be denoted by q_r (see [8]).

Karakaya and Savaş [9] defined the sequence spaces $\widehat{BV}_\theta(p)$ and $\widehat{\widehat{BV}}_\theta(p)$ as follows:

$$\widehat{BV}_\theta(p) = \left\{ x : \sum_{r=1}^{\infty} \left| \varphi_{rn}(x) \right|^{p_r} \text{converges uniformly in } n \right\},$$

$$\widehat{\widehat{BV}}_\theta(p) = \left\{ x : \sup_n \sum_{r=1}^{\infty} \left| \varphi_{rn}(x) \right|^{p_r} < \infty \right\},$$

where

$$\varphi_{r,n}(x) = \frac{1}{h_r + 1} \sum_{j=k_{r-1}+1} x_{j+n} - \frac{1}{h_r} \sum_{j=k_{r-1}+1}^{k_r} x_{j+n}, \quad r > 1.$$

Straightforward calculation shows that

$$\varphi_{r,n}(x) = \frac{1}{h_r(h_r+1)} \sum_{u=1}^{h_r} u(x_{k_{r-1}+u+1+n} - x_{k_{r-1}+u+n})$$

and

$$\varphi_{r-1,n}(x) = \frac{1}{h_r(h_r-1)} \sum_{u=1}^{h_r-1} (x_{k_{r-1}+u+1+n} - x_{k_{r-1}+u+n}).$$

Note that for any sequences x, y and scalar λ, we have

$$\varphi_{r,n}(x+y) = \varphi_{r,n}(x) + \varphi_{r,n}(y) \quad \text{and} \quad \varphi_{r,n}(\lambda x) = \lambda \varphi_{r,n}(x).$$

An Orlicz function is a function $M : [0,\infty) \to [0,\infty)$, which is continuous, nondecreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. (For details, see Krasnoselskii and Rutickii [10].)

It is well known that if M is a convex function and $M(0) = 0$, then $M(\lambda x) \le \lambda M(x)$ for all λ with $0 < \lambda < 1$.

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct the sequence space

$$\ell_M = \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M is a Banach space with the norm

$$\|x\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\},$$

and this space is called an Orlicz sequence space. For $M(t) = t^p$, $1 \leq p < \infty$, the space ℓ_M coincides with the classical sequence space ℓ_p.

Definition 1.1 Any two Orlicz functions M_1 and M_2 are said to be equivalent if there are positive constants α and β, and x_0 such that $M_1(\alpha x) \leq M_2(x) \leq M_1(\beta x)$ for all x with $0 \leq x \leq x_0$ (see Kamthan and Gupta [12]).

Later on, different types of sequence spaces were introduced by using an Orlicz function by Mursaleen *et al.* [13], Choudhary and Parashar [14], Tripathy and Mahanta [15] , Altinok *et al.* [16], Bhardwaj and Singh [17], Et *et al.* [18] and many others.

A sequence space E is said to be solid (or normal) if $(\alpha_k x_k) \in E$ whenever $(x_k) \in E$ for all sequences (α_k) of scalars with $|\alpha_k| \leq 1$.

It is well known that a sequence space E is normal implies that E is monotone.

Definition 1.2 Let q_1, q_2 be seminorms on a vector space X. Then q_1 is said to be stronger than q_2 if whenever (x_n) is a sequence such that $q_1(x_n) \to 0$, then also $q_2(x_n) \to 0$. If each is stronger than the others, q_1 and q_2 are said to be equivalent (one may refer to Wilansky [19]).

Lemma 1.3 *Let q_1 and q_2 be seminorms on a linear space X. Then q_1 is stronger than q_2 if and only if there exists a constant T such that $q_2(x) \leq Tq_1(x)$ for all $x \in X$ (see, for instance, Wilansky* [19]).

