Scientific Research and Essays Vol. 6(23), pp. 4925-4935, 16 October, 2011
Available online at http://www.academicjournals.org/SRE
DOI: 10.5897/SRE11.364
ISSN 1992-2248 ©2011 Academic Journals

Full Length Research Paper

Recursive identification of Hammerstein ARMAX model with general discontinuous nonlinearity

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Accepted 22 August, 2011

This paper deals with the recursive parameter identification of the Hammerstein ARMAX (HARMAX) model with General model of discontinuous and asymmetric Nonlinearity (GNL) containing hysteresis, saturation, preload and dead-zone. This model contains different parameters the choice of which may generate several different nonlinearities. The linear and nonlinear structures are known, the linear block parameters are unknown. However the nonlinear block contains nine parameters only one of them is known. The estimation of the nonlinearity parameters, the internal variables relative to the nonlinearity as well as the linear model parameters are ensured by Recursive Extended Least Squares algorithm (RELS). The provided model is validated by simulation results.

Key words: Identification, discontinuous general nonlinearity, HARMAX model, parameter estimation, RELS algorithm.

INTRODUCTION

Hammerstein and Wiener models have been used to handle nonlinear systems (Chaari et al., 2008; Chen, 2004; Ding and Chen, 2005; Giri et al., 2001, 2008; Rejeb et al., 2010, 2011; Vörös, 1997, 2001, 2005; Liu and Bai, 2007). These models contain two cascaded linear and nonlinear blocks. When the nonlinearity is static, a particular study should be done (Bai, 2002; Vörös, 1997, 2003; Rejeb et al., 2010; Liu and Bai, 2007). In (Bai, 2002) six nonlinearities have been treated such as the saturation, the preload, the relay, the dead-zone, the hysteresis-relay and the hysteresis. In (Liu and Bai, 2007) only saturation and preload nonlinearities have been connected to linear block to build Hammerstein model. In literature, many authors treat the case of discontinuous nonlinearities. However these nonlinearities are handled separately (Bai, 2002; Rejeb et al., 2010; Liu and Bai, 2007). In (Vörös, 2001), the author has associated the dead-zone and preload nonlinearities to propose a Wiener model. In (Vörös, 2003), the same author has associated the same nonlinearities to propose a Hammerstein model. In (Chaari et al., 2008), the hysteresis and saturation nonlinearities are handled together to provide a Wiener model. Vörös (2010) has proposed a new form of dynamic backlash nonlinearity.

In this paper we propose a GNL which may generate all the possible combinations of elementary discontinuous nonlinearities. The idea consists on parametrizing a general nonlinear block by some parameters the choice of which selects the desired nonlinearity. The linear block which is described by an ARMAX model follows the considered nonlinearity. The paper is organized as follows: Section 2 gives a proposition of GNL. Section 3 presents the HARMAX model which consists on the association of this nonlinearity with a linear dynamic block characterized by ARMAX model. The estimation of model parameters and the internal variables is discussed in section 4 using the RELS algorithm. Section 5 is

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Abbreviations: HARMAX, Hammerstein ARMAX; **RELS**, Recursive Extended Least Squares algorithm; **GNL**, General model of Nonlinearity; $\mathbf{G_{NL}}$, General Nonlinearity; **ENL**, Exact Nonlinearity; **NMSE**, Normalized Mean Squared Error.

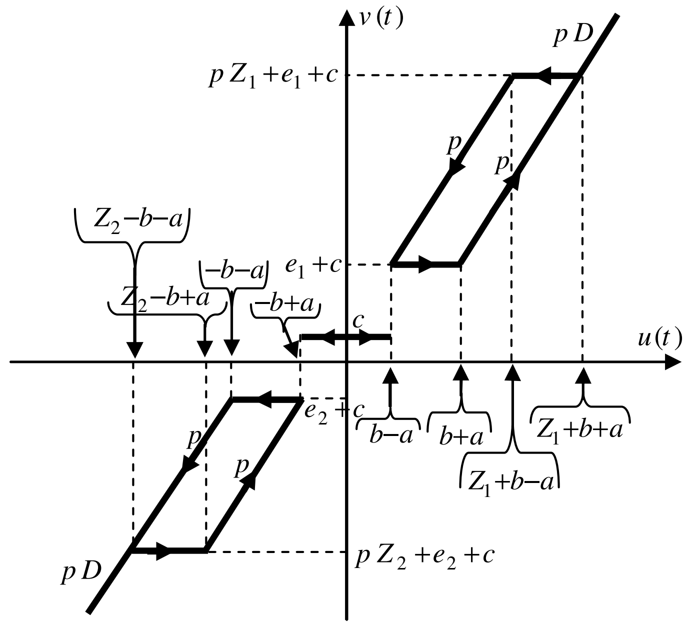

Figure 1. A general nonlinearity.

devoted to simulation results to evaluate the effectiveness of the proposed method. An emphasis of this effectiveness is built through a comparative study between the results of the proposed algorithm for some nonlinearities derived from the GNL and their corresponding exact nonlinearities.

MATERIALS AND METHODS

A general nonlinearity description

A general discontinuous and asymmetric nonlinearity can be characterized by a mapping function $g[.]$ as:

$$v(t) = g[u(t)] \tag{1}$$

Where $u(t)$ and $v(t)$ are the input and the output of the nonlinear block, respectively. In Figure 1, we illustrate the evolution of the output $v(t)$ with respect to the input $u(t)$. This General Nonlinearity (GNL) contains an asymmetric piecewise function with an hysteresis, a saturation, a preload and a dead-zone each of them may vary depending on some parameters. It is characterized by the unknown parameters $p,\ a,\ b,\ Z_1,\ Z_2,\ e_1,\ e_2,\ c$ and a known binary parameter D which enables or not the existence of the slope. Z_1 and Z_2 characterize the saturation, $a \geq 0$, $b \geq 0$ and $a \leq b$ that define the width and the center of the hysteresis as well as the dead-zone width, p is the hysteresis slope, e_1 and e_2 define the preload thresholds and $c \geq 0$ gives the threshold value.

