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Full Length Research Paper

Improving forward solution for 2D block electrical impedance tomography using modified equations

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Electrical impedance tomography is a simple and economic medical imaging technology which permits regional assessment of the electrical properties of organs within the body based on measurements made from electrodes on the surface of the body. Block method is a new solution for electrical impedance tomography used to enhance image resolution and to improve reconstruction algorithm. Image reconstruction by block method is an ill-posed and nonlinear problem also has memory and time consuming process which can be improved by using modified equations. Improving forward solution for block electrical impedance tomography method can make linear equations for image reconstruction algorithm.

Key words: Electrical impedance tomography, numerical analysis, forward solution, nonlinear equations, computational methods, medical imaging.

INTRODUCTION

Electrical Impedance Tomography (EIT) is a medical imaging technology based on measurements made from electrodes on the surface of the body. EIT is cheaper and smaller than other medical imaging systems and requires no ionizing radiation. Different tissues of a body have different conductivities. The knowledge of the map of the internal electrical properties has a number of advantages in the many of medical diagnosis. EIT is a useful method for medical imaging of pulmonary embolism and blood clots in the lungs (Frerichs et al., 2002), breast (Hartov et al., 2004), neural system studies (Polydorides et al., 2002), breath system studies (Wu et al., 2007; Lionheart et al., 2008), vascular system studies, brain imaging and other medical issues (Holder, 2004).

EIT problem includes forward and inverse problems. Presenting an appropriate solution for EIT inverse problem is the main propose of EIT methods. However, EIT inverse problem depends on EIT forward problem. Reconstruction algorithm for EIT or EIT inverse problem can be solved by iterative algorithms (Kaipio et al., 2000; Borcea, 2001; Goharian et al., 2009), or non-iterative linearization algorithms (Allers and Santosa, 1991; Isaacson and Cheney, 1991; Blue, 1997), or layer-stripping algorithms (Cheney et al., 1992; Kulkarni et al., 2009) and direct algorithms (Mueller et al., 2002; Abbasi et al., 2009). Linear methods are suitable for low contrast imaging, and nonlinear methods are used when more accurate imaging results are required. A semi-linear method can be used to preserve some properties of the nonlinear inverse solver and at the same time can have some advantages in computational time (Soleimani, 2008). In block method, it is assumed that the subject has a two-dimensional (2D) rectangular shape and is made of fixed size blocks and all of the particles of each block have the same electrical properties (electrical conductivities) (Abbasi et al., 2009). Image reconstruction by block method is an ill-posed and nonlinear inverse problem. Nonlinear equations on EIT inverse problem cause memory and time consuming process. Nonlinear equations on EIT inverse problem depends on nonlinear equations on EIT forward problem. Linearization techniques are widely used and require the repeated solution of a linear forward problem (Soleimani et al., 2005). Modifying forward block method, equations can improve the performance of method. In this paper, block method is presented in brief and then modification on equations is proposed and finally results are discussed.

BLOCK METHOD

In block method, is a rectangular shape subject that is divided into

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$m \times n$, similar size blocks with same electrical impedances in each block (Abbasi et al., 2009). As shown in Figure 1, a rectangular subject has been aligned in Cartesian system and each block has been named as B(i,j) (block in the i th row and j th column of Cartesian system).

The first index, i is called the transverse index and the second is the normal index, j . The transverse index is the horizontal distance along the x-axis, measured from the origin. The normal index is the vertical distance along the y-axis, also measured from the origin. Reasonably, we would have $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$. In this manner and according to previous assumptions, the specific conductivity $\sigma[i, j]$ is assigned to any particle of block B(i,j). In this block set, any block has electrical interchanges with neighboring blocks or injector electrodes. Now assume that the monotonic current densities $J_1[i, j]$ and $J_3[i, j]$ enter to left and bottom sides of block B(i,j), respectively, and similarly, the current densities $J_2[i, j]$ and $J_4[i, j]$ exit the right and upper sides, respectively, and the terms of $e_0[i, j]$, $e_1[i, j]$, $e_2[i, j]$, $e_3[i, j]$ and $e_4[i, j]$ are the potentials of nodes placed in the middle of left, right, bottom and upper sides of the block B(i,j), respectively (Figures 2 and 3).