Let $p = (p_r)$ be a sequence of strictly positive real numbers, X be a seminormed space over the field $\mathbb{C}$ of complex numbers with the seminorm q, M be an Orlicz function and $s \geq 0$ be a fixed real number. Then we define the sequence space $\widehat{BV}_\theta(M, p, q, s)$ as follows:

$$\widehat{BV}_\theta(M, p, q, s) = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} r^{-s} \left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho} \right) \right) \right]^{p_r} < \infty \right.$$
$$\left. \text{for some } \rho > 0 \text{ uniformly in } n \right\}.$$

It is clear that $q(\frac{\varphi_{rn}(x)}{\rho}) = \frac{q(\varphi_{rn}(x))}{\rho}$ for any seminorm q and any $\rho > 0$.

We get the following sequence spaces from $BV_\theta(M, p, q, s)$ by choosing some of the special p, M and s:

For $M(x) = x$ we get

$$\widehat{BV}_\theta(p,q,s) = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} r^{-s} \big[\big(q(\varphi_{rn}(x)) \big) \big]^{p_r} < \infty \text{ uniformly in } n \right\}$$

for $p_k = 1$, for all $r \in \mathbb{N}$, we get

$$\begin{aligned} &\widehat{BV}_\theta(M,q,s) \\ &\quad = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} r^{-s} \left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho} \right) \right) \right] < \infty \text{ for some } \rho > 0 \text{ uniformly in } n \right\} \end{aligned}$$

for $s = 0$ we get

$$\begin{aligned} &\widehat{BV}_\theta(M,p,q) \\ &\quad = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} \left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho} \right) \right) \right]^{p_r} < \infty \text{ for some } \rho > 0 \text{ uniformly in } n \right\} \end{aligned}$$

for $M(x) = x$ and $s = 0$ we get

$$\widehat{BV}_\theta(p,q) = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} \big[\big(q(\varphi_{rn}(x)) \big) \big]^{p_r} < \infty \text{ uniformly in } n \right\}$$

for $p_r = 1$, for all $r \in \mathbb{N}$, and $s = 0$ we get

$$\begin{aligned} &\widehat{BV}_\theta(M,q) \\ &\quad = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} \left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho} \right) \right) \right] < \infty \text{ for some } \rho > 0 \text{ uniformly in } n \right\} \end{aligned}$$

for $M(x) = x$, $p_r = 1$, for all $r \in \mathbb{N}$, and $s = 0$ we have

$$BV_\theta(q) = \left\{ x = (x_k) \in X : \sum_{r=1}^{\infty} q\big(\varphi_{rn}(x)\big) < \infty, \text{ uniformly in } n \right\}.$$

The following inequalities will be used throughout the paper. Let $p = (p_r)$ be a bounded sequence of strictly positive real numbers with $0 < p_r \le \sup p_r = H$, $D = \max(1, 2^{H-1})$, then

$$|a_r + b_r|^{p_r} \le D\big\{ |a_r|^{p_r} + |b_r|^{p_r} \big\}, \tag{1.2}$$

where $a_r, b_r \in \mathbb{C}$.

2 Main results

In this section we prove the general results of this paper on the sequence space $\widehat{BV}_\theta(M,p,q,s)$, those characterize the structure of this space.

Theorem 2.1 *The sequence space $\widehat{BV}_\theta(M,p,q,s)$ is a linear space over the field $\mathbb{C}$ of complex numbers.*

Proof Omitted. □

Theorem 2.2 *For any Orlicz function M and a bounded sequence $p = (p_r)$ of strictly positive real numbers, $\widehat{BV}_\theta(M,p,q,s)$ is a paranormed space (not necessarily totally paranormed), paranormed by*

$$g(x) = \inf\left\{\rho^{p_r/H} : \left(\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_k}\right)^{\frac{1}{H}} \le 1, \\ r = 1,2,3,\ldots, n = 1,2,3,\ldots\right\},$$

where $H = \max(1, \sup p_r)$.

