

Full Length Research Paper

Some finite integrals involving multivariable polynomials, H-function of one variable and H-function of 'r' variables

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Accepted 9 May, 2011

In the present paper we establish the integrals involving the product of two general classes of polynomial, H-function of one variable and H-function of 'r' variables. These integrals are unified in nature and we can derive from them by a large number of integrals involving simpler functions and polynomials as their particular cases.

Key words: Multivariable class of polynomials, H-function of one variable, H-function of 'r' variables.

INTRODUCTION

Recently, the integrals involving general class of polynomial with H-function (Sharma, 2006) are evaluated. In the present paper we establish the integrals involving the product of two multivariable polynomials, H-function of one variable and H-function of 'r' variables.

We shall utilize the following formulae in the present investigation. The H-function of one variable is given by Buschman and Srivastava (1990).

$$H_{P,Q}^{M,N}[z] = H_{P,Q}^{M,N} \left[z \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{M+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi i} \int_L \phi(s) z^s ds \quad (1)$$

where

$$\phi(s) = \frac{\prod_{j=1}^M \Gamma(b_j - \beta_j s) \prod_{j=1}^N \{\Gamma(1 - a_j + \alpha_j s)\}^{A_j}}{\prod_{j=M+1}^Q \{\Gamma(1 - b_j + \beta_j s)\}^{B_j} \prod_{j=N+1}^P \Gamma(a_j - \alpha_j s)}$$

which contains fractional powers of some of the gamma functions. Here, and through out the paper a_j ($j = 1, \dots, P$) and b_j ($j = 1, \dots, Q$) are complex parameters, α_j ($j = 1, \dots, P$), β_j ($j = 1, \dots, Q$) (not all zero simultaneously) and the exponents A_j ($j = 1, \dots, N$) and B_j ($M+1, \dots, Q$) can take non-integer values. The contour L in Equation (1) is imaginary axis $\text{Re}(s) = 0$. It is suitably indented in order to avoid the singularities of the gamma functions and to keep those singularities on appropriate sides. Again, for A_j ($j = 1, \dots, N$) not an integer, the poles of the gamma functions of the numerator in $\phi(s)$ are converted to branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j - \beta_j s)$ ($j = 1, \dots, M$) and $\Gamma(1 - a_j + \alpha_j s)$ ($j = 1, \dots, M$) pair, the branch cuts can be chosen so that the path of integration can be distorted in the usual manner.

Evidently, when the exponents A_j and B_j are all positive integers, the H-function reduces to the well-known Fox's H-function (Fox, 1961; Srivastava et al., 1982). The basic properties and the following sufficient conditions for the absolute convergence of the defining integral for the H-function have been given by Buschman and Srivastava (1990). Special kind of H-function of one variable is defined as follows (Mukherjee and Prasad, 1972):

$$H_{p,q+1}^{m+1,n} \left[ax^\sigma \left| \begin{matrix} (a_j, \alpha_j)_{1,p} \\ (b_0, \beta_0), (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] = \frac{1}{\beta_0} \sum_{r=0}^{\infty} F(r) x^{\sigma p_r} \quad (2)$$

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Where:

$$F(r) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j \rho_r) \prod_{j=1}^n \Gamma(1 - a_j + \alpha_j \rho_r)}{\prod_{j=m+1}^q \Gamma(1 - b_j + \beta_j \rho_r) \prod_{j=n+1}^p \Gamma(a_j - \alpha_j \rho_r)} \frac{(-1)^r a^{\rho_r}}{r!}$$

