

# Hydromagnetic thermal boundary layer flow of a perfectly conducting fluid

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## Abstract

Using successive approximation technique, an analysis of unsteady hydromagnetic boundary layer flow with thermal relaxation of perfectly conducting, viscous incompressible fluid past a semi-infinite porous plate in presence of heat absorbing sinks is carried out. The expression for velocity and temperature fields, local skin friction coefficient, local heat transfer coefficient etc have been derived when the free stream velocity exponentially depends on time. The effects of different parameters entering into the problem are shown graphically and discussed numerically.

**Key Words:** Boundary layer, porous plate, sinks, hydromagnetic, perfectly conducting

## 1. Introduction

Unsteady boundary layer plays an important role in many engineering problems like start-up process and periodic fluid motion. Unsteady boundary layer has different behavior due to extra time dependent terms, which will influence the fluid motion pattern and boundary layer separation [1]. The typical examples of unsteady boundary layers in the history of fluid mechanics are the Rayleigh problem and Stokes oscillating plate [2, 3]. The magnetohydrodynamics (MHD) boundary layer flow of an electrically conducting fluid through porous medium has gained considerable importance in the field of astrophysics, geophysics, biophysics and engineering.

Important aspects of biophysics have derived from physiology, especially in studies involving the conduction of nerve impulses [4]. It is known that the extra cellular fluid has a high concentration of positively charged sodium ions ( $\text{Na}^+$ ) outside the neuron cell and a high concentration of negatively charged chloride ( $\text{Cl}^-$ ) as well as lower concentration of positively charged potassium ( $\text{K}^+$ ) inside, giving rise to a potential called resting potential, usually measured at about  $-75$  millivolts. The stimulation of the cell by any physical effect (heat, electric current, light etc) cause a nerve impulse consisting of sodium ions pumped into the cell, potassium ions pumped out, from which the cell membrane reaches a depolarization stage at which an electric signal is transmitted to other nerve cells. The action potential is conducted at speeds that range from 1 to 100 m/sec [4]. This extracellular fluid can be considered a perfect conducting fluid.

Historically Rossow [5] was the first to study the hydrodynamic behavior of the boundary layer on a semi-infinite plate in the presence of a uniform transverse magnetic field. Later the boundary layer flow for an electrically conducting fluid have been discussed by many authors [6–12]. The different solutions of boundary layer flow problems are found in the works of Zakaria [13–17]. Varshney and Kumar [18] studied magnetohydrodynamic boundary layer flow of non-Newtonian fluid past a flat plate. Recently Das and Jana [19] considered MHD boundary layer flow and heat transfer of viscoelastic fluid past a stretching plate through a porous medium.

The objective of the present paper is the study of unsteady hydromagnetic boundary layer flow with thermal relaxation of perfectly conducting fluid past a semi-infinite porous plate in the presence of heat absorbing sinks.

## 2. Mathematical formulation of the problem

The boundary layer equations for two dimensional unsteady flow of a viscous incompressible perfectly conducting fluid past a semi-infinite porous plate in presence of a transverse magnetic field and heat absorbing sinks is considered. The x-axis is taken along the body and y-axis normal to it. Also it is assumed that the velocity at large distance from the body will depend only on time  $t$  and  $x$ .

Let  $\vec{H}_0$  be the strength of constant magnetic field acts in the direction of y-axis. This produces an induced magnetic field  $\vec{h}$  and induced electric field  $\vec{E}$ , which satisfy the linearized equations of electromagnetic field, valid for slowly moving media of a perfectly conductor [17],

$$\nabla \times \vec{h} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \tag{1}$$

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}, \tag{2}$$

$$\vec{E} = -\mu_0 (\vec{V} \times \vec{H}_0), \tag{3}$$

$$\nabla \cdot \vec{h} = 0 \tag{4}$$

where  $\vec{J}$  is the electric current density,  $\mu_0$  and  $\varepsilon_0$  are the magnetic and electric permeabilities,  $\vec{V} = (u, v, 0)$  is the velocity vector of the fluid,  $\vec{H}_0 = (0, H, 0)$  is applied magnetic field and  $\vec{h} = (h_1, h_2, 0)$  is the induced magnetic field. The vector  $\vec{E}$  and  $\vec{J}$  will have non-vanishing components only in the z-direction, i.e.  $\vec{E} = (0, 0, E)$ ,  $\vec{J} = (0, 0, J)$ .