The output of the GNL $v(t)$ represented by Figure 1 can be developed in piecewise as follows:

$$v(t)=\begin{cases} pZ_1+e_1+c & \text{if} \quad Z_1+b-a<u(t)<Z_1+b+a \text{ and } \Delta u(t)<0 \\ p(u(t)-a-b)+e_1+c & \text{if} \quad b+a\leq u(t)\leq Z_1+b+a \text{ and } \Delta u(t)>0 \\ p(u(t)-a+b)+e_2+c & \text{if} \quad Z_2-b+a\leq u(t)\leq -b+a \text{ and } \Delta u(t)>0 \\ p(u(t)+a-b)+e_1+c & \text{if} \quad b-a\leq u(t)\leq Z_1+b-a \text{ and } \Delta u(t)<0 \\ p(u(t)+a+b)+e_2+c & \text{if} \quad Z_2-b-a\leq u(t)\leq -b-a \text{ and } \Delta u(t)<0 \\ pZ_2+e_2+c & \text{if} \quad Z_2-b-a<u(t)<Z_2-b+a \text{ and } \Delta u(t)>0 \\ pD(u(t)-Z_1-b-a)+pZ_1+e_1+c & \text{if} \quad u(t)>Z_1+b+a\ \forall\, \Delta u(t) \\ pD(u(t)-Z_2+b+a)+pZ_2+e_2+c & \text{if} \quad u(t)<Z_2-b-a\ \forall\, \Delta u(t) \\ e_1+c & \text{if} \quad b-a<u(t)<b+a \text{ and } \Delta u(t)>0 \\ e_2+c & \text{if} \quad -b-a<u(t)<-b+a \text{ and } \Delta u(t)<0 \\ c & \text{if} \quad -b+a<u(t)<b-a\ \forall\, \Delta u(t) \end{cases} \tag{2}$$

With $\Delta u(t) = u(t) - u(t-1)$.

The set of the above piecewises defining $v(t)$ can be summarized in one equation as:

$$\begin{aligned} v(t) &= p\,u(t) f_1(t) - p\,a f_2(t) - p\,b f_3(t) + p\,Z_1 f_4(t) \\ &\quad + p\,Z_2 f_5(t) + e_1 f_6(t) + e_2 f_7(t) + c \end{aligned} \tag{3}$$

Where the functions α and $f_i(t),\ i = 1,\dots,7$ are defined as:

$$\alpha = h[u(t) - u(t-1)] \ = h[\Delta u(t)] \tag{4}$$

$$\begin{aligned} f_1(t) &= \alpha[(1-(h[u(t)-(Z_1+b+a)]+h[(b+a)-u(t)])) \\ &\quad +(1-(h[(Z_2-b+a)-u(t)]+h[u(t)-(-b+a)]))] \\ &\quad +(1-\alpha)[(1-(h[u(t)-(Z_1+b-a)]+h[(b-a)-u(t)])) \\ &\quad +(1-(h[(Z_2-b-a)-u(t)]+h[u(t)-(-b-a)]))] \\ &\quad +D\,(h[u(t)-(Z_1+b+a)]+h[(Z_2-b-a)-u(t)]) \end{aligned} \tag{5}$$

$$\begin{aligned} f_2(t) &= \alpha[(1-(h[u(t)-(Z_1+b+a)]+h[(b+a)-u(t)])) \\ &\quad +(1-(h[(Z_2-b+a)-u(t)]+h[u(t)-(-b+a)]))] \\ &\quad -(1-\alpha)[(1-(h[u(t)-(Z_1+b-a)]+h[(b-a)-u(t)])) \\ &\quad +(1-(h[(Z_2-b-a)-u(t)]+h[u(t)-(-b-a)]))] \\ &\quad +D\,(h[u(t)-(Z_1+b+a)]-h[(Z_2-b-a)-u(t)]) \end{aligned} \tag{6}$$

$$\begin{aligned} f_3(t) &= \alpha[(1-(h[u(t)-(Z_1+b+a)]+h[(b+a)-u(t)])) \\ &\quad -(1-(h[(Z_2-b+a)-u(t)]+h[u(t)-(-b+a)]))] \\ &\quad +(1-\alpha)[(1-(h[u(t)-(Z_1+b-a)]+h[(b-a)-u(t)])) \\ &\quad -(1-(h[(Z_2-b-a)-u(t)]+h[u(t)-(-b-a)]))] \\ &\quad +D\,(h[u(t)-(Z_1+b+a)]-h[(Z_2-b-a)-u(t)]) \end{aligned} \tag{7}$$

$$\begin{aligned} f_4(t) &= \alpha h[u(t)-(Z_1+b+a)]+(1-\alpha)h[u(t)-(Z_1+b-a)] \\ &\quad -D\,(h[u(t)-(Z_1+b+a)]) \end{aligned} \tag{8}$$

$$\begin{aligned} f_5(t) &= \alpha h[(Z_2-b+a)-u(t)]+(1-\alpha)h[(Z_2-b-a)-u(t)] \\ &\quad -D\,(h[(Z_2-b-a)-u(t)]) \end{aligned} \tag{9}$$

$$f_6(t) = h[u(t)-(b-a)] \tag{10}$$

$$f_7(t) = h[(-b+a)-u(t)] \tag{11}$$

$h[\beta]$ is a switching function

Table 1. Different generated nonlinearities.

