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Full Length Research Paper

A comparison between the variational iteration method and Adomian decomposition method for the FitzHugh-Nagumo equations

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In this work, the variational iteration method and the decomposition method are used to determine the solutions of the FitzHugh-Nagumo (FN) equations. The two algorithms are illustrated by studying an initial value problem. The obtained results show that only few terms are required to deduce approximated solutions which are found to be accurate and efficient.

Key words: FitzHugh-Nagumo equations, Adomian decomposition, Lagrange multiplier, variational iteration method.

INTRODUCTION

The pioneering work of Hodgkin and Huxley (1952), and subsequent investigations, has established that good mathematical models for the conduction of nerve impulses along an axon can be given. These models take the form of a system of ordinary differential equations, coupled to a diffusion equation. Simpler models, which seem to describe the qualitative behavior, have been proposed by FitzHugh and Nagumo (FN) (Cohen, 1971; Hastings, 1975). This paper is devoted to the study of the FN system:

$$\begin{aligned} v_t(x,t) &= v_{xx}(x,t) - f(v(x,t)) - w(x,t), \\ w_t(x,t) &= b\,v(x,t) - \gamma\, w(x,t), \end{aligned} \tag{1a}$$

with given initial condition:

$$v(x,0) = \begin{cases} a e^{\alpha_1 x}, & x \le 0 \\ (a - \frac{1}{p_1'})e^{\alpha_1 x} - \frac{1}{p_2'}e^{\alpha_2 x} - \frac{1}{p_3'}e^{\alpha_3 x}, & 0 \le x \le z_1, \\ \frac{e^{-\alpha_2 z_1} - 1}{p_2'}e^{\alpha_2 x} + \frac{e^{-\alpha_3 z_1} - 1}{p_3'}e^{\alpha_3 x}, & z_1 \le x \end{cases} \tag{1b}$$

$$w(x,0) = (v(x,0))_{xx} - (v(x,0))_t - f(v(x,0)),$$

Where b and γ are positive constants and $f(v(x,t))$ is nonlinear function. Existence and uniqueness for this system is given in 1978 by Rauch and Smoller, in which they showed that small solutions $v(x,t)$ decay to 0 as $t \to \infty$ and large pulses produce a traveling wave. We consider the FN equations in the form:

$$\begin{aligned} v_t(x,t) &= v_{xx}(x,t) - f(v(x,t)) - w(x,t), \\ w_t(x,t) &= b\,v(x,t), \end{aligned} \tag{2}$$

and the function $f(v(x,t))$ is given by McKean (1970) such as:

$$f(v(x,t)) = v(x,t) - H(v(x,t) - a), 0 \le a \le \frac{1}{2}, \tag{3}$$

Where H is the Heaviside step function:

$$H(s) = \begin{cases} 0 & s < 0 \\ 1 & s \ge 0 \end{cases} \tag{4}$$

The exact solution of this system is given by:

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$$v(x,t)=\begin{cases} ae^{\alpha_1 z}, & z\le 0 \\ (a-\frac{1}{p_1'})e^{\alpha_1 z}-\frac{1}{p_2'}e^{\alpha_2 z}-\frac{1}{p_3'}e^{\alpha_3 z}, & 0\le z\le z_1, \\ \frac{e^{-\alpha_2 z_1}-1}{p_2'}e^{\alpha_2 z}+\frac{e^{-\alpha_3 z_1}-1}{p_3'}e^{\alpha_3 z}, & z_1\le z \end{cases} \quad (5)$$

$$w(x,t)=v_{xx}(x,t)-v_t(x,t)-f(v(x,t)), \quad (6)$$

Where $z=x+ct$, c is the speed of the traveling wave and $\alpha_i, i=1,2,3$ are the zeros of the polynomial:

$$p(\alpha)=\alpha^3-c\,\alpha^2-\alpha-\frac{b}{c}, \quad (7)$$

and:

$$p_i{}'=p'(\alpha_i), \quad i=1,2,3 \quad (8)$$

A numerical scheme for FN equations (Elibeck et al., 1981; Rinzel and Keller, 1973; Khalifa, 1979) is the "Hopscotch" finite difference scheme first proposed by Gordon (1975), and further developed by Gourlay and McGuire (1970a, b, 1971). Other possible schemes which were considered are (1) finite difference schemes (Rinzel, 1977), (2) Galerkin-type schemes (Cannon and Ewing, 1977) and (3) collocation schemes with quadratic and cubic splines (Khalifa, 1979). In this paper, we use the variational iteration and Adomian decomposition methods to find the numerical solutions of the FN equations which will be useful in numerical studies. In our numerical study, we consider the case of $b=0.1$ and $a=0.3$, also:

$$\begin{aligned} c &= 0.7122, \\ \alpha_1 &= 1.46192629534582, \\ \alpha_2 &= -0.1639653991443764, \\ \alpha_3 &= -0.5857608638090818, \\ z_1 &= 4.5976770121482735, \end{aligned} \quad (9)$$

With these parameters, we now can use the exact travelling wave solution (Equation 5) to test the suggested numerical methods.