The governing equations under the boundary layer approximation and due to the inclusion of relaxation time are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha^2}{H_0} \left( \frac{\partial h_1}{\partial y} - \frac{\partial h_2}{\partial x} - \mu_0 \varepsilon_0 H_0 \frac{\partial u}{\partial t} \right) - \frac{\nu u}{k}, \tag{6}$$

$$\frac{\partial h_1}{\partial t} = H_0 \frac{\partial u}{\partial y}, \tag{7}$$

$$\frac{\partial h_2}{\partial t} = -H_0 \frac{\partial u}{\partial x}, \quad (8)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \tau_0 \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + S(T - T_\infty) \quad (9)$$

where  $\rho$  is the density of the fluid,  $\nu$  is the kinematics viscosity,  $C_p$  is the specific heat at constant pressure  $p$ ,  $\alpha$  is the Alfven velocity given by  $\alpha^2 = \frac{\mu_0 H_0^2}{\rho}$ ,  $k$  is the permeability of porous medium,  $T$  is the temperature of the fluid in the boundary layer and  $T_\infty$  is the temperature of the fluid far away from the plate,  $S$  is the sink strength and  $\lambda$  is the thermal conductivity and  $\tau_0$  is the relaxation time.

The boundary conditions are given by

$$\left. \begin{aligned} u = 0, v = 0, T - T_\infty = T_0 U(x, t) \quad \text{at } y = 0 \\ u \rightarrow U_\infty, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{aligned} \right\} \quad (10)$$

where  $T_0$  is the mean temperature of the surface of the body and  $U_\infty$  is the velocity of the fluid at large distance from the plate.

The pressure term in (6) can be expressed in terms of the free stream velocity  $U_\infty$  which is a function of  $x$  and  $t$  only. Thus equation (6) will become a generalized Bernoulli's equation as [15]

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial U_\infty}{\partial t} + U_\infty \frac{\partial U_\infty}{\partial x} + \frac{\alpha^2}{H_0} \frac{\partial h_{2\infty}}{\partial x} + \alpha^2 \mu_0 \varepsilon_0 \frac{\partial U_\infty}{\partial t} + \frac{\nu U_\infty}{k}, \quad (11)$$

where  $h_{2\infty}$  is the component of induced magnetic field at large distance from the body.

In virtue of (11), equation (6) becomes

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U_\infty}{\partial t} + U_\infty \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha^2}{H_0} \left\{ \frac{\partial h_1}{\partial y} - \frac{\partial h_2}{\partial x} + \frac{\partial h_{2\infty}}{\partial x} - \mu_0 \varepsilon_0 H_0 \left( \frac{\partial u}{\partial t} \right. \right. \\ \left. \left. - \frac{\partial U_\infty}{\partial t} \right) \right\} - \frac{\nu}{k} (u - U_\infty). \end{aligned} \quad (12)$$

Eliminating  $h_1$  and  $h_2$  from (12), using (7), (8) and applying the boundary layer approximation the following equation is obtained:

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} + u \frac{\partial^2 u}{\partial t \partial x} + \frac{\partial u}{\partial t} \frac{\partial u}{\partial x} + v \frac{\partial^2 u}{\partial t \partial y} + \frac{\partial v}{\partial t} \frac{\partial u}{\partial y} = \frac{\partial^2 U_\infty}{\partial t^2} + U_\infty \frac{\partial^2 U_\infty}{\partial t \partial x} + \frac{\partial U_\infty}{\partial t} \frac{\partial U_\infty}{\partial x} + \nu \frac{\partial^3 u}{\partial t \partial y^2} \\ + \alpha^2 \left\{ \frac{\partial^2 u}{\partial y^2} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} (u - U_\infty) \right\} - \frac{\nu}{k} \frac{\partial}{\partial t} (u - U_\infty). \end{aligned} \quad (13)$$