GNL	Choice of parameter
G_{NL}	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 \neq 0, e_2 \neq 0, c \neq 0, D = 1$
NL_1	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 \neq 0, e_2 \neq 0, c = 0, D = 1$
NL_2	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 1$
NL_3	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 \neq 0, e_2 \neq 0, c = 0, D = 0$
NL_4	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 0$
NL_5	$p \neq 0, a \neq 0, b = 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 0$
NL_6	$p \neq 0, a \neq 0, b = 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 1$
NL_7	$p \neq 0, a \neq 0, b \neq 0, Z_1 = 0, Z_2 \neq 0, e_1 \neq 0, e_2 = 0, c = 0, D = 1$
NL_8	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 = 0, e_1 = 0, e_2 \neq 0, c = 0, D = 1$
NL_9	$p \neq 0, a \neq 0, b \neq 0, Z_1 = 0, Z_2 \neq 0, e_1 = 0, e_2 \neq 0, c = 0, D = 1$
NL_{10}	$p \neq 0, a \neq 0, b \neq 0, Z_1 \neq 0, Z_2 = 0, e_1 \neq 0, e_2 = 0, c = 0, D = 1$
NL_{11}	$p \neq 0, a = 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 \neq 0, e_2 \neq 0, c = 0, D = 0$
NL_{12}	$p \neq 0, a = 0, b = 0, Z_1 \neq 0, Z_2 \neq 0, e_1 \neq 0, e_2 \neq 0, c = 0, D = 0$
NL_{13}	$p \neq 0, a = 0, b \neq 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 0$
NL_{14}	$p \neq 0, a = 0, b = 0, Z_1 \neq 0, Z_2 \neq 0, e_1 = 0, e_2 = 0, c = 0, D = 0$
NL_{15}	$p \neq 0, a = 0, b \neq 0, Z_1 = 0, Z_2 = 0, e_1 = 0, e_2 = 0, c \neq 0, D = 1$
NL_{16}	$p \neq 0, a = 0, b \neq 0, Z_1 = 0, Z_2 = 0, e_1 = 0, e_2 = 0, c = 0, D = 1$
NL_{17}	$p \neq 0, a = 0, b = 0, Z_1 = 0, Z_2 = 0, e_1 \neq 0, e_2 \neq 0, c = 0, D = 1$

$$h[\beta] = \begin{cases} 1 & if \quad \beta > 0 \\ 0 & if \quad \beta \leq 0 \end{cases} \tag{12}$$

The GNL described by Figure 1 is obtained when all the parameters in Equation (3) are different from zero. Different choices of parameters, lead to the definition of different nonlinearities. In this paper, we are interested in the nonlinearities (NL_i , $i = 1,..., 17$) summarized in Table 1 and represented by Figure 2.

The HARMAX model with GNL

The HARMAX model is sketched by Figure 3 where the GNL is connected to an ARMAX model. Where $u(t)$ is the model input, $v(t)$ is the internal variable (considered unknown) which is the nonlinearity output, $\tilde{y}(t)$ is the noise-free model output, $w(t)$ is the noise block output, $e(t)$ is an additive white noise with zero mean and variance σ^2 and $y(t)$ is the model output. The output of the nonlinear block $v(t)$ is described by Equation (3), and the HARMAX model output $y(t)$ is:

$$y(t) = \tilde{y}(t) + w(t) \tag{13}$$

Where $\tilde{y}(t)$ and $w(t)$ satisfy:

$$\tilde{y}(t) = \frac{q^{-d}\, B(q^{-1})}{A(q^{-1})} v(t) \ , \quad w(t) = \frac{C(q^{-1})}{A(q^{-1})} e(t) \tag{14}$$

d is the delay, and $A(q^{-1})$, $B(q^{-1})$, $C(q^{-1})$ are polynomials of q^{-1} defined as:

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_m q^{-m} \tag{15}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_n q^{-n} \tag{16}$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_\ell q^{-\ell} \tag{17}$$

And the output $y(t)$ becomes:

$$y(t) = \sum_{i=0}^{n} b_i\, v(t-d-i) - \sum_{j=1}^{m} a_j\, y(t-j) + \sum_{i=1}^{\ell} c_i\, e(t-i) + e(t) \tag{18}$$

A direct substitution of Equation (3) into Equation (18) may result in a very complex equation of HARMAX model with cross-multiplied parameters of linear and nonlinear blocks as all the parameters b_i, $i = 0,..., n$ will be combined with the parameters characterizing