Proof Clearly $g(x) = g(-x)$. By using Theorem 2.1 and then using Minkowski's inequality, we get $g(x+y) \le g(x) + g(y)$.

Since $q(\bar{\theta}) = 0$ and $M(0) = 0$, we get $\inf\{\rho^{p_r/H}\} = 0$ for $x = \Theta$, where $\bar{\Theta}$ is the zero sequence of X.

Finally, we prove that scalar multiplication is continuous. Let λ be any numbers. By definition,

$$g(\lambda x) = \inf\left\{\rho^{p_r/H} : \left(\sum_{r} r^{-s}\left[M\left(q\left(\frac{\lambda\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} \le 1, \\ r = 1,2,3,\ldots, n = 1,2,3,\ldots\right\}.$$

Then

$$g(\lambda x) = \inf\left\{(\lambda r)^{p_r/H} : \left(\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{r}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} \le 1, \\ r = 1,2,3,\ldots, n = 1,2,3,\ldots\right\},$$

where $r = \frac{\rho}{|\lambda|}$. Since $|\lambda|^{p_r} \le \max(1, |\lambda|^H)$, it follows that $|\lambda|^{p_r/H} \le (\max(1, |\lambda|^H))^{\frac{1}{H}}$.

Hence

$$g(\lambda x) = \left(\max\left(1, |\lambda|^H\right)\right)^{\frac{1}{H}} \inf\left\{r^{p_r/H} : \left(\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{r}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} \le 1, \\ r = 1,2,3,\ldots, n = 1,2,3,\ldots\right\},$$

which converges to zero as $g(x)$ converges to zero in $\widehat{BV}_\theta(M,p,q,s)$. Now suppose that $\lambda_n \to 0$ and x is in $BV_\sigma(M,p,q,s)$. For arbitrary $\varepsilon > 0$, let N be a positive integer such that

$$\sum_{r=N+1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r} < \frac{\varepsilon}{2}$$

for some $\rho > 0$, all n. This implies that

$$\left(\sum_{r=N+1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} \leq \frac{\varepsilon}{2}$$

for some $\rho > 0$, $r > N$ and all n.

Let $0 < |\lambda| < 1$, using convexity of M and all n, we get

$$\sum_{r=N+1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\lambda\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r} < \sum_{r=N+1}^{\infty} r^{-s}\left[|\lambda| M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r} < \left(\frac{\varepsilon}{2}\right)^{H}.$$

Since M is continuous everywhere in $[0,\infty)$, then

$$f(t) = \sum_{r=1}^{N} r^{-s}\left[M\left(q\left(\frac{t\varphi_{rn}(x)}{\rho}\right)\right)\right]$$

is continuous at 0. So there is $1 > \delta > 0$ such that $|f(t)| < \frac{\varepsilon}{2}$ for $0 < t < \delta$. Let K be such that $|\lambda_i| < \delta$ for $i > K$, then for $i > K$, all n,

$$\left(\sum_{r=1}^{N} r^{-s}\left[M\left(q\left(\frac{\lambda_i\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} < \frac{\varepsilon}{2}.$$

Thus

$$\left(\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\lambda_i\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r}\right)^{\frac{1}{H}} < \varepsilon$$

for $i > K$ and n, so that $g(\lambda x) \to 0$ $(\lambda \to 0)$. □

Theorem 2.3 *Let M, M_1, M_2 be Orlicz functions q, q_1, q_2 seminorms and $s, s_1, s_2 \geq 0$. Then*

(i) $\widehat{BV}_\theta(M_1,p,q,s) \cap \widehat{BV}_\theta(M_2,p,q,s) \subseteq \widehat{BV}_\theta(M_1+M_2,p,q,s)$,

(ii) *If $s_1 \leq s_2$ then* $\widehat{BV}_\theta(M,p,q,s_1) \subseteq \widehat{BV}_\theta(M,p,q,s_2)$,

(iii) $\widehat{BV}_\theta(M,p,q_1,s) \cap \widehat{BV}_\theta(M,p,q_2,s) \subseteq \widehat{BV}_\theta(M,p,q_1+q_2,s)$,

(iv) *If q_1 is stronger than q_2, then* $\widehat{BV}_\theta(M,p,q_1,s) \subseteq \widehat{BV}_\theta(M,p,q_2,s)$.