To generate an EIT image, a series of electrodes are attached to the body. Various currents can be injected through these electrodes and the produced voltages can be measured. By current injection, voltage measuring and using reconstruction algorithm, conductivity distribution inside the subject would be calculated (Lionheart, 2004; Holder, 2004; Babaeizadeh et al., 2007). EIT forward problem involves constructing a block model and calculating a set of voltages (or currents or both) produced on the boundary when other set of currents are injected (or voltages are applied) on the other boundary (Babaeizadeh et al., 2007). For B(i,j) block following equations are true (Vosoughi Vahdat and Niknam, 2003):

$\forall i \in N, 1 \le i \le m$ and $\forall j \in N, 1 \le j \le n$

$$e_1[i, j] - e_O[i, j] = \frac{1}{8}\Delta\left(\frac{3J_1[i, j] + J_2[i, j]}{\sigma[i, j]}\right) \quad (1)$$

$$e_O[i, j] - e_2[i, j] = \frac{1}{8}\Delta\left(\frac{J_1[i, j] + 3J_2[i, j]}{\sigma[i, j]}\right) \quad (2)$$

$$e_3[i, j] - e_O[i, j] = \frac{1}{8}\Delta\left(\frac{3J_3[i, j] + J_4[i, j]}{\sigma[i, j]}\right) \quad (3)$$

$$e_O[i, j] - e_4[i, j] = \frac{1}{8}\Delta\left(\frac{J_3[i, j] + 3J_4[i, j]}{\sigma[i, j]}\right) \quad (4)$$

$$J_1[i, j] + J_3[i, j] = J_2[i, j] + J_4[i, j] \quad (5)$$

where, Δ is the lengths of the block in both X and Y directions (the transverse and vertical sizes of the block).

Modified block method

Applying block method, Equations 1 - 5 as a forward solution for EIT causes nonlinear and time consuming inverse solution (Abbasi et al., 2009). Modifying EIT forward equation can lead to generate faster and performance inverse solution algorithm. In this section modification on EIT equations will be discussed. According to Equations 1, 3 and 5 next equation can be written as following:

$$e_1[i, j] + e_3[i, j] - 2e_O[i, j] = \frac{1}{8}\Delta\left(\frac{4J_1[i, j] + 4J_3[i, j]}{\sigma[i, j]}\right) \quad (6)$$

Also, Equation 1 can be modified as Equation 7:

$$2e_1[i, j] - 2e_O[i, j] = 6f_1[i, j] + 2f_2[i, j] \quad (7)$$

Where, $f_k[i, j]$ is an auxiliary equation

$$\left(f_k[i, j] = \frac{1}{8}\Delta\frac{J_k[i, j]}{\sigma[i, j]} \right)$$

Equations 6 and 7 result:

$$e_1[i, j] - e_3[i, j] = 2f_1[i, j] + 2f_2[i, j] - 4f_3[i, j] \quad (8)$$

Equations 1 and 2 can be modified as following:

$$e_1[i, j] - e_2[i, j] = 4f_1[i, j] + 4f_2[i, j] \quad (9)$$

According to equations 8 and 9 modified equations can be written as following:

$$e_2[i, j] = -e_1[i, j] + 2e_3[i, j] - 8f_3[i, j] \quad (10)$$

$$f_2[i, j] = \frac{1}{2}e_1[i, j] - \frac{1}{2}e_3[i, j] - f_1[i, j] + 2f_3[i, j] \quad (11)$$

Equation 12 explains Equation 10 and 11 in matrix format:

$$\begin{bmatrix} f_2[i, j] \\ e_2[i, j] \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} f_1[i, j] \\ e_1[i, j] \end{bmatrix} + \begin{bmatrix} 2 & -\frac{1}{2} \\ -8 & 2 \end{bmatrix} \begin{bmatrix} f_3[i, j] \\ e_3[i, j] \end{bmatrix} \quad (12)$$

Replacing auxiliary equation in Equation 12 results in final modified equation:

$$\begin{bmatrix} J_2[i, j] \\ e_2[i, j] \end{bmatrix} = \begin{bmatrix} -1 & \frac{4\sigma[i, j]}{\Delta} \\ 0 & -1 \end{bmatrix} \begin{bmatrix} J_1[i, j] \\ e_1[i, j] \end{bmatrix} + \begin{bmatrix} 2 & -\frac{4\sigma[i, j]}{\Delta} \\ -\frac{\Delta}{\sigma[i, j]} & 2 \end{bmatrix} \begin{bmatrix} J_3[i, j] \\ e_3[i, j] \end{bmatrix} \quad (13)$$