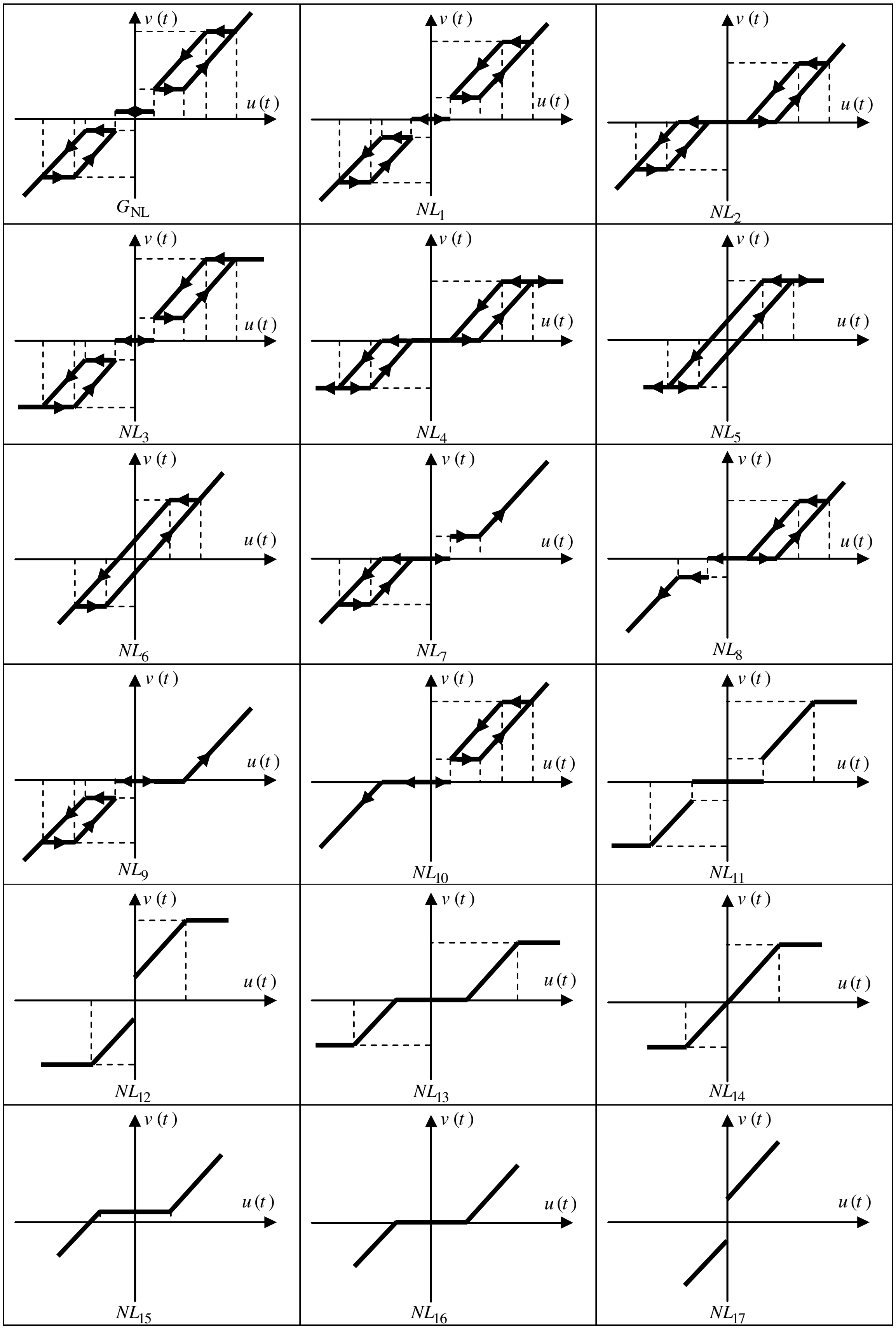

Figure 2. Different cases of nonlinearities

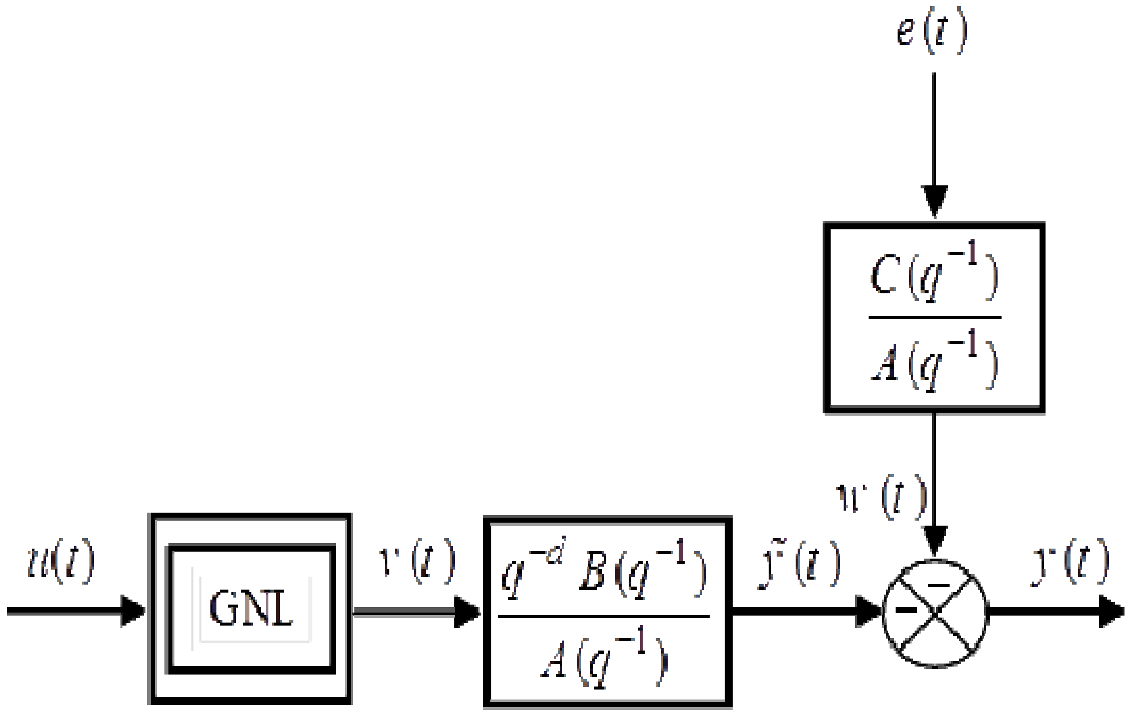

Figure 3. The HARMAX model.

the nonlinearity. Therefore the total number of the HARMAX model parameters is $L_0 = r\,n + m + \ell$, with r the number of nonlinearity parameters. To overcome this problem, we use the separation principle (Vörös, 2003). Assuming that $b_0 = 1$, the first sum of Equation (18) which contains internal variable can be split as:

$$y(t) = v(t-d) + \sum_{i=1}^{n} b_i\, v(t-d-i) - \sum_{j=1}^{m} a_j\, y(t-j) + \sum_{i=1}^{\ell} c_i\, e(t-i) + e(t) \tag{19}$$

Replacing $v(t-d)$ by its value given by Equation (3), the HARMAX model output is:

$$\begin{aligned} y(t) &= p\,u(t-d) f_1(t-d) - p\,a f_2(t-d) - p\,b f_3(t-d) \\ &+ p\,Z_1 f_4(t-d) + p\,Z_2 f_5(t-d) + e_1 f_6(t-d) \\ &+ e_2 f_7(t-d) + c + \sum_{i=1}^{n} b_i\, v(t-d-i) \\ &- \sum_{j=1}^{m} a_j\, y(t-j) + \sum_{i=1}^{\ell} c_i\, e(t-i) + e(t) \end{aligned} \tag{20}$$

Equation (20) and Equations (3, 5 to 11) defining the internal variable $v(t)$ and the functions $f_i(t),\ i = 1,...,7$ represent a new form of discontinuous HARMAX model, where all the parameters to be estimated are separated. The total number of the HARMAX model parameters becomes ($L = r + n + m + \ell$) as there are no combinations of parameters in the Equation (20).