Provided: (i) $\beta_0 > 0$, $\beta < \text{Re}(b_0/\beta_0) < \delta$, (ii) $|\arg a| < \frac{1}{2} \lambda \pi$, $(\lambda, A > 0)$,

where:

$$\lambda = \sum_{j=1}^n \alpha_j - \sum_{j=n+1}^p \alpha_j + \sum_{j=1}^m \beta_j - \sum_{j=m+1}^q \beta_j$$

$$A = \sum_{j=1}^q \beta_j - \sum_{j=1}^p \alpha_j.$$

$$\rho_r = \frac{b_0 + r}{\beta_0}$$

The H-function of 'r' variable is given by Srivastava and Panda (1976):

$$H[x_1, \dots, x_r]$$

$$= \left[\begin{matrix} (a_j; \alpha_{1j}, \dots, \alpha_{rj})_{1,p} : (c_{1j}, C_{1j})_{1,p_1} ; \dots ; (c_{rj}, C_{rj})_{1,p_r} \\ x_1 \\ \vdots \\ x_r \\ b_j; \beta_{1j}, \dots, \beta_{rj})_{1,q} : (d_{1j}, D_{1j})_{1,q_1} ; \dots ; (d_{rj}, D_{rj})_{1,q_r} \end{matrix} \right] \quad (3)$$

$$= \frac{1}{(2\pi i)^r} \int_{L_1} \dots \int_{L_r} \theta(s_1, \dots, s_r) \prod_{k=1}^r \varphi_k(s_k) x_k^{s_k} ds_k,$$

$$i = \sqrt{-1}$$

where:

$$\theta(s_1, \dots, s_r) = \frac{\prod_{j=1}^n \Gamma(1 - a_j + \sum_{k=1}^r \alpha_j^{(k)} s_k)}{\prod_{j=n+1}^p \Gamma(a_j - \sum_{k=1}^r \alpha_j^{(k)} s_k) \prod_{j=1}^q \Gamma(1 - b_j + \sum_{k=1}^r \beta_j^{(k)} s_k)}$$

$$\varphi_k(s_k) = \frac{\prod_{j=1}^m \Gamma(d_j^{(k)} - D_j^{(k)} s_k) \prod_{j=1}^n \Gamma(1 - c_j^{(k)} + C_j^{(k)} s_k)}{\prod_{j=m_k+1}^q \Gamma(1 - d_j^{(k)} + D_j^{(k)} s_k) \prod_{j=n_k+1}^p \Gamma(c_j^{(k)} - C_j^{(k)} s_k)},$$

and $n, p, q, m_k, n_k, p_k, q_k, k$ are non-negative integers such that $0 \leq n \leq p, q \geq 0$,

$0 \leq m_k \leq q_k$ and $0 \leq n_k \leq p_k, k = 1, \dots, r$ and $\alpha_j^{(k)}, \beta_j^{(k)}, C_j^{(k)}, D_j^{(k)}$ are all positive.

The contour L_k lies in the complex plane s_k is of Mellin-Barnes type which runs from $-i\infty$ to $+i\infty$ with indentations, if necessary to ensure that all poles of

$$\Gamma(d_j^{(k)} D_j^{(k)} s_k), j = 1, \dots, n_k \text{ and } \Gamma(1 - a_j^{(k)} + \sum \alpha_j^{(k)} s_k),$$

$j = 1, \dots, n$ are to the left of L_k .

The second class of multivariable polynomials given by Srivastava (1985: 686) is defined as follows:

$$S_{V_1, \dots, V_t}^{U_1, \dots, U_t} [x_1, \dots, x_t] = \sum_{k_1=0}^{[V_1/U_1]} \dots \sum_{k_t=0}^{[V_t/U_t]} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots, V_t, k_t) \frac{x_1^{k_1}}{k_1!} \dots \frac{x_t^{k_t}}{k_t!}$$

$$V = 0, 1, 2, \dots \quad (4)$$

The first class of multivariable polynomials introduced by Srivastava and Garg (1987: 686) (Equation 4) is defined as follows:

$$S_V^{U_1, \dots, U_t} [x_1, \dots, x_t] = \sum_{k_1, \dots, k_t=0}^{U_1 k_1 + \dots + U_t k_t \leq V} (-V)_{U_1 k_1 + \dots + U_t k_t} A(V; k_1, \dots, k_t) \frac{x_1^{k_1}}{k_1!} \dots \frac{x_t^{k_t}}{k_t!} \quad (5)$$