Equation 13 is the modified equation for block method. In this equation $J_2[i, j]$ and $e_2[i, j]$ are directly calculated from $J_1[i, j]$, $J_3[i, j]$, $e_1[i, j]$ and $e_3[i, j]$. In EIT forward

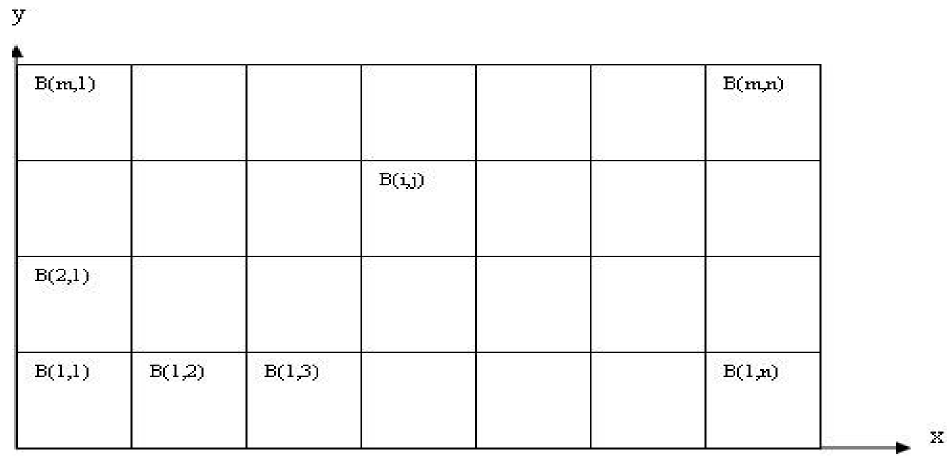

Figure 1. A schematic of a rectangular shape subject divided to $m \times n$ similar blocks.

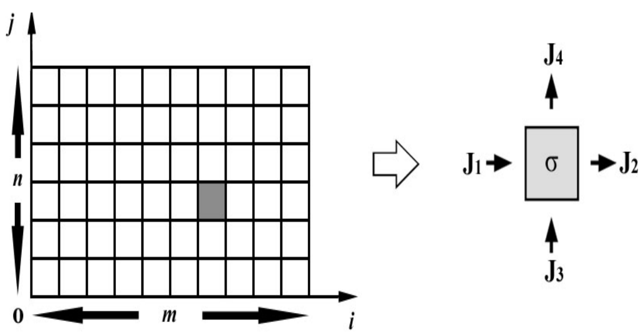

Figure 2. A general schematic of a blocked body in EIT analysis and a sample block.

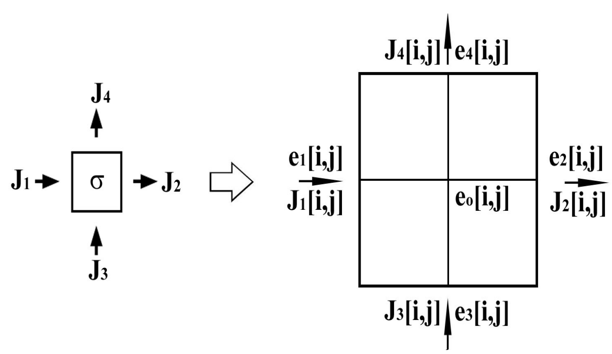

Figure 3. A general block in the body and its input-output current densities and potentials of nodes.