Recursive identification algorithm

The HARMAX model with GNL contains non-measurable internal variable which must be estimated. A RELS algorithm has been used to estimate the HARMAX model parameters. The output $y(t)$ of Equation (20) can be tightened in a matrix form as:

$$y(t) = \varphi^T(t)\,\theta + e(t) \tag{21}$$

Where $\varphi^T(t)$ and θ are the $L-$dimensional observation and parameter vectors given by:

$$\begin{aligned} \varphi^T(t) = [&u(t-d) f_1(t-d),\ -f_2(t-d),\ -f_3(t-d) \\ &, f_4(t-d), f_5(t-d), f_6(t-d), f_7(t-d), 1 \\ &, v(t-d-1),..., v(t-d-n), -y(t-1),.. \\ &., -y(t-m), e(t-1),..., e(t-\ell)] \end{aligned} \tag{22}$$

$$\begin{aligned} \theta = [&p, pa, pb, pZ_1, pZ_2, e_1, e_2, c, b_1,..., b_n \\ &, a_1,..., a_m, c_1,..., c_\ell]^T \end{aligned} \tag{23}$$

RELS Algorithm

$$\hat{\theta}(0) = \left[0.0001,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ldots,\ 0,\ 0,\ldots,\ 0,\ 0,\ldots,\ 0\right]^T,\ \hat{P}(0) = 10^3 I \quad (24a)$$

$$\begin{aligned}\hat{\varphi}^T(t) = [&u(t-d)\hat{f}_1(t-d),\ -\hat{f}_2(t-d),\ -\hat{f}_3(t-d),\ \hat{f}_4(t-d),\ \hat{f}_5(t-d),\ \hat{f}_6(t-d),\ \hat{f}_7(t-d),\ 1\\ &,\hat{v}(t-d-1),\ldots,\hat{v}(t-d-n),\ -y(t-1),\ldots,\ -y(t-m),\ \varepsilon(t-1),\ldots.,\ \varepsilon(t-\ell)]\end{aligned} \quad (24b)$$

$$\varepsilon(t) = y(t) - \hat{\varphi}^T(t)\hat{\theta}(t-1) \quad (24c)$$

$$P(t) = \left(\frac{1}{\lambda}\right)\left[P(t-1) - \frac{P(t-1)\hat{\varphi}(t)\hat{\varphi}^T(t)P(t-1)}{\lambda + \hat{\varphi}^T(t)P(t-1)\hat{\varphi}(t)}\right] \quad (24d)$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P(t-1)\hat{\varphi}(t)}{\lambda + \hat{\varphi}^T(t)P(t-1)\hat{\varphi}(t)}\varepsilon(t) \quad (24e)$$

$$\begin{aligned}\hat{\theta}(t) = [&\hat{p}(t),\ \hat{p}(t)\hat{a}(t),\ \hat{p}(t)\hat{b}(t),\ \hat{p}(t)\hat{Z}_1(t),\ \hat{p}(t)\hat{Z}_2(t),\ \hat{e}_1(t),\ \hat{e}_2(t),\ \hat{c}(t)\\ &,\hat{b}_1(t),\ldots,\hat{b}_n(t),\ \hat{a}_1(t),\ldots,\hat{a}_m(t),\ \hat{c}_1(t),\ldots,\hat{c}_\ell(t)]^T\end{aligned} \quad (24f)$$

where λ is a forgetting factor and $\hat{v}(t-d)$ is:

$$\begin{aligned}\hat{v}(t-d) = {}&\hat{p}(t-1)u(t-d)\hat{f}_1(t-d) - \hat{p}(t-1)\hat{a}(t-1)\hat{f}_2(t-d) - \hat{p}(t-1)\hat{b}(t-1)\hat{f}_3(t-d)\\ &+ \hat{p}(t-1)\hat{Z}_1(t-1)\hat{f}_4(t-d) + \hat{p}(t-1)\hat{Z}_2(t-1)\hat{f}_5(t-d) + \hat{e}_1(t-1)\hat{f}_6(t-d)\\ &+ \hat{e}_2(t-1)\hat{f}_7(t-d) + \hat{c}(t-1)\end{aligned} \quad (25)$$

with

$$\begin{aligned}\hat{f}_1(t-d) = {}&\alpha[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]+h[(\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]))\\ &+(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)+\hat{a}(t-1))]))]\\ &+(1-\alpha)[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)-\hat{a}(t-1))]+h[(\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]))\\ &+(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)-\hat{a}(t-1))]))]\\ &+D\,(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]+h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)])\end{aligned} \quad (26)$$

$$\begin{aligned}\hat{f}_2(t-d) = {}&\alpha[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]+h[(\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]))\\ &+(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)+\hat{a}(t-1))]))]\\ &-(1-\alpha)[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)-\hat{a}(t-1))]+h[(\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]))\\ &+(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)-\hat{a}(t-1))]))]\\ &+D\,(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]-h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)])\end{aligned} \quad (27)$$

$$\begin{aligned}\hat{f}_3(t-d) = {}&\alpha[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]+h[(\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]))\\ &-(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)+\hat{a}(t-1))]))]\\ &+(1-\alpha)[(1-(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)-\hat{a}(t-1))]+h[(\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]))\\ &-(1-(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]+h[u(t-d)-(-\hat{b}(t-1)-\hat{a}(t-1))]))]\\ &+D\,(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]-h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)])\end{aligned} \quad (28)$$