Introducing the non-dimensional quantities

$$\left. \begin{aligned} \bar{x} = \frac{U_0 x}{\nu}; \bar{y} = \frac{U_0 y}{\nu}; \bar{t} = \frac{U_0^2 t}{\nu}; \bar{u} = \frac{u}{U_0}; \bar{v} = \frac{v}{U_0}; \delta = \frac{d_1}{\lambda}; \bar{h}_1 = \frac{h_1}{H_0}; \bar{h}_2 = \frac{h_2}{H_0}; \\ \bar{T} = \frac{T - T_\infty}{T_0}; \bar{T}_0 = \frac{U_0^2 T_0}{\nu}; \bar{\alpha} = \frac{\alpha}{U_0}; \bar{U}_\infty = \frac{U_\infty}{U_0}; K = \frac{U_0^2 k}{\nu^2}; \bar{S} = \frac{S \nu}{U_0^2}; P_r = \frac{\rho C_p \nu}{\lambda}, \end{aligned} \right\} \quad (14)$$

and taking  $U_\infty = U_0 U(x, t)$ , equations (5), (9) and (13) can be written (after dropping the bar notation) as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (15)$$

$$a \frac{\partial^2 u}{\partial t^2} + u \frac{\partial^2 u}{\partial t \partial x} + \frac{\partial u}{\partial t} \frac{\partial u}{\partial x} + v \frac{\partial^2 u}{\partial t \partial y} + \frac{\partial v}{\partial t} \frac{\partial u}{\partial y} = a \frac{\partial^2 U}{\partial t^2} + U \frac{\partial^2 U}{\partial t \partial x} + \frac{\partial U}{\partial t} \frac{\partial U}{\partial x} + \frac{\partial^3 u}{\partial t \partial y^2} + \alpha^2 \frac{\partial^2 u}{\partial y^2} - \frac{1}{K} \frac{\partial}{\partial t} (u - U), \quad (16)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{P_r} \frac{\partial^2 T}{\partial y^2} - \tau_0 \frac{\partial}{\partial t} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + ST, \quad (17)$$

where  $P_r$  is the Prandtl number,  $K$  is the permeability of the porous medium,  $S$  is the strength of the sink,  $a = 1 + \frac{\alpha^2}{c^2}$  and  $c$  is the speed of light given by  $c^2 = \frac{1}{\varepsilon_0 \mu_0}$ .

Also the boundary conditions become

$$\left. \begin{aligned} u = 0, v = 0, T - T_\infty = U(x, t) \text{ at } y = 0, \\ u \rightarrow U(x, t), T \rightarrow 0 \text{ as } y \rightarrow \infty. \end{aligned} \right\} \quad (18)$$

### 3. Method of solution

The successive approximation method [17] is used to solve the unsteady boundary layer equations (15)–(17). A coordinate system which is at rest with respect to the plate and the magnetohydrodynamics flow of a perfectly conducting fluid moves with respect to the plane surface is considered so that the velocity components  $u$ ,  $v$  and temperature  $T$  possess a series solution of the form

$$u(x, y, t) = \sum_{i=0}^{\infty} u_i(x, y, t), v(x, y, t) = \sum_{i=0}^{\infty} v_i(x, y, t), T(x, y, t) = \sum_{i=0}^{\infty} T_i(x, y, t), \quad (19)$$

where  $u_i = 0(\varepsilon^i)$ ,  $i$  is an integer and  $\varepsilon$  is a small number.