From table of integrals we have (Gradshteyn and Ryzhik, 2001: 314, Equation 3):

$$\int_{-1}^1 (1-x)^p (1+x)^q dx = 2^{p+q+1} B(p+1, q+1) \quad (6)$$

$$\int_0^\infty \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-p-1} dx = \frac{\sqrt{\pi} \Gamma(p + \frac{1}{2})}{2a(4ab + c)^{p + \frac{1}{2}} \Gamma(p+1)}$$

$$\text{Re}(p) + \frac{1}{2} > 0 \quad (7)$$

MAIN RESULTS

First integral

$$\int_{-1}^1 (1-x)^p (1+x)^q S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 (1-x)^{m_1} (1+x)^{n_1}, \dots, y_t (1-x)^{m_t} (1+x)^{n_t} \right] dx$$

(8)

$$\times H_{P,Q}^{M,N} \left[z(1-x)^g (1+x)^h \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= 2^{\rho+\sigma+1} \sum_{k_1=0}^{[V_1/U_1]} \cdots \sum_{k_t=0}^{[V_t/U_t]} (-V_1)_{U_1 k_1} \cdots (-V_t)_{U_t k_t} A(V_1, k_1; \cdots; V_t, k_t) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_t^{k_t}}{k_t!} 2^{i=1} \sum_{i=1}^t (m_i + n_i) k_i$$

$$\times H_{P+2, Q+1}^{M, N+2} \left[z^{2h+g} \left| \begin{matrix} (-\sigma - \sum_{i=1}^t m_i k_i, h; 1), (-\rho - \sum_{i=1}^t n_i k_i, g; 1), (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (-\sigma - \rho - \sum_{i=1}^t (m_i + n_i) k_i - 1, h + g; 1) \end{matrix} \right. \right]$$

where $m_i > 0$ ($i = 1, \dots, t$), $n_i > 0$ ($i = 1, \dots, t$), $h \geq 0$, $g \geq 0$ (not both are zero simultaneously)

Second integral

$$\int_{-1}^1 (1-x)^{\rho} (1+x)^{\sigma} S_V^{U_1, \dots, U_t} \left[y_1 (1-x)^{m_1} (1+x)^{n_1}, \dots, y_t (1-x)^{m_t} (1+x)^{n_t} \right] \quad (9)$$

$$\times H_{P,Q}^{M,N} \left[z(1-x)^g (1+x)^h \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] dx$$

$$= 2^{\rho+\sigma+1} \sum_{k_1, \dots, k_t=0}^{U_1 k_1 + \dots + U_t k_t \leq V} (-V)_{U_1 k_1 + \dots + U_t k_t} A(V; k_1, \dots, k_t) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_t^{k_t}}{k_t!} 2^{i=1} \sum_{i=1}^t (m_i + n_i) k_i$$

$$\times H_{P+2, Q+1}^{M, N+2} \left[z^{2h+g} \left| \begin{matrix} (-\sigma - \sum_{i=1}^t m_i k_i, h; 1), (-\rho - \sum_{i=1}^t n_i k_i, g; 1), (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (-\sigma - \rho - \sum_{i=1}^t (m_i + n_i) k_i - 1, h + g; 1) \end{matrix} \right. \right]$$

Provided the conditions stated in Equation (8) are satisfied.

Third integral

$$\int_0^{\infty} \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_1}, \dots, y_t \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_t} \right] \quad (10)$$

$$\times H_{p,q+1}^{m+1,n} \left[\left(a_0 \left(ax + \frac{b}{x} \right)^2 + c \right)^{-\sigma} \left| \begin{matrix} (e_j, E_j)_{1,p} \\ (f_0, F_0), (f_j, F_j)_{1,q} \end{matrix} \right. \right] H \left[z_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_1}, \dots, z_r \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_r} \right] dx$$