$$\begin{aligned}\hat{f}_4(t-d) = {}&\alpha\,h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))]+(1-\alpha)h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)-\hat{a}(t-1))]\\ &-D\,(h[u(t-d)-(\hat{Z}_1(t-1)+\hat{b}(t-1)+\hat{a}(t-1))])\end{aligned} \quad (29)$$

$$\begin{aligned}\hat{f}_5(t-d) = {}&\alpha h[(\hat{Z}_2(t-1)-\hat{b}(t-1)+\hat{a}(t-1))-u(t-d)]+(1-\alpha)h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)]\\ &-D\,(h[(\hat{Z}_2(t-1)-\hat{b}(t-1)-\hat{a}(t-1))-u(t-d)])\end{aligned} \quad (30)$$

$$\hat{f}_6(t-d) = h[u(t-d)-(\hat{b}(t-1)-\hat{a}(t-1))] \quad (31)$$

Table 2. Parameters estimation of the HARMAX model with G_{NL} and NL_i , $i = 1, 4, 5, 6, 8, 9, 11, 13, 16, 17$

		$\hat{p}$	$\hat{a}$	$\hat{b}$	$\hat{Z}_1$	$\hat{Z}_2$	$\hat{e}_1$	$\hat{e}_2$	$\hat{c}$	$\hat{b}_1$	$\hat{a}_1$	$\hat{a}_2$	$\hat{c}_1$
G_{NL}	ex.	1.8000	0.4000	1.2000	1.5000	-1.0000	1.0000	-0.5000	0.5000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8013	0.3994	1.2024	1.4987	-1.0002	1.0023	-0.5003	0.4985	0.1497	-0.1998	0.3498	0.2060
NL_1	ex.	1.8000	0.4000	1.2000	1.5000	-1.0000	1.0000	-0.5000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.7987	0.4006	1.1973	1.4999	-1.0007	0.9978	-0.4994	-0.0004	0.1491	-0.2007	0.3501	0.1709
NL_4	ex.	1.8000	0.4000	1.2000	1.5000	-1.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8025	0.4007	1.2013	1.4980	-1.0005	-0.0056	-0.0033	0.0051	0.1496	-0.2002	0.3504	0.1961
NL_5	ex.	1.8000	0.4000	0.0000	1.5000	-1.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8000	0.4014	0.0031	1.4961	-0.9943	0.0116	-0.0059	-0.0041	0.1517	-0.1984	0.3497	0.1642
NL_6	ex.	1.8000	0.4000	0.0000	1.5000	-1.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8004	0.4005	-0.0001	1.5005	-0.9990	0.0016	0.0014	-0.0022	0.1503	-0.1997	0.3499	0.1651
NL_8	ex.	1.8000	0.4000	1.2000	1.5000	0.0000	0.0000	-0.5000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.7989	0.4009	1.1972	1.4982	-0.0036	-0.0021	-0.4989	-0.0006	0.1488	-0.2008	0.3502	0.1622
NL_9	ex.	1.8000	0.4000	1.2000	0.0000	-1.0000	0.0000	-0.5000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.7992	0.4003	1.2002	0.0000	-0.9985	-0.0008	-0.5025	0.0002	0.1489	-0.2009	0.3503	0.2002
NL_{11}	ex.	1.8000	0.0000	1.2000	1.5000	-1.0000	1.0000	-0.5000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8015	0.0001	1.1882	1.5087	-1.0138	0.9853	-0.4719	-0.0037	0.1494	-0.2007	0.3503	0.1764
NL_{13}	ex.	1.8000	0.0000	1.2000	1.5000	-1.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.7958	0.0009	1.1836	1.5184	-1.0182	-0.0261	0.0297	-0.0005	0.1496	-0.2004	0.3501	0.2548
NL_{16}	ex.	1.8000	0.0000	1.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8007	0.1270	1.0733	0.0008	0.0014	0.0018	0.0028	-0.0011	0.1500	-0.1998	0.3500	0.2240
NL_{17}	ex.	1.8000	0.0000	0.0000	0.0000	0.0000	1.0000	-0.5000	0.0000	0.1500	-0.2000	0.3500	0.2000
	es.	1.8007	-0.0026	-0.0613	0.0494	0.0139	0.7565	-0.5117	0.1270	0.1494	-0.2003	0.3498	0.1405

$$\hat{f}_7(t-d) = h[(-\hat{b}(t-1) + \hat{a}(t-1)) - u(t-d)] \quad (32)$$

The internal variable $v(t-d)$ and the functions $f_i(t-d)$, $i = 1,..., 7$ depend on the vector θ, for that $\varphi(t) = \varphi(\theta, t)$. The latter are unknown. Therefore they are replaced in vector $\hat{\varphi}^T(t)$ by their estimates $\hat{v}(t-d)$ and $\hat{f}_i(t-d)$. the estimation of θ is assured by the RELS algorithm.

RESULTS

The proposed algorithm for the recursive identification of the HARMAX model with GNL has been implemented and tested in simulations. The considered HARMAX model is given by:

The nonlinear block is defined by Figure 1 with $p = 1.8$; $a = 0.4$; $b = 1.2$; $Z_1 = 1.5$; $Z_2 = -1$; $e_1 = 1$
$e_2 = -0.5$; $c = 0.5$
The linear block is given by the difference equation

$$y(t) = v(t-1) + 0.15v(t-2) + 0.2y(t-1) - 0.35y(t-2) + 0.2e(t-1) + e(t)$$

Thereafter, we propose to demonstrate the efficiency of the GNL In fact, in Table 3, we compare the estimated parameters of the HARMAX model for a particular nonlinearity issued from the GNL with those of the HARMAX model containing the Exact Nonlinearity (ENL) (see (A.1 - A.6)). Three cases of the GNL presented in Figure 2 are selected (NL_{14}, NL_{15}, NL_{16}) that are described in Table 1.