Proof Omitted □

Corollary 2.4 *Let M be an Orlicz function, then we have*

(i) *If $q_1 \cong$(equivalent to) q_2, then* $\widehat{BV}_\theta(M,p,q_1,s) = \widehat{BV}_\theta(M,p,q_2,s)$,

(ii) $\widehat{BV}_\theta(M,p,q) \subseteq \widehat{BV}_\theta(M,p,q,s)$,

(iii) $\widehat{BV}_\theta(M,q) \subseteq \widehat{BV}_\theta(M,q,s)$.

Theorem 2.5 *Suppose that $0 < m_k \leq t_k < \infty$ for each $k \in \mathbb{N}$. Then $\widehat{BV}_\theta(M,m,q) \subseteq \widehat{BV}_\theta(M,t,q)$.*

Proof Let $x \in \widehat{BV}_\theta(M, m, q)$. Then there exists some $\rho > 0$ such that

$$\sum_{r=1}^{\infty}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{m_k} < \infty \quad \text{uniformly in } n.$$

This implies that $M(q(\frac{\varphi_{rn}(x)}{\rho})) \leq 1$ for sufficiently large values of k, say $k \geq k_0$ for some fixed $k_0 \in \mathbb{N}$. Since $m_k \leq t_k$, for each $k \in \mathbb{N}$ we get

$$\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{t_k} \leq \left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{m_k}$$

for all $k \geq k_0$, and therefore

$$\sum_{r=1}^{\infty}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{t_r} \leq \sum_{r=1}^{\infty}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{m_k}.$$

Hence we have

$$\sum_{r=1}^{\infty}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{t_r} < \infty,$$

so $x \in \widehat{BV}_\theta(M, t, q)$. This completes the proof. □

The following result is a consequence of the above result.

Corollary 2.6

(i) *If* $0 < p_r \leq 1$ *for each* r, *then* $\widehat{BV}_\theta(M, p, q) \subseteq \widehat{BV}_\theta(M, q)$,

(ii) *If* $p_r \geq 1$ *for all* r, *then* $\widehat{BV}_\theta(M, q) \subseteq \widehat{BV}_\theta(M, p, q)$.

Theorem 2.7 *Let* M_1 *and* M_2 *be any two of Orlicz functions. If* M_1 *and* M_2 *are equivalent, then* $\widehat{BV}_\theta(M_1, p, q, s) = \widehat{BV}_\theta(M_2, p, q, s)$.

Proof Proof follows from Definition 1.1. □

Theorem 2.8 *The sequence space* $\widehat{BV}_\theta(M, p, q, s)$ *is solid.*

Proof Let $x \in \widehat{BV}_\theta(M, p, q, s)$, *i.e.*,

$$\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r} < \infty.$$

Let (α_r) be sequence of scalars such that $|\alpha_r| \leq 1$ for all $r \in \mathbb{N}$. Then the result follows from the following inequality:

$$\sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\alpha_r\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r} \leq \sum_{r=1}^{\infty} r^{-s}\left[M\left(q\left(\frac{\varphi_{rn}(x)}{\rho}\right)\right)\right]^{p_r}.$$ □

Corollary 2.9 *The sequence space* $\widehat{BV}_\theta(M, p, q, s)$ *is monotone.*

Competing interests
The authors declare that they have no competing interest.

Authors' contributions
MI, YA and ME have contributed to all parts of the article. All authors read and approved the final manuscript.

Author details
[1]Department of Statistics, Firat University, Elazığ, 23119, Turkey. [2]Department of Mathematics, Firat University, Elazığ, 23119, Turkey.

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