$$= \frac{\sqrt{\pi} a_0^{-\sigma}}{2aF_0(4ab+c)} \sum_{k_1=0}^{[V_1/U_1]} \cdots \sum_{k_t=0}^{[V_t/U_t]} (-V_1)_{U_1 k_1} \cdots (-V_t)_{U_t k_t} A(V_1, k_1; \dots; V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} (4ab+c)^{i-1}}{k_i!} \sum_{u=0}^{\infty} G(u) \rho_u$$

$$\times H_{P+1, Q+1}^{0, N+1} : I : J \left[z_1 (4ab+c)^{-q_1}, \dots, z_r (4ab+c)^{-q_r} \left| \begin{matrix} A : C \\ B : D \end{matrix} \right. \right]$$

where:

$$A = \left(\frac{1}{2} - \eta - \sigma - \sum_{i=1}^t m_i k_i; q_1, \dots, q_r \right) (a_k : \alpha_{1k}, \dots, \alpha_{rk})_{1,P},$$

$$B = \left(-\eta - \sigma - \sum_{i=1}^t m_i k_i; q_1, \dots, q_r \right) (b_k : \beta_{1k}, \dots, \beta_{rk})_{1,Q},$$

$$C = (c_{1k}, C_{1k})_{1,p_1}, \dots, (c_{rk}, C_{rk})_{1,p_r}$$

$$D = (d_{1k}, D_{1k})_{1,q_1}, \dots, (d_{rk}, D_{rk})_{1,q_r},$$

$$I = m_1, n_1; \dots; m_r, n_r, J = p_1, q_1; \dots; p_r, q_r$$

$$G(u) = \frac{\prod_{j=1}^m \Gamma(f_j - F_j \rho_u) \prod_{j=1}^n \Gamma(1 - e_j + E_j \rho_u)}{\prod_{j=m+1}^q \Gamma(1 - f_j + F_j \rho_u) \prod_{j=n+1}^p \Gamma(e_j - E_j \rho_u)} \frac{(-1)^u}{u!} a^{\rho_u}$$

$$\rho_u = \frac{f_0 + u}{F_0}$$

Proof

To establish integral in Equation (8), we express H-function occurring in its left-hand side in terms of Mellin-Barnes contour integral given by Equation (1), the second class of polynomial given by Equation (4). Then interchange the order of integration of summations and integration, we arrive at the following after a little simplification:

$$\sum_{k_1, \dots, k_t=0}^{U_1 k_1 + \dots + U_t k_t \leq V} (-V)_{U_1 k_1 + \dots + U_t k_t} A(V; k_1, \dots, k_t) \frac{y_1^{k_1}}{k_1!} \cdots \frac{y_t^{k_t}}{k_t!}$$

$$\times \frac{1}{2\pi i} \int_L \phi(s) z^s \int_{-1}^1 (1-x)^{\rho+gs+\sum_{i=1}^t m_i k_i} (1+x)^{\sigma+hs+\sum_{i=1}^t n_i k_i} dx ds$$

Now we evaluate the above integral with help of integral [5]. Interpreting the resulting contour integral of H-function we can easily arrive at desired result Equation (8).

The second integral can be established on the same lines similar to the proof of Equation (8)

To evaluate Equation 10, we first express the polynomial containing in its left hand side in the series form given by Equation (4) and write multivariable H-function, H-function of one variable using Equations (3) and (1) respectively. Next, interchange the order of integrations and summations, we arrive at the following

after a little simplification:

$$\sum_{k_1=0}^{\lfloor V_1/U_1 \rfloor} \dots \sum_{k_t=0}^{\lfloor V_t/U_t \rfloor} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots; V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} a_0^{-\sigma}}{k_i! F_0} \sum_{u=0}^{\infty} G(u) \rho_u$$

$$\frac{1}{(2\pi i)^t} \int_{L_1} \dots \int_{L_r} \theta(s_1, \dots, s_r) \phi_1(s_1) \dots \phi_r(s_r) z_1^{s_1} \dots z_r^{s_r} \int_0^{\infty} \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta - \sum_{i=1}^t m_i k_i - \sum_{j=1}^r q_j s_j - \sigma - 1} dx ds_1 \dots ds_r$$

where s_1, \dots, s_r denote the variables of aforementioned Mellin-Barnes contour integral of H-function. Now we evaluate the x-integral with the help of (6). Interpreting the resulting contour integral in terms of the H-function of 'r' variables, we easily arrive at the desired result Equation (10).