The recursive identification was carried out for $N = 2000$ samples of uniformly distributed random inputs with $|u(t)| < 5$. The forgetting factor $\lambda = 0.99$. The noise $e(t)$ is Gaussian white with zero mean and variance $\sigma^2 = 0.0001$.

In Table 2, we summarise for some cases of nonlinearities (G_{NL} and NL_i , $i = 1,4,5,6,8,9,11,13,16,17$) and for each parameter of the vector θ, the exact value (ex.) and the final estimated value (es.).
The results were carried out with $N = 2000$ samples of uniformly distributed random input with $|u(t)| < 5$ and a Gaussian white noise with zero mean and two variances $\sigma_1^2 = 0.001$ and $\sigma_2^2 = 0.5$ for the HARMAX model with GNL and ENL.

In Table 3, we present the parameters estimated of the HARMAX model with GNL and ENL and for each parameter the exact value (ex.) and the final estimated values for σ_1^2 and σ_2^2.

In Figures 4 to 6, we illustrate the evolution of the Normalized Mean Squared Error (*NMSE*) for the HARMAX model with the cases of GNL mentioned

Table 3. Parameters estimation of the HARMAX model with GNL and ENL

			$\hat{p}$	$\hat{a}$	$\hat{b}$	$\hat{Z}_1$	$\hat{Z}_2$	$\hat{e}_1$	$\hat{e}_2$	$\hat{c}$	$\hat{b}_1$	$\hat{a}_1$	$\hat{a}_2$	$\hat{c}_1$
	ENL	ex.	1.8000	—	—	1.5000	-1.0000	—	—	—	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.8145	—	—	1.4850	-0.9891	—	—	—	0.1496	-0.2004	0.3501	0.2436
		σ_2^2	1.8120	—	—	1.4672	-0.9202	—	—	—	0.1755	-0.1695	0.3468	0.2954
	GNL	ex.	1.8000	0.0000	0.0000	1.5000	-1.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.8090	-0.0003	-0.0765	1.5636	-1.0676	-0.1371	0.1336	0.0030	0.1499	-0.2002	0.3501	0.2299
NL_{14}		σ_2^2	1.8956	0.0072	-0.1079	1.5098	-0.8942	-0.3833	-0.1198	0.1606	0.1670	-0.1735	0.3462	0.2926
	ENL	ex.	1.8000	—	1.2000	—	—	—	—	0.5000	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.7980	—	1.1986	—	—	—	—	0.4992	0.1512	-0.1987	0.3497	0.2183
		σ_2^2	1.7213	—	1.1261	—	—	—	—	0.5775	0.1190	-0.2234	0.3665	0.2047
	GNL	ex.	1.8000	0.0000	1.2000	0.0000	0.0000	0.0000	0.0000	0.5000	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.7979	0.1500	1.0472	0.0047	-0.0015	0.0020	0.0071	0.4957	0.1510	-0.1987	0.3497	0.2187
NL_{15}		σ_2^2	1.7428	-0.1327	1.2207	-0.2023	-0.1972	-0.0913	0.2090	0.5281	0.1259	-0.2166	0.3625	0.2071
	ENL	ex.	1.8000	—	1.2000	—	—	—	—	—	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.7958	—	1.1938	—	—	—	—	—	0.1504	-0.1986	0.3495	0.1564
		σ_2^2	1.7915	—	1.1830	—	—	—	—	—	0.1310	-0.2322	0.3507	0.2203
	GNL	ex.	1.8000	0.0000	1.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1500	-0.2000	0.3500	0.2000
		σ_1^2	1.7963	0.0094	1.1899	-0.0077	-0.1302	0.0007	-0.0136	0.0036	0.1503	-0.1987	0.3496	0.1515
NL_{16}		σ_2^2	1.8236	0.2663	1.0195	0.0520	-0.0212	0.1536	-0.0463	-0.0556	0.1221	-0.2379	0.3510	0.2240

previously in Table 3 ($NL_{14}, NL_{15}, NL_{16}$) for both noise variances $\sigma_1^{\ 2}$ and $\sigma_2^{\ 2}$.

$$NMSE\,(\%) = 100\,\frac{\sum_{t=1}^{N}(y(t)-\hat{y}(t))^2}{\sum_{t=1}^{N}(y(t))^2}$$

DISCUSSION

The test of the proposed RELS algorithm for parameter identification of HARMAX model with each case of nonlinearities described by Figure 2 has shown good results. In Table 2, we chose to present only some cases of these nonlinearities. Observing this Table, we remark that all the parameters estimated characterizing the GNL and the linear block converge well towards their exact values. This convergence is also good even in the other cases of nonlinearities where there are some parameters are zero. For example, in NL_6 four parameters are equal zero ($b = 0, e_1 = 0, e_2 = 0, c = 0$), these final estimated values converge to zero ($\hat{b} = -0.0001, \hat{e}_1 = 0.0016, \hat{e}_2 = 0.0014,\ \hat{c} = -0.0022$).

The GNL can provide 18 cases of nonlinearities while varying the parameters which characterize them. For each one of these nonlinearities, we can, thus, find an ENL while removing the parameters and the components which equal zero in the GNL, e.g. for the nonlinearities ($NL_{14}, NL_{15}, NL_{16}$) (see (A.1 - A.6)).

In Table 3, we present all the parameters, for those which are zero in the GNL, we do not find them in the ENL and we can note them by ($^{-}$).

