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Full Length Research Paper

Wavelet method for a class of space and time fractional telegraph equations

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In this paper, an operational matrix of integration based on Haar wavelets (HW) is introduced, and a procedure for applying the matrix to solve space and time fractional telegraph equations is formulated. The space and time fractional derivatives are considered in the Caputo sense. The accuracy and effectiveness of the proposed method is demonstrated by the five test problems. Approximate solutions of the space and time fractional telegraph equations are compared with the other numerical solutions and the exact solutions. The proposed scheme can be used in a wide class of linear and nonlinear reaction-diffusion equations. These calculations demonstrate that the accuracy of the Haar wavelet is quite high even in the case of a small number of grid points. The present method is a very reliable, simple, small computation costs, flexible and convenient alternative method. The power of the manageable method is confirmed.

Key words: Haar wavelet, fractional differential equation, decomposition method, telegraph equation, operational matrix of integration.

INTRODUCTION

Wavelets have been applied extensively in scientific and engineering fields. In this paper, we use Haar wavelets to solve the space and time fractional telegraph equations, which appeared in many engineering, such as the continuous-time random walks, modeling of anomalous diffusive and sub-diffusive systems, unification of diffusion and wave propagation phenomenon and simplification of the results (Agrawal, 2002). The nature of the diffusion is characterized by the temporal scaling of the mean square displacement $\langle r^2(t)\rangle \sim t^{\alpha}$. For standard diffusion, $\alpha = 1$, whereas in anomalous sub-diffusion, $\alpha < 1$ and in anomalous super-diffusion, $\alpha > 1$. Both types of anomalous diffusion have been unified in continuous time random walks models with spatial and temporal memories (Henry and Wearne, 2000).

Fractional differential equations are generalized from classical integer-order ones, which are obtained by replacing integer-order derivatives by fractional ones. Their advantages compared with integer-order differential equations are the capability of simulating natural physical process and dynamic system more accurately. Therefore, fractional diffusion equations are largely used in describing abnormal slowly-diffusion phenomenon, and fractional diffusion equations are always used in describing abnormal convection phenomenon of liquid in medium. Therefore, space and time fractional telegraph equations are increasingly studied, but it is difficult to do theoretic analyzing and numerical solving for them.

Nigmatullin (1986) pointed out that many of the universal electromagnetic, acoustic and mechanical responses can be modeled accurately using the fractional diffusive-wave equations. Anh and Leonenko (2000) analyzed fractional diffusion equations with random initial conditions. Angulo et al. (2000) established diffusion equations with space fractional derivatives. The space and time fractional telegraph equations have recently been considered by Orsingher and Zhao (2003) and Orsingher and Beghlin (2004), respectively.

Zhuang and Liu (2005) showed an explicit difference

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approximations for space and time fractional telegraph diffusion equation. There is a lot of work that discussed boundary problems of fractional differential equations. Zhou et al. (2009a, b) proved the existence and uniqueness of solutions for boundary problems of fractional differential equations. The telegraph equation is used in signal analysis for transmission and propagation of electrical signals and also used modeling reaction diffusion. Momani (2005) established Adomian decomposition method (ADM) for solving the space and time fractional telegraph equations and Yıldırım (2010) used the homotopy perturbation method (HPM) for the same problem. Ali (2010) established the He's variational iteration method for solving the fractional telegraph equations. Odibata and Momani (Odibata and Momani, 2008; Momani and Odibat, 2007) applied the generalized differential transform method for linear partial differential equations of fractional order.

In this paper, making use of good properties of Haar wavelet and the operational matrix, we consider the following space and time fractional telegraph equations,

$$\frac{\partial^\alpha U}{\partial x^\alpha}=\frac{\partial^2 U}{\partial t^2}+\frac{\partial U}{\partial t}+g(t), \quad t\ge 0,\ 0<\alpha\le 2, \tag{1}$$

subject to the initial and boundary conditions,

$$\begin{aligned} &U(0,t)=f_1(t), \quad t\ge 0,\\ &\frac{\partial U(0,t)}{\partial x}=f_2(t),\ t\ge 0,\\ &U(x,0)=S(x), \quad 0<x<1 \end{aligned} \tag{2}$$