Special cases

Take $A(V_1, k_1; \dots; V_t, k_t) = A_1(V_1, k_1) \dots A_t(V_t, k_t)$ in Equation (8), the multivariable polynomial $S_{V_1, \dots, V_t}^{U_1, \dots, U_t}[x_1, \dots, x_t]$ reduced to the product of t well known general class of polynomials $S_V^U[x]$ (Srivastava, 1972: 1, Equation 1) and the result Equation (8), reduced to following form:

$$\int_{-1}^1 (1-x)^{\rho} (1+x)^{\sigma} \prod_{i=1}^t S_{V_i}^{U_i} \left[y_i (1-x)^{m_i} (1+x)^{n_i} \right] H_{P,Q}^{M,N} \left[z(1-x)^g (1+x)^h \left| \begin{matrix} (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q} \end{matrix} \right. \right] dx \quad (11)$$

$$= 2^{\rho+\sigma+1} \prod_{j=1}^t \sum_{k_j=0}^{\lfloor V_j/U_j \rfloor} (-V_j)_{U_j k_j} A_j(V_j, k_j) \frac{y_j^{k_j}}{k_j!} 2^{\sum_{i=1}^t (m_i + n_i) k_i}$$

$$\times H_{P+2, Q+1}^{M, N+2} \left[z 2^{h+g} \left| \begin{matrix} (-\sigma - \sum_{i=1}^t m_i k_i, h; 1), (-\rho - \sum_{i=1}^t n_i k_i, g; 1), (a_j, \alpha_j; A_j)_{1,N}, (a_j, \alpha_j)_{N+1,P} \\ (b_j, \beta_j)_{1,M}, (b_j, \beta_j; B_j)_{M+1,Q}, (-\sigma - \rho - \sum_{i=1}^t (m_i + n_i) k_i - 1, h + g; 1) \end{matrix} \right. \right]$$

If in Equation (8), we take

$$A(V_1, k_1; \dots; V_t, k_t) = \frac{(\beta_1)_{k_1} \phi_1 + \dots + k_t \phi_t}{(\nu_1)_{k_1} \psi_1 + \dots + k_t \psi_t} \quad \text{then the}$$

polynomial $S_{V_1, \dots, V_t}^{U_1, \dots, U_t}[x_1, \dots, x_t]$ reduced to the first class of multivariable hypergeometric polynomials defined by Srivastava and Garg, 1987 and we easily arrive at an integral involving the polynomial.

Also, if in Equation (9), we take

$$A(V_1, k_1; \dots; V_t, k_t) = \frac{(\beta_1)_{k_1} \phi_1 + \dots + k_t \phi_t}{(\nu_1)_{k_1} \psi_1 + \dots + k_t \psi_t} \quad \text{then the}$$

polynomial $S_{V_1, \dots, V_t}^{U_1, \dots, U_t}[x_1, \dots, x_t]$ reduced to the second class of multivariable hypergeometric polynomials defined by Srivastava and Garg (1987) and we easily arrive at an integral involving the polynomial.

Put $\alpha_j = \beta_j = A_j = B_j = 1$

$$\int_{-1}^1 (1-x)^{\rho} (1+x)^{\sigma} S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 (1-x)^{m_1} (1+x)^{n_1}, \dots, y_t (1-x)^{m_t} (1+x)^{n_t} \right] G_{P,Q}^{M,N} \left[z(1-x)^g (1+x)^h \left| \begin{matrix} (a_j)_{1,P} \\ (b_j)_{1,Q} \end{matrix} \right. \right] dx \quad (12)$$