Then the continuity equation (15) gives

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} = 0, \quad i = 0, 1, 2, \dots \quad (20)$$

Substituting the series solution (19) into (16) and (17), and equating to zero of the same order terms, the following equations are obtained:

$$a \frac{\partial^2 u_0}{\partial t^2} - \frac{\partial^3 u_0}{\partial t \partial y^2} - \alpha^2 \frac{\partial^2 u_0}{\partial y^2} = a \frac{\partial^2 U}{\partial t^2} - \frac{1}{K} \frac{\partial}{\partial t} (u_0 - U), \quad (21)$$

$$a \frac{\partial^2 u_1}{\partial t^2} - \frac{\partial^3 u_1}{\partial t \partial y^2} - \alpha^2 \frac{\partial^2 u_1}{\partial y^2} = U \frac{\partial^2 U}{\partial t \partial x} + \frac{\partial U}{\partial t} \frac{\partial U}{\partial x} - u_0 \frac{\partial^2 u_0}{\partial t \partial x} - v_0 \frac{\partial^2 u_0}{\partial t \partial y} - \frac{\partial u_0}{\partial t} \frac{\partial u_0}{\partial x} - \frac{\partial v_0}{\partial t} \frac{\partial u_0}{\partial y} - \frac{1}{K} \frac{\partial u_1}{\partial t}, \quad (22)$$

$$\frac{\partial T_0}{\partial t} + \tau_0 \frac{\partial^2 T_0}{\partial t^2} - \frac{1}{P_r} \frac{\partial^2 T_0}{\partial y^2} = ST_0, \quad (23)$$

$$\frac{\partial T_1}{\partial t} + \tau_0 \frac{\partial^2 T_1}{\partial t^2} - \frac{1}{P_r} \frac{\partial^2 T_1}{\partial y^2} = -u_0 \frac{\partial}{\partial x} (T_0 + \tau_0 \frac{\partial T_0}{\partial t}) - v_0 \frac{\partial}{\partial y} (T_0 + \tau_0 \frac{\partial T_0}{\partial t}) + ST_1 \quad (24)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} u_i = 0, v_i = 0, T_0 = U(x, t), T_i = 0, i = 0, 1, 2, \dots, \text{for } y = 0 \\ u_0 \rightarrow U(x, t), u_i \rightarrow 0, T_i \rightarrow 0, i = 0, 1, 2, \dots, \text{as } y \rightarrow \infty. \end{aligned} \right\} \quad (25)$$

Due to the complexity of higher order approximation in the present analysis, only first two terms of the series solutions (19) are taken. This type solutions [17] is satisfactory in the phases of the non-periodic motion after it has been started from rest ( till the moment when the first separation of boundary layer occurs) and in the case of periodic motion when the amplitude of oscillation is small.

#### 4. Solution of the problem

To solve the problem it is assumed that the free stream flow at large distance from the surface is of the form

$$U(x, t) = e^{\omega t} V(x). \quad (26)$$

It is assumed that where  $\omega > 0$ , the exact solution of the equation (20) and (22) is of the form

$$u_0(x, y, t) = e^{\omega t} V(x) f_1'(y), \quad (27)$$

$$T_0(x, y, t) = e^{\omega t} V(x) \phi_1(y). \quad (28)$$

Then equation (20) yields

$$v_0(x, y, t) = -e^{\omega t} \frac{dV}{dx} f_1(y). \quad (29)$$

Using equations (27) and (28), one obtain from equations (21) and (23), the differential equations for  $f_1(y)$ ,  $\phi_1(y)$  as

$$f_1''' - k_1^2 f_1' = -k_1^2, \quad (30)$$

$$\phi_1'' - k_2^2 \phi_1 = 0, \quad (31)$$

where  $k_1^2 = \frac{K\alpha\omega^2 + \omega}{K(\omega + \alpha^2)}$ ,  $k_2^2 = Pr(S + \omega + \tau_0\omega^2)$ .

The corresponding boundary conditions become

$$\left. \begin{aligned} f_1 = 0, f_1' = 0, \phi_1 = 1, \text{for } y = 0 \\ f_1' \rightarrow 1, \phi_1 \rightarrow 0, \text{as } y \rightarrow \infty. \end{aligned} \right\} \quad (32)$$