where α is a parameter describing the order of the fractional space-derivative and $U(x,t)$ is assumed to be a causal function of space, that is, vanishing for $x<0$. The fractional derivative is considered in the Caputo sense. The general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses. In the case of $\alpha=1$, the fractional equation reduces to the classical telegraph equation. Also, we shall examine the time-fractional telegraph equation:

$$\frac{\partial^{2\alpha} U}{\partial x^{2\alpha}}+\lambda\frac{\partial^\alpha U}{\partial t^\alpha}=v\frac{\partial^2 U}{\partial x^2}, \quad t\ge 0,\ 0<\alpha\le 1, \tag{3}$$

where λ and v are arbitrary constants and $U(x,t)$ is assumed to be a causal function of time, that is, vanishing for $t<0$. In the case of $\alpha=1$, the time-fractional equation reduces to the classical telegraph equation. We establish a clear procedure for solving the fractional telegraph equations via Haar wavelet. In solving ordinary differential equations (ODEs), Chen and Hsiao (1997) derived an operational matrix of integration based on the Haar wavelet method. Lepik (2007a, b, 2005, 2010) solved higher order as well as nonlinear ODEs and some nonlinear evolution equations using Haar wavelet method. Hariharan and Kannan (2009, 2010a, b, c) introduced the Haar wavelet method for solving some nonlinear ODEs and reaction-diffusion problems. Chen et al. (2010) showed the Haar wavelet method for solving some fractional convection-diffusion equations. In the present paper, a new direct computational method for solving space and time fractional telegraph equations is introduced. This method consists of reducing the problem to a set of algebraic equations by first expanding the term, which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of the Haar functions.

Identification and optimization procedures of the solutions are greatly reduced or simplified. Since the integration of the Haar functions vector is a continuous function, the solutions obtained are continuous function. This method gives us the implicit form of the approximate solutions of the problems. In this method, a few sparse matrices can be obtained, and there are no complex integrals or methodology. Therefore, the present method is useful for obtaining the implicit form of the approximations of linear or nonlinear differential equations, and round off errors and necessity of large computer memory are significantly minimized. Therefore, this paper suggests the use of this technique for solving the space and time fractional telegraph equation problems. Illustrative examples are given to demonstrate the application of the proposed method.

DEFINITIONS OF FRACTIONAL DERIVATIVES AND INTEGRALS

Here, we present some notations, definitions and preliminary facts that will be used further in this work. Fractional calculus is 300 years old topic. The first serious attempt to give logical definition is due to Liouville. Since then, several definitions of fractional integrals and derivatives have been proposed. These definitions include the Riemann-Liouville, Caputo, Weyl, Hadamard, Marchaud, Riesz, Grunwald-Letnikov and Erdelyi-Kober. The Caputo fractional derivative allows the utilization of initial and boundary conditions involving integer order derivatives, which have clear physically interpretations. Therefore, in this paper we shall use the Caputo derivative D^α proposed by Caputo in his work on the theory of viscoelasticity.

In the development of theories of fractional derivatives and integrals, it appears that many definitions, such as Riemann-Liouvlle and Caputo fractional differential-integral definition as follows.

1. Riemann-Liouville definition:

$$ {}_{a}^{R}D_{t}^{\alpha}f(t)=\begin{cases}\dfrac{d^{m}f(t)}{dt^{m}}, & \alpha=m\in N;\\ \dfrac{d^{m}}{dt^{m}}\dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_{a}^{t}\dfrac{f(T)}{(t-T)^{\alpha-m+1}}dT, & 0\le m-1<\alpha<m.\end{cases} \quad (4)$$

Fractional integral of order α is as follows:

$$ {}_{a}^{R}I_{t}^{\alpha}f(t)=\frac{1}{\Gamma(-\alpha)}\int_{0}^{t}(t-T)^{-\alpha-1}f(T)dT, \quad \alpha<0. \quad (5)$$

2. Caputo definition:

$$ {}_{a}^{c}D_{t}^{\alpha}f(t)=\begin{cases}\dfrac{d^{m}f(t)}{dt^{m}}