THE FORMALISM

We introduce the main points of each of the two methods, where details can be found in these studies (He and Wu, 2006a, b; He, 2004, 2006a, b, 1998, 1999, 2000; Odibat and Momani, 2006; Bildik and Konuralp, 2006; Yusufoglu, 2007; Sweilam and Khader, 2007; Tari et al., 2007; Abdou and Soliman, 2005a, b; Soliman and Abdou, 2007; Soliman, 2005; Wazwaz, 2006a, b; Wazwaz and Gorguis, 2004; Adomian, 1994).

Variational iteration method (VIM)

VIM is the general Lagrange method**,** in which an extremely accurate approximation at some special point can be obtained, but not an analytical solution. To illustrate the basic idea of the VIM we consider the following general partial differential equation:

$$L_t u(x,t)+L_x u(x,t)+N\,u(x,t)+g(t,x)=0, \quad (10)$$

Where L_t and L_x are linear operators of t and x, respectively, and N is a nonlinear operator. According to the VIM, we can express the following correction functional in $t-$and $x-$directions, respectively, as follows:

$$u_{n+1}(x,t)=u_n(x,t)+\int_{t_0}^{t}\lambda(L_s u_n(x,s)+(L_x+N)\tilde{u}_n(x,s)+g(x,s))ds, \quad (11a)$$

$$u_{n+1}(x,t)=u_n(x,t)+\int_{x_0}^{x}\mu(L_s u_n(t,s)+(L_t+N)\tilde{u}_n(t,x)+g(t,s))ds, \quad (11b)$$

Where λ and μ are general Lagrange multipliers, which can be identified optimally via the variational theory, and $\tilde{u}_n(x,t)$ are restricted variations which mean that $\delta\tilde{u}_n(x,t)=0$. By this method, it is first required to determine Lagrange multipliers λ and μ that will be identified optimally. The successive approximations $u_{n+1}(x,t)$, $n\ge 0$ of the solution $u(x,t)$ will be readily obtained upon using the determined Lagrange multipliers and any selective function $u_0(x,t)$. Consequently, the solution is given by:

$$u(x,t)=\lim_{n\to\infty}u_n(x,t) \quad (12)$$

The aforementioned analysis yields the following theorem.

Theorem 1

The VIM solution of the partial differential Equation 10 can be determined by Equation 12 with the iterations (Equation 11a or 11b).

Adomian decomposition method (ADM)

Applying the inverse operator $L^{-1}(.)=\int_0^t (.)dt$ to both sides of Equation 10 and using the initial condition we get:

$$u_0(x,t)=u(x,0),$$
$$u_{n+1}(x,t)=\int_0^t (-L_x u_n(x,t)-A_n-g(x,t))dt,\ n\geq 0, \quad (13)$$

Where the nonlinear operator $N(u)=\sum_{n=0}^{\infty}A_n$ is the Adomian polynomial determined by:

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}(\sum_{i=0}^{\infty}\lambda^i u_i(x,t))\Big|_{\lambda=0}, \quad n=0,1,2..., \quad (14)$$

We next decompose the unknown function $u(x,t)$ by a sum of components defined by the following decomposition series:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \quad (15)$$

The aforementioned analysis yields the following theorem:

Theorem 2

The ADM solution of the partial differential Equation 10 can be determined by the series of Equation 15 with the iterations (Equation 13).

APPLICATIONS

We solve the FN equations using two methods, namely, VIM and ADM.