$$= 2^{\rho+\sigma+1} \sum_{k_1=0}^{\lfloor V_1/U_1 \rfloor} \dots \sum_{k_t=0}^{\lfloor V_t/U_t \rfloor} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots; V_t, k_t) \frac{y_1^{k_1}}{k_1!} \dots \frac{y_t^{k_t}}{k_t!} 2^{\sum_{i=1}^t (m_i + n_i) k_i}$$

$$\times H_{P+2, Q+1}^{M, N+2} \left[z 2^{h+g} \left| \begin{matrix} (-\sigma - \sum_{i=1}^t m_i k_i, h; 1), (-\rho - \sum_{i=1}^t n_i k_i, g; 1), (a_j)_{1,P} \\ (b_j)_{1,Q}, (-\sigma - \rho - \sum_{i=1}^t (m_i + n_i) k_i - 1, h + g; 1) \end{matrix} \right. \right]$$

The multivariable H-function occurring in these results can be suitably specialized to yield a wide variety of special functions (or product of such functions) of one or more variables. Again by suitably specializing the coefficients of general class of multivariable polynomials, these can be reduced to other multivariable hypergeometric polynomials of one or more variables. Thus our result would give the corresponding results involving a large number of simpler functions and polynomials.

On taking $A(V_1, k_1; \dots; V_t, k_t) = A_1(V_1, k_1) \dots A_t(V_t, k_t)$ in Equation (10), the multivariable polynomial $S_{V_1, \dots, V_t}^{U_1, \dots, U_t}[x_1, \dots, x_t]$ reduces to the product of t well

known general class of polynomials $S_V^U[x]$ (Fox, 1961: 1, Equation 1) and the Equation (10), becomes:

$$\int_0^{\infty} \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} \prod_{i=1}^t S_{V_i}^{U_i} \left[y_i \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_i} \right] \quad (13)$$

$$\times H_{P, q+1}^{m+1, n} \left[\left(a_0 \left(ax + \frac{b}{x} \right)^2 + c \right)^{-\sigma} \left| \begin{matrix} (e_j, E_j)_{1,p} \\ (f_0, F_0), (f_j, F_j)_{1,q} \end{matrix} \right. \right] H \left[z_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_1}, \dots, z_r \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_r} \right] dx$$

$$= \frac{\sqrt{\pi} a_0^{-\sigma}}{2aF_0(4ab+c)} \sum_{k_1=0}^{\lfloor V_1/U_1 \rfloor} \dots \sum_{k_t=0}^{\lfloor V_t/U_t \rfloor} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots; V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} (4ab+c)^{-m_i k_i}}{k_i!} \sum_{u=0}^{\infty} G(u) \rho_u$$

$$\times H_{P+1, Q+1}^{0, N+1} : I \left[z_1 (4ab+c)^{-q_1}, \dots, z_r (4ab+c)^{-q_r} \left| \begin{matrix} A:C \\ B:D \end{matrix} \right. \right]$$

Provided the conditions similar to that of Equation (10). On taking $m = n = q = 0$, $\sigma = -1$, $F_0 = 1$ in Equation (10), we get integral transform involving product of general class of polynomials with H-function of 'r' variables:

$$\int_0^\infty \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_1}, \dots, y_t \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_t} \right] dx \quad (14)$$

$$\times H_{0,1}^{1,0} \left[\left(a_0 \left(ax + \frac{b}{x} \right)^2 + c \right) \left| \begin{matrix} (e_j, E_j)_{l,p} \\ (f_0, F_0), (f_j, F_j)_{l,q} \end{matrix} \right. \right] H \left[z_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_1}, \dots, z_r \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_r} \right] dx$$

$$= \frac{\sqrt{\pi} a_0^{-\sigma}}{2a(4ab+c)} \frac{[V_1/U_1]}{\sum_{k_1=0}^{\eta+\sigma+\frac{1}{2}}} \dots \frac{[V_t/U_t]}{\sum_{k_t=0}^{\eta+\sigma+\frac{1}{2}}} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots, V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} (4ab+c)^{-\sum_{i=1}^t m_i k_i}}{k_i!}$$

$$\times \sum_{u=0}^{\infty} G(u) \rho_u H_{P+1, Q+1}^{0, N+1 : I : J} \left[z_1 (4ab+c)^{-q_1}, \dots, z_r (4ab+c)^{-q_r} \left| \begin{matrix} A : C \\ B : D \end{matrix} \right. \right]$$

Provided the conditions similar to that of Equation (10) with $m = n = p = q = 0$ and $\sigma = -1$, $F_0 = 1$.