problem using block method, it is assumed that $J_1[i,1]$ and $e_1[i,1]$ for $\forall i \in N, 1 \le i \le m$ and, $J_3[1,j]$ and $e_3[1,j]$ for $\forall j \in N, 1 \le j \le n$ and, $\sigma[i,j]$ for $\forall i \in N, 1 \le i \le m$ and $\forall j \in N, 1 \le j \le n$ are known parameters and all others $J\ [i,j]$ and $e\ [i,j]$ are unknown parameters and must be calculated by EIT forward block method (Abbasi et al., 2009). For block B(1,1) $J_1[1,1]$, $J_3[1,1]$, $e_1[1,1]$ and $e_3[1,1]$ are known boundary parameters and $J_2[1,1]$ and $e_2[1,1]$ can be calculated from Equation 13. For B(1,2) $J_3[1,2]$ and $e_3[1,2]$ are known boundary parameters, also, $J_1[1,2] = J_2[1,1]$ and $e_1[1,2] = e_2[1,1]$ where $J_2[1,1]$ and $e_2[1,1]$ are valid from B(1,1). Therefore, $J_2[1,2]$ and $e_2[1,2]$ can be calculated from Equation 13 directly. Equation 13 calculates $J_2[i,j]$ and $e_2[i,j]$ for all blocks, one by one in first row. Using Equations 1 – 5, $J_3[2,j]$ and $e_3[2,j]$ can be calculated from first row ($J_3[2,j] = J_4[1,j]$ and $e_3[2,j] = e_4[1,j]$). Unknown parameters for the second row can be calculated by Equation 13 directly for all blocks, one by one. Equation 13 calculates $J_2[i,j]$ and $e_2[i,j]$ for all blocks, one by one in all rows. Since Equation 13 is a linear description for block forward method, it can remove nonlinear problems for inverse problem in EIT (Abbasi et al., 2009).

RESULTS AND DISCUSSION

A numerical example has been developed for illustrating the algorithm. In this example, a 2*2 (n = 2 and m = 2) block has been presented. Real amount of conductivities in blocks are 15, 15, 30, and 15 ($\sigma[1,1]$=15, $\sigma[1,2]$=15, $\sigma[2,1]$=30, $\sigma[2,2]$=15). For both algorithms, calculated conductivities are near to real amount of them (Figure 4). However, algorithm time for proposed method is 0.13785 s. and for previous block method is 4.0222 s. Although, both methods present acceptable conductivity values, proposed method is a faster algorithm and requires low memory.

Proposed algorithm and previous block method has been compared for different block sizes. For different size

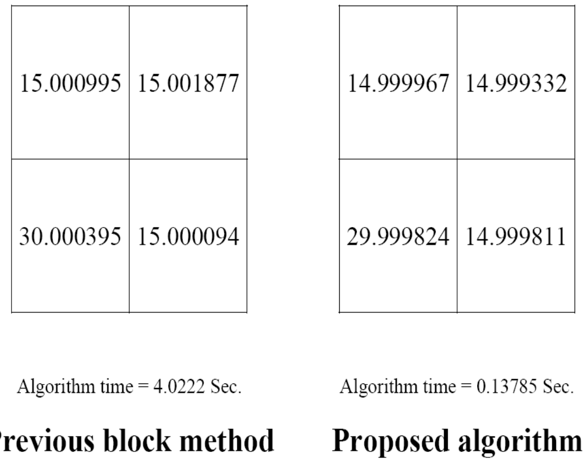

Figure 4. Conductivity distribution (15, 15, 30, 15) and algorithm time for 2*2 (n = 2 and m = 2) block.

of blocks for example, 3*3 (n = 3 and m = 3), 4*4 (n = 4 and m = 4), 5*5 (n = 5 and m = 5) proposed algorithm is at least 20 times faster than previous block method which is very considerable for image processing algorithm time. Image processing is always time consuming process and decreasing time is valuable and the proposed algorithm is a faster algorithm.

The results show that the proposed algorithm is able to reconstruct images with the same performance as previous method. The results also show that as the number of the block increases, the proposed algorithm tends to be more efficient and faster and useful, because image processing time in many numbers of blocks is very considerable. Proposed algorithm is a fast and robust approach that can be used for EIT ill-posed problems and it requires very low memory. The implication of these is that the proposed algorithm is faster and therefore, more efficient. The algorithm is also recommended for large number of blocks.

Conclusion

Block method is a new approach in EIT. Nonlinear equations in EIT block method require modifications to improve the performance of the algorithm. Modifications in EIT equations improve the performance of method. This paper has improved on EIT block method using the linear technique and decreasing algorithm time. Results for 3*3, 4*4, 5*5 and 5*6 blocks has shown that the proposed algorithm is efficient. The algorithm is recommended for all sizes of blocks but much more efficient as the number of blocks increases.

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