Solutions of (30)and (31) and use of the boundary conditions (32) give the following results:

$$f_1(y) = y + \frac{1}{k_1} (e^{-k_1 y} - 1), \quad (33)$$

$$\phi_1(y) = e^{-k_2 y} - 1. \quad (34)$$

Again assuming the solution of (22) is of the form

$$u_1(x, y, t) = e^{2\omega t} V \frac{dV}{dx} f_2'(y), \quad (35)$$

an exact solution of (24) is obtained if it is considered that  $T_1(x, y, t)$  is of the form [15]

$$T_1(x, y, t) = e^{2\omega t} V \frac{dV}{dx} \phi_2(y). \quad (36)$$

Then using (35) and (36), one obtains from equations (22) and (24), the differential equations for  $f_2(y)$  and  $\phi_2(y)$  can be written as

$$f_2''' - k_3^2 f_2' = \omega_1 (f_1'^2 - f_1 f_1'' - 1), \quad (37)$$

$$\phi_2'' - k_4^2 \phi_2 = \frac{k_2^2}{\omega} (\phi_1 f_2' - \phi_1' f_1), \quad (38)$$

where  $k_3^2 = (\frac{1}{K} + 2a\omega)\omega_1$ ,  $\omega_1 = \frac{2\omega}{2\omega + \alpha^2}$  and  $k_4^2 = P_r(4\tau_0\omega^2 + 2\omega - S)$ .

The corresponding boundary conditions become

$$\left. \begin{aligned} f_2 = 0, f_2' = 0, \phi_2 = 0, \text{ for } y = 0 \\ f_2' \rightarrow 0, \phi_2 \rightarrow 0, \text{ as } y \rightarrow \infty. \end{aligned} \right\} \quad (39)$$

With boundary conditions (39), solutions to (37) and (38) are

$$f_2(y) = A_1 + A_2 e^{-k_3 y} + A_4 (A_3 + y) e^{-k_1 y}, \quad (40)$$

$$\phi_2(y) = A_5 e^{-k_4 y} + A_6 e^{-(k_1 + k_2)y} + A_7 (A_8 - y) e^{-k_2 y}, \quad (41)$$

where

$$\begin{aligned} A_1 &= \frac{\omega_1}{k_1^2 - k_3^2}, & A_5 + A_6 + A_7 A_8 &= 0, \\ A_2 &= \frac{A_1}{k_3} (1 - k_1 A_3), & A_6 &= \frac{k_2^2 (k_2 - k_1)}{\omega k_1 \{(k_1 + k_2)^2 - k_4^2\}}, \\ A_3 &= \frac{2(2k_1^2 - k_3^2)}{k_1^2 - k_3^2}, & A_7 &= \frac{k_2^2}{\omega (k_2^2 - k_4^2)}, \\ A_4 &= \frac{A_1}{k_3} \{A_3 (k_1 - k_3) - 1\}, & A_8 &= A_7 \left\{ \frac{k_1 - k_2}{k_1 k_2} + \frac{2k_2^2}{k_2^2 - k_4^2} \right\}. \end{aligned} \quad (42)$$

The non-dimensional form of the equations (7) and (8) are

$$\frac{\partial h_1}{\partial t} = \frac{\partial u}{\partial y}, \quad (43)$$

$$\frac{\partial h_2}{\partial t} = -\frac{\partial u}{\partial x}. \quad (44)$$

Then from equations (27), (35), (43) and (44), the components of the induced magnetic field are given by

$$h_1(x, y, t) = \frac{V e^{\omega t}}{2\omega} (2f_1'' + \varepsilon e^{\omega t} \frac{dV}{dx} f_2''), \quad (45)$$

$$h_2(x, y, t) = -\frac{e^{\omega t}}{2\omega} \left\{ 2\frac{dV}{dx} f'_1 + \varepsilon e^{\omega t} f'_2 \frac{d}{dx} \left( V \frac{dV}{dx} \right) \right\} \quad (46)$$

After obtaining velocity and temperature field, some important flow characteristics of the problem can be obtained, viz. wall shear stress  $\tau$  and local heat flux  $q$ , as given below.