The VIM for the FN equations

Consider the FN equations in the form:

$$\begin{aligned} &v_t - v_{xx} + f(v) + w = 0,\\ &w_t - bv = 0 \end{aligned} \quad (16)$$

Then, the VIM formulae take the forms:

$$\begin{aligned} &v_{n+1}(x,t)=v_n(x,t)+\int_0^t \lambda(v_s(x,s)-\tilde{v}_{xx}(x,s)+f(\tilde{v}(x,s))+\tilde{w}(x,s))ds,\\ &w_{n+1}(x,t)=w_n(x,t)+\int_0^t \mu(w_s(x,s)-b\tilde{v}(x,s))ds, \end{aligned} \quad (17)$$

Where $v_0(x,t)=v(x,0)$, $w_0(x,t)=w(x,0)$ and $n\geq 0$. This yields the stationary conditions:

$$\lambda'(s)=0,\ \lambda+1\big|_{s=t}=0, \mu'(s)=0,\ \mu+1\big|_{s=t}=0, \quad (18)$$

Hence, the Lagrange multipliers are

$$\lambda(s)=\mu(s)=-1, \quad (19)$$

Substituting these values of Lagrange multipliers into the functional correction (Equation 17) gives the iterations formulae:

$$\begin{aligned} &v_{n+1}(x,t)=v_n(x,t)+\int_0^t \lambda(v_s(x,s)-v_{xx}(x,s)+f(v(x,s))+w(x,s))ds,\\ &w_{n+1}(x,t)=w_n(x,t)+\int_0^t \mu(w_s(x,s)-bv(x,s))ds, \end{aligned} \quad (20)$$

We start with initial approximations as:

$$v_0(x,t)=\begin{cases} ae^{\alpha_1 x}, & x\leq 0\\ (a-\frac{1}{p_1'})e^{\alpha_1 x}-\frac{1}{p_2'}e^{\alpha_2 x}-\frac{1}{p_3'}e^{\alpha_3 x}, & 0\leq x\leq z_1,\\ \frac{e^{-\alpha_2 z_1}-1}{p_2'}e^{\alpha_2 x}+\frac{e^{-\alpha_3 z_1}-1}{p_3'}e^{\alpha_3 x}, & z_1\leq x \end{cases} \quad (21)$$
$$w_0(x,t)=(v_0(x,t))_{xx}-(v_0(x,t))_t-f(v_0(x,t)),$$

and then, the first iterations are:

$$v_1(x,t)=\begin{cases} v11, & z\leq 0\\ v12, & 0\leq z\leq z_1\\ v13 & z_1\leq z \end{cases} \quad (22a)$$

$$v11 = 0.312355e^{1.46193x}(0.960445+t),$$
$$v12 = e^{-0.749726x}(e^{0.585761x}(1.45816-0.170279t)+e^{2.21165x}(-0.000361868-0.000376771t)$$
$$+e^{0.163965x}(-1.1578+0.483011t)),$$
$$v13 = e^{-0.749726x}(e^{0.163965x}(15.9522-6.65492t)+e^{0.585761x}(-1.64071+0.191596t)),$$

$$w_1(x,t)=\begin{cases} w11, & z\le 0 \\ w12, & 0\le z\le z_1 \\ w13 & z_1\le z \end{cases} \quad \text{(22b)}$$

$$w11 = 0.03e^{1.46193x}(0.960445+t),$$
$$w12 = e^{-0.749726x}(e^{0.749726x}+e^{0.163965x}(0.277531-0.11578t)+e^{2.21165x}(-0.000034755$$
$$-0.0000361868t)+e^{0.585761x}(-1.24868+0.145816t),$$
$$w13 = e^{-0.749726x}(e^{0.585761x}(1.405-0.164071t)+e^{0.163965x}(-3.82383+1.59522t)),$$

etc.

The VIM produces the solutions $v(x,t), w(x,t)$ as:

$$v(x,t)=\lim_{n\to\infty} v_n(x,t),\quad w(x,t)=\lim_{n\to\infty} w_n(x,t), \qquad (23)$$

Where $v_n(x,t)$, $w_n(x,t)$, will be determined in a recursive manner.