Put $\alpha_{ij} = \beta_{ij} = C_{ij} = D_{ij} = 1$ ($i = 1, \dots, r$) Equation (10) to get integral transform involving product of general class of polynomials, H-function of one variable and G-function of 'r' variables:

$$\int_0^\infty \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_1}, \dots, y_t \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_t} \right] dx \quad (15)$$

$$\times H_{p,q+1}^{m+1,n} \left[\left(a_0 \left(ax + \frac{b}{x} \right)^2 + c \right) \left| \begin{matrix} (e_j, E_j)_{l,p} \\ (f_0, F_0), (f_j, F_j)_{l,q} \end{matrix} \right. \right] G \left[z_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_1}, \dots, z_r \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_r} \right] dx$$

$$= \frac{\sqrt{\pi} a_0^{-\sigma}}{2a\beta_0(4ab+c)} \frac{[V_1/U_1]}{\sum_{k_1=0}^{\eta+\sigma+\frac{1}{2}}} \dots \frac{[V_t/U_t]}{\sum_{k_t=0}^{\eta+\sigma+\frac{1}{2}}} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots, V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} (4ab+c)^{-\sum_{i=1}^t m_i k_i}}{k_i!}$$

$$\times \sum_{u=0}^{\infty} G(u) \rho_u H_{P+1, Q+1}^{0, N+1 : I : J} \left[z_1 (4ab+c)^{-q_1}, \dots, z_r (4ab+c)^{-q_r} \left| \begin{matrix} A : C \\ B : D \end{matrix} \right. \right]$$

Provided the conditions are similar to that of Equation (10) with $\alpha_{ij} = \beta_{ij} = C_{ij} = D_{ij} = 1$

Take $N = P = Q = 0$ in Equation (10), we get:

$$\int_0^\infty \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-\eta-1} S_{V_1, \dots, V_t}^{U_1, \dots, U_t} \left[y_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_1}, \dots, y_t \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-m_t} \right] dx \quad (16)$$

$$\times H_{p,q+1}^{m+1,n} \left[\left(a_0 \left(ax + \frac{b}{x} \right)^2 + c \right) \left| \begin{matrix} (e_j, E_j)_{l,p} \\ (f_0, F_0), (f_j, F_j)_{l,q} \end{matrix} \right. \right] H_{p_1, q_1}^{m_1, n_1} \left[z_1 \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_1} \right] \times \dots \times H_{p_r, q_r}^{m_r, n_r} \left[z_r \left[\left(ax + \frac{b}{x} \right)^2 + c \right]^{-q_r} \right] dx$$

$$= \frac{\sqrt{\pi} a_0^{-\sigma}}{2aF_0(4ab+c)} \frac{[V_1/U_1]}{\sum_{k_1=0}^{\eta+\sigma+\frac{1}{2}}} \dots \frac{[V_t/U_t]}{\sum_{k_t=0}^{\eta+\sigma+\frac{1}{2}}} (-V_1)_{U_1 k_1} \dots (-V_t)_{U_t k_t} A(V_1, k_1; \dots, V_t, k_t) \prod_{i=1}^t \frac{y_i^{k_i} (4ab+c)^{-\sum_{i=1}^t m_i k_i}}{k_i!}$$

$$\times \sum_{u=0}^{\infty} G(u) \rho_u H_{1,1}^{0,1 : I : J} \left[z_1 (4ab+c)^{-q_1}, \dots, z_r (4ab+c)^{-q_r} \left| \begin{matrix} A : C \\ B : D \end{matrix} \right. \right]$$

Provided that the conditions are similar to that of Equation (10) with $N = P = Q = 0$.

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