### Wall shear stress

The wall shear stress in non-dimensional form is given by the function

$$\tau = \rho U_0^2 \left( \frac{\partial u}{\partial y} \right)_{y=0}. \quad (47)$$

From equations (27), (35), one obtains

$$\tau = \rho U_0^2 V(x) e^{\omega t} \left\{ k_1 + \varepsilon e^{\omega t} \frac{dV}{dx} [(k_1^2 A_3 A_4 - 2k_1 A_4) + k_3^2 A_2] \right\}. \quad (48)$$

Thus the local skin friction coefficient  $C_f$  is given by

$$C_f = \frac{\tau}{\frac{1}{2}\rho U_0^2} = 2V(x) e^{\omega t} \left\{ k_1 + \varepsilon e^{\omega t} \frac{dV}{dx} [(k_1^2 A_3 A_4 - 2k_1 A_4) + k_3^2 A_2] \right\}. \quad (49)$$

### Local heat flux

The local heat flux in non-dimensional form is given by

$$q = -\frac{\kappa U_0}{\gamma} (T_0 - T_\infty) \left( \frac{\partial T}{\partial y} \right)_{y=0}. \quad (50)$$

Thus equations (28) and (36) yield

$$q = \frac{\kappa U_0}{\gamma} V(x) e^{\omega t} (T_0 - T_\infty) \left\{ k_2 + \varepsilon e^{\omega t} \frac{dV}{dx} [(k_4 A_5 + (k_1 + k_2) A_6 + A_7 (k_2 A_8 + 1))] \right\}. \quad (51)$$

So the local heat transfer coefficient is given by

$$h(x, t) = \frac{q(x, t)}{T_0 - T_\infty} = \frac{\kappa U_0}{\gamma} V(x) e^{\omega t} \left\{ k_2 + \varepsilon e^{\omega t} \frac{dV}{dx} [(k_4 A_5 + (k_1 + k_2) A_6 + A_7 (k_2 A_8 + 1))] \right\}. \quad (52)$$

## 5. Numerical results and discussion

In order to get the physical insight of the problem, it is assumed that the stream velocity is of the form

$$U_\infty(x, y, t) = U_0 e^{\omega t} x^n,$$

where  $c$  and  $n$  are fixed constants.

The velocity component  $u$ , temperature  $T$ , local skin friction coefficient  $C_f$  and local heat transfer coefficient  $h$  have been discussed numerically through graphs for different values of permeability parameter  $K$ , Alfven velocity  $\alpha$ , sink strength  $S$ , etc.

The velocity profiles are shown in Figures 1 and 2, in which, respectively, the effect of Alfven velocity and permeability parameter on the velocity component for  $n = 1$  are clearly shown. In these figures, the dotted lines denotes the flow when  $t = 0.5$  and the solid lines denote the flow for  $t = 1.0$ . It is observed that the velocity decreases with increasing both the values of  $\alpha$ ,  $K$  but an increase in the value of  $t$  leads to an increase in velocity.

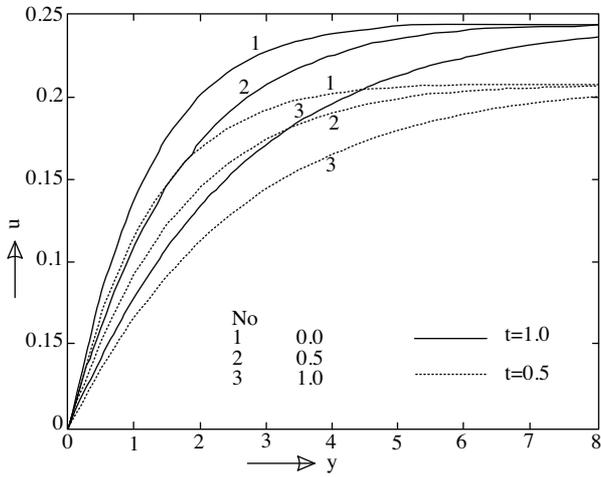


Figure 1. Effect of Alfven velocity  $\alpha$  on velocity distribution.

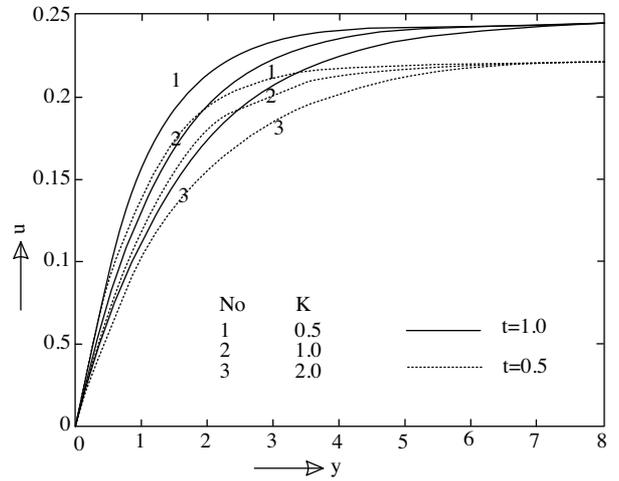


Figure 2. Effect of permeability parameter  $K$  on velocity distribution.

The temperature profiles are illustrated in Figures 3 and 4 for various values of sink strength and permeability parameter. Here, the dotted lines represents the solution of this flow when  $t = 1.0$  and the solid lines represents the flow when  $t = 0.5$ . It can be seen from these figures that temperature field increases

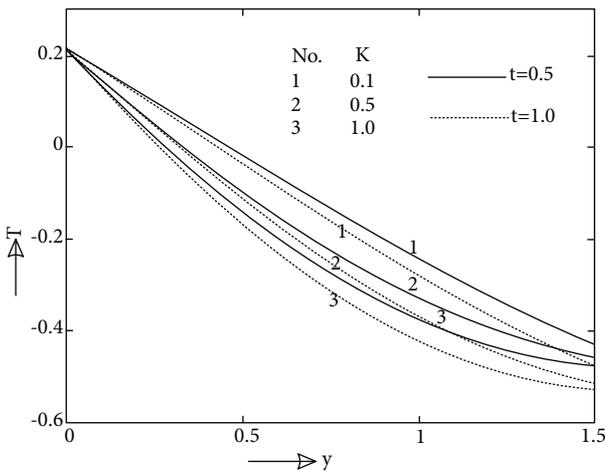


Figure 3. Effect of permeability parameter  $K$  on temperature distribution.

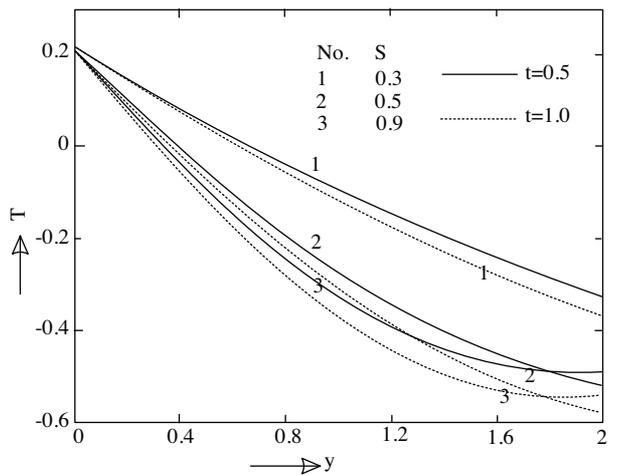
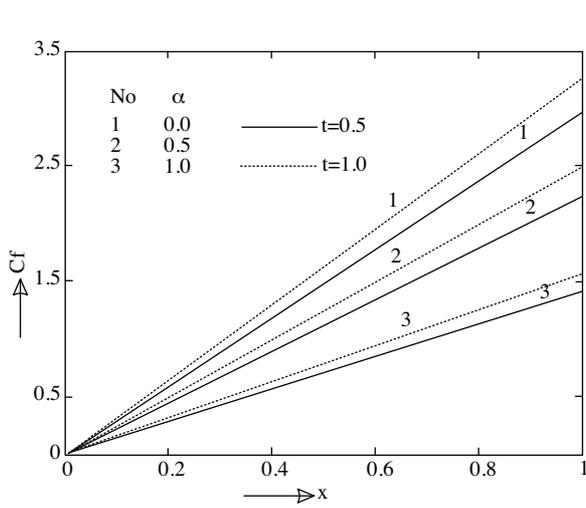


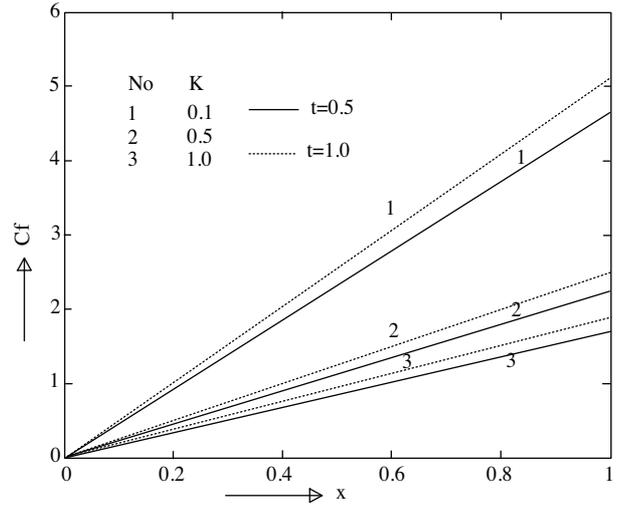
Figure 4. Effect of sink strength  $S$  on temperature distribution.

with decreasing both the values of  $t$  and  $K$ . But an increases in the value of  $S$  leads to decrease in the temperature field within the boundary layer region, while the temperature far away from the plate increases.

The skin friction coefficient is plotted against  $x$  in Figures 5 and 6. The effects of Alfvén velocity and permeability parameter on skin friction coefficient are shown in the figures and it is observed that skin friction coefficient decreases with increase in  $\alpha$  and  $K$ . Also skin friction coefficient increases with increasing  $t$ .

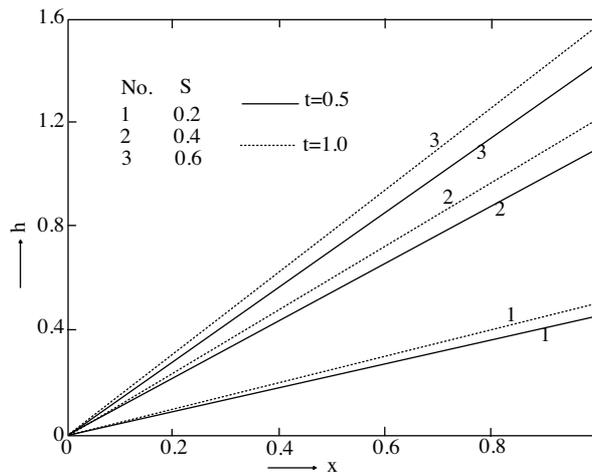


**Figure 5.** Effect of Alfvén velocity  $\alpha$  on skin friction coefficient.



**Figure 6.** Effect of permeability parameter  $K$  on skin friction coefficient.

The effect of sink strength on heat transfer coefficient are exhibited by the curves shown in Figure 7. An increase in the value of sink strength leads to increase in the heat transfer coefficient. Also the heat transfer coefficient is found to increase when  $t = 1.0$  as compared to  $t = 0.5$ .



**Figure 7.** Effect of sink strength  $S$  on heat transfer coefficient.

## 6. Conclusions

In the present investigation, the problem of unsteady magnetohydrodynamic thermal boundary layer flow of perfectly conducting fluid past a semi-infinite porous plate in presence of heat absorbing sinks has been formulated and solved using successive approximation technique. The results are analyzed numerically through graphs for finding the effect of different parameters, such as permeability parameter, Alfven velocity parameter, sink strength etc on velocity, temperature field and other characteristics. The specific conclusions derived from this study are summarized as follows.

- Increasing the Alfven velocity and permeability parameter decelerates the motion of the fluid but the effect is reverse for time  $t$ .
- The temperature distribution of the fluid decreases with increasing the permeability parameter and time. Near the boundary region the temperature decreases as sink strength increases but the effect is reverse far away from the plate.
- Increasing the Alfven velocity and permeability parameter lead to reduce the skin friction coefficient whereas the effect is opposite for time.
- The rate of heat transfer at the plate increases with increasing the sink strength and time.

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