The ADM for the FN equations

Consider the FN equations in the form:

$$\begin{aligned} Lv &= v_{xx}-f(v)-w, \\ Lw &= bv, \end{aligned} \qquad (24)$$

Where $L(.)=\dfrac{\partial(.)}{\partial t}$. Operating by $L^{-1}(.)=\int_0^t (.)\,dt$ on both sides of Equations 24, we get:

$$\begin{aligned} v(x,t) &= v(x,0)+\int_0^t (v_{xx}(x,t)-f(v(x,t))-w(x,t))dt, \\ w(x,t) &= w(x,0)+\int_0^t (bv(x,t)dt \end{aligned} \qquad (25)$$

The ADM assumes that the unknown functions $v(x,t)$ and $w(x,t)$ can be expressed by an infinite series in the forms:

$$\begin{aligned} v(x,t) &= \sum_{n=0}^{\infty} v_n(x,t), \\ w(x,t) &= \sum_{n=0}^{\infty} w_n(x,t), \end{aligned} \qquad (26)$$

Where $v_n(x,t)$, and $w_n(x,t)$ can be determined by using the recurrence relations:

$$\begin{aligned} v_{n+1}(x,t) &= \int_0^t (v_{n\,xx}(x,t)-f(v_n(x,t))-w_n(x,t))dt, \\ w_{n+1}(x,t) &= \int_0^t (bv_n(x,t)dt, \qquad n=0,1,\ldots \end{aligned} \qquad (27)$$

Where:

$$f(v_n(x,t))=\begin{cases} v_n(x,t), & v_n(x,t)<a \\ v_n(x,t)-1, & v_n(x,t)\ge a \end{cases} \qquad (28)$$

Such that:

$$\begin{aligned} v_0(x,t) &= v(x,0), \\ w_0(x,t) &= w(x,0) \end{aligned} \qquad (29)$$

Then, the first iterations are:

Table 1. Comparison between the exact and approximate (VIM) solutions for the FN equations at time T= 5.

x	v_{VIM}	v_{exact}	w_{VIM}	w_{exact}
-7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
-3.561	0.3	0.3	0.0288134	0.0288134
-0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	-0.25639	-0.25639	0.382524	0.382524
8.439	-0.215232	-0.215232	0.193024	0.193024
16.439	-0.0616494	-0.0616494	16.439	16.439
22.439	-0.023095	-0.023095	0.0197795	0.0197795
48.439	-0.000325199	-0.000325199	0.000278481	0.000278481

$$v_1(x,t)=\begin{cases} 0.312355e^{1.46193}, & z\le 0 \\ t+(-1+0.483011e^{-0.585761x}-0.170279e^{-0.163965x} & \\ -0.000376771e^{1.46193x})t, & 0\le z\le z_1 \\ (-6.65492e^{-0.585761x}+0.191596e^{-0.163965x})t, & z_1\le z \end{cases}, \quad (30a)$$

$$w_1(x,t)=\begin{cases} 0.03e^{1.46193x}, & z\le 0 \\ 0.1(-1.1578e^{-0.585761x}+1.45816e^{-0.163965x} & \\ -0.000361868e^{1.46193x})t, & 0\le z\le z_1 \\ 0.1(15.9522e^{-0.585761x}-1.64071e^{-0.163965x})t, & z_1\le z \end{cases}, \quad (30b)$$

etc.

The ADM yields the solutions $v(x,t), w(x,t)$ as:

$$v(x,t)=\sum_{n=0}^{\infty} v_n(x,t),\ \ w(x,t)=\sum_{n=0}^{\infty} w_n(x,t), \qquad (31)$$

Where $v_n(x,t),\ w_n(x,t),$ will be determined in a recursive manner.

TEST PROBLEM FOR THE FN EQUATIONS

We discuss the solutions of the FN equations using the two considered VIM and ADM methods.

The VIM

Solve the FN Equations 2 using the VIM with finite iterations at time $T=5$. A comparison between the computed solutions and the exact solutions at different values of x are given in Table 1. We note that the VIM solutions converge to the exact solutions, especially when n is increased. Figure 1 shows the behavior of the VIM solutions of FN equations at time $T=5$. If the exact solutions are plotted on Figure 1, we will find out that the VIM and exact solutions curves are indistinguishable.

The ADM

Consider the same problems and use the ADM with the same initial conditions and use the technique discussed the formalism. A comparison between the exact solutions and ADM solutions are as shown in Table 2 and it seems that the errors are very small. Figure 2 shows the numerical solutions of the FN equations. The results listed in Table 3 are representing the maximum errors at different times of VIM and ADM which shows that the ADM is better than VIM in the solutions of FN equations. Now we show a comparison between our schemes and other methods as shown in Table 4.