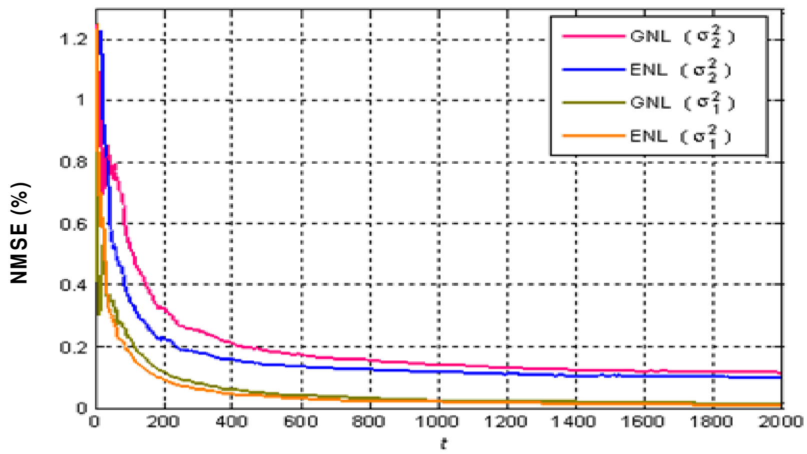

Figure 4. NMSE for the HARMAX model with NL_{14}

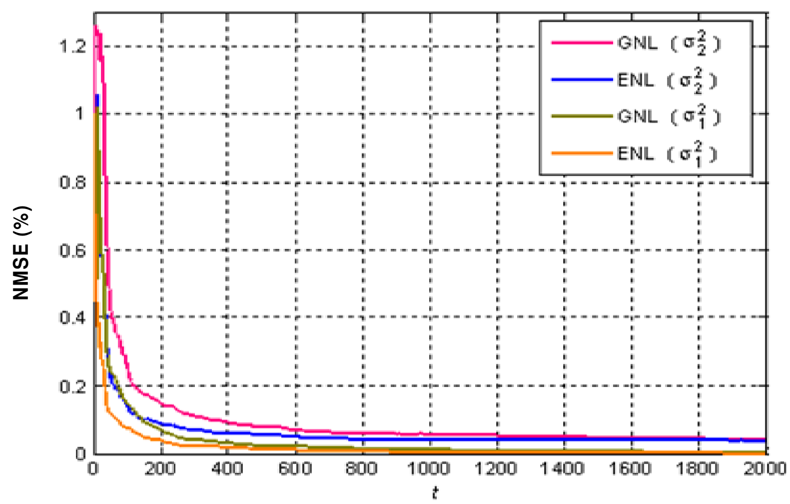

Figure 5. NMSE for the HARMAX model with NL_{15}.

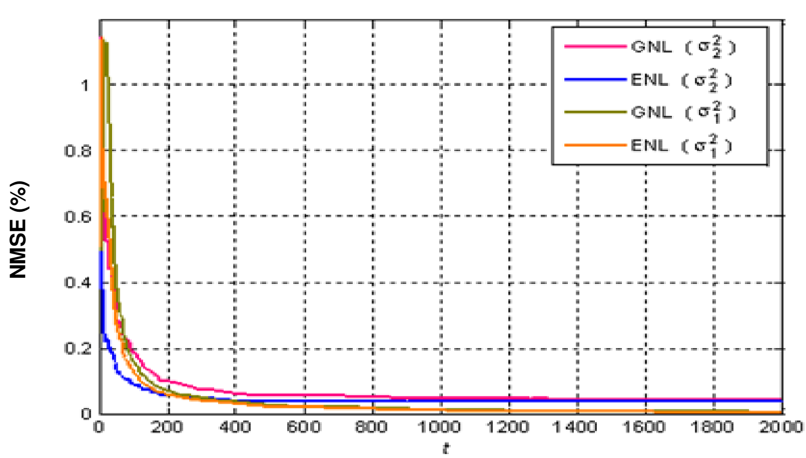

Figure 6. NMSE for the HARMAX model with NL_{16}.

Comparing the estimated parameters of the HARMAX model with GNL and ENL for both noise variances σ_1^2 and σ_2^2, we notice that they are very close and converge towards their exact values. Indeed, an evaluation of the *NMSE* shows the clear convergence.
After a deep discussion, the GNL can be validated.

Conclusions

In this paper, a RELS algorithm is presented for the identification of HARMAX model parameters with the GNL. The new form of this nonlinearity contains hysteresis, saturation, preload and dead-zone. This form enables us to obtain a multitude of nonlinearities according to the choice of some parameters equal or different from zero. The decomposition technique based on the separation principle enables to separate the parameters of the linear block from those of the nonlinear one. It is assumed here that the structure of this model is known and that all the parameters are unknown including the points of discontinuity, but one which is known and binary. The proposed algorithm has been tested in simulation. Moreover a comparison between the estimated parameters for a specific choice of the nonlinearity with those corresponding to the ENL has been achieved.

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Appendix

The outputs of the nonlinear block containing the exact nonlinearities (NL_{14}, NL_{15}, NL_{16}) can be written respectively as follows:

For NL_{14}:

$$v(t) = p\,u(t) f_{11}(t) + p\,Z_1\,h_{11}(t) + p\,Z_2\,h_{12}(t) \tag{A.1}$$

$$\begin{aligned} f_{11}(t) &= 1 - (h[u(t) - Z_1] + h[Z_2 - u(t)]) \\ h_{11}(t) &= h[u(t) - Z_1] \\ h_{12}(t) &= h[Z_2 - u(t)] \end{aligned} \tag{A.2}$$

For NL_{15}:

$$v(t) = p\,u(t) f_{21}(t) - p\,b\,f_{22}(t) + c \tag{A.3}$$

$$\begin{aligned} f_{21}(t) &= h[u(t) - b] + h[-b - u(t)] \\ f_{22}(t) &= h[u(t) - b] - h[-b - u(t)] \end{aligned} \tag{A.4}$$

For NL_{16}:

$$v(t) = p\,u(t) f_{31}(t) - p\,b\,f_{32}(t) \tag{A.5}$$

$$\begin{aligned} f_{31}(t) &= h[u(t) - b] + h[-b - u(t)] \\ f_{32}(t) &= h[u(t) - b] - h[-b - u(t)] \end{aligned} \tag{A.6}$$