It is clear that the suggested methods for solving FN equation are the best methods than all other methods. Also all other methods give the solution as a discrete solution, but our methods give the solution as a function

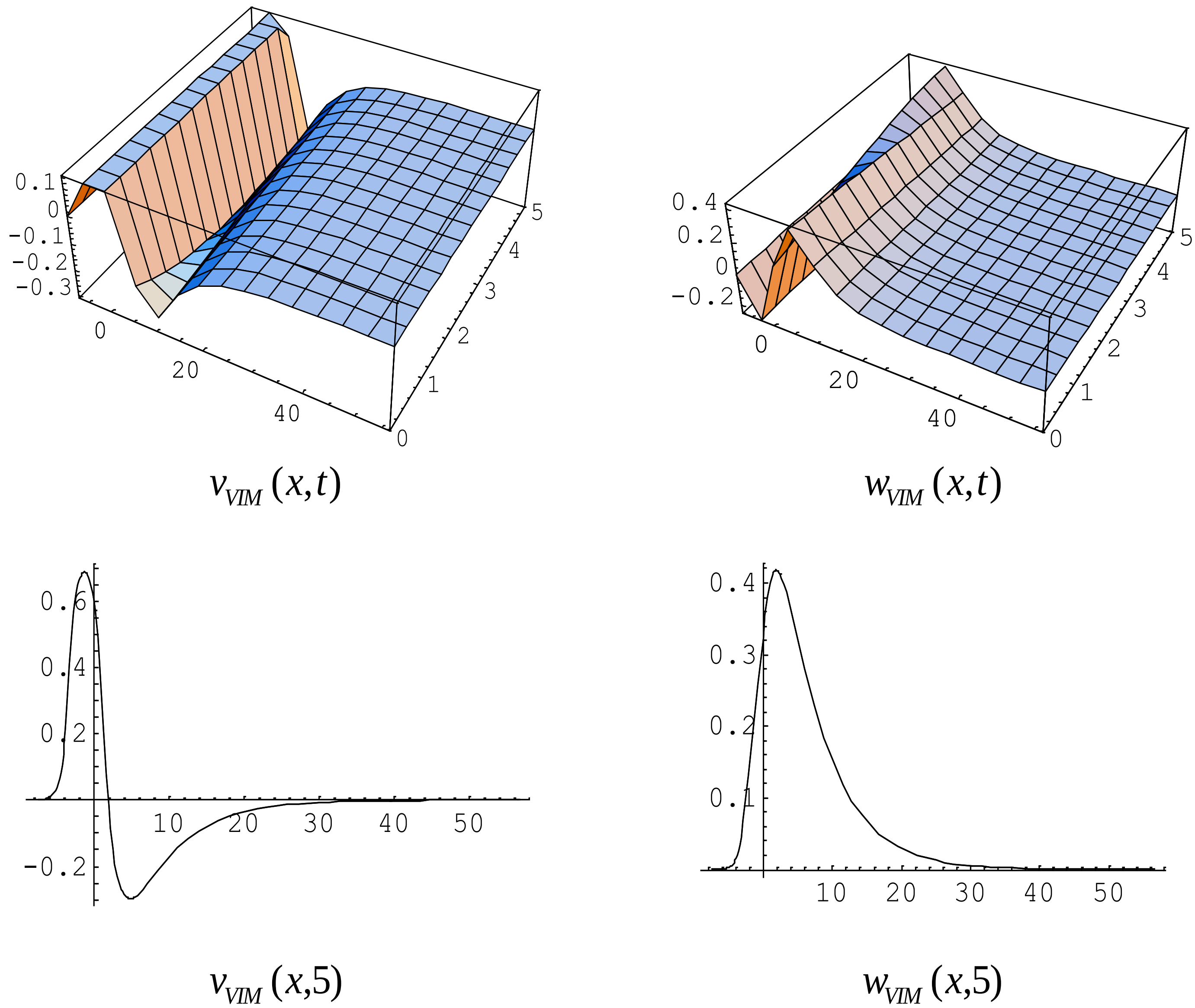

Figure 1. The approximated solutions for $v(x,t), w(x,t)$ at time T= 5.

Table 2. Comparison between the exact solutions and approximation solutions (ADM) for F-N equations at time T= 5.

x	v_{ADM}	v_{exact}	w_{ADM}	w_{exact}
-7.561	0.000865955	0.000865955	0.0000831702	0.0000831702
-3.561	0.3	0.3	0.0288134	0.0288134
-0.561	0.662823	0.662823	0.28156	0.28156
1.439	0.130074	0.130074	0.414489	0.414489
3.439	-0.25639	-0.25639	0.382524	0.382524
8.439	-0.215232	-0.215232	0.193024	0.193024
16.439	-0.0616494	-0.0616494	16.439	16.439
22.439	-0.023095	-0.023095	0.0197795	0.0197795
48.439	-0.000325199	-0.000325199	0.000278481	0.000278481

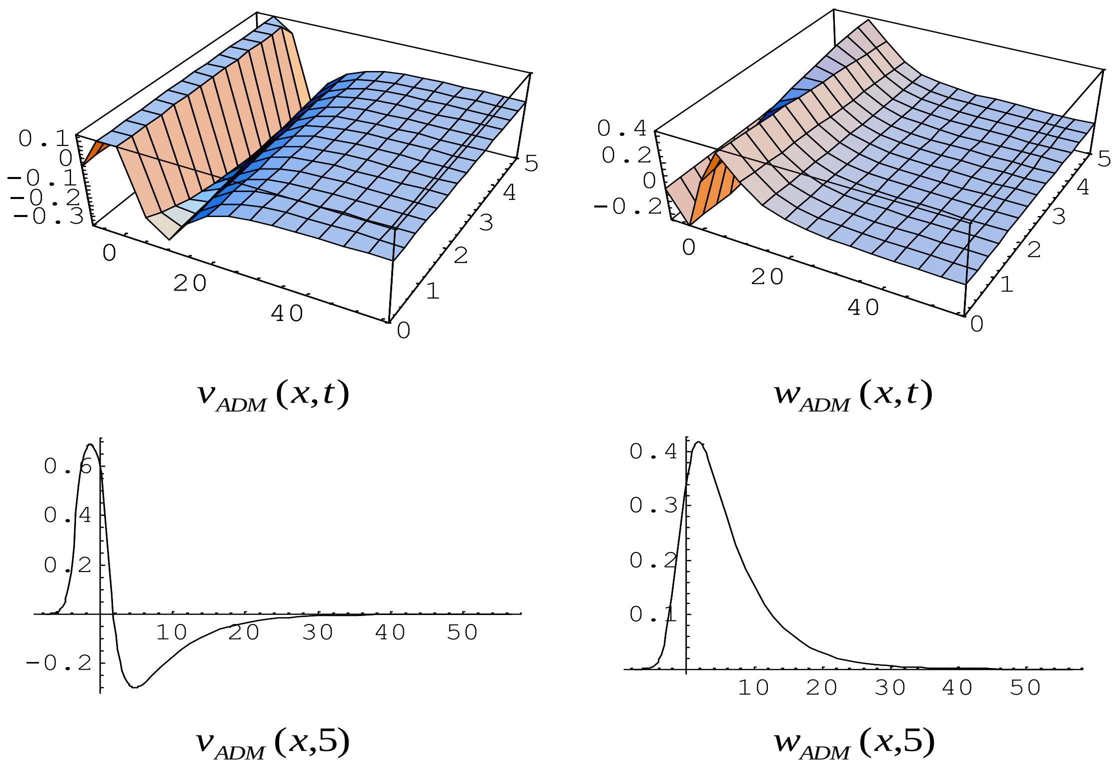

Figure 2. The approximation solutions $v(x,t), w(x,t)$.

Table 3. The maximum errors of our suggested methods VIM and ADM.

Time	**VIM**		**ADM**	
	Max. errors for $v(x,t)$	**Max. errors for $w(x,t)$**	**Max. errors for $v(x,t)$**	**Max. errors for $w(x,t)$**
2.0	3.66374E-15	4.02456E-16	3.71925E-15	4.71845E-16
4.0	1.14429E-9	1.09902E-10	1.15225E-9	1.10668E-10
6.0	3.37523E-7	3.24191E-8	3.37523E-7	3.24191E-8

Table 4. Comparison between VIM, ADM and other methods.

Method	**T=1.60**	**T=10.0**
Finite difference		
C-N	0.848E-2	0.189
Hopscotch [9]	0.557E-2	0.0506
Collocation method		
Quadratic [6]	0.758E-2	0.138
Cubic [6]	0.589E-2	0.12
VIM	3.33067E-16	0.000316341
ADM	4.44089E-16	0.000316341

of x and t.

Conclusion

Solutions for the FN equations using VIM and ADM methods have been generated. All numerical results obtained using few terms of the VIM and ADM show very good agreement with the exact solutions. Comparing our results with those of previous several methods shows that the considered techniques are more reliable, powerful and promising. We believe that the accuracy of the VIM and ADM, recommend it to be a much wider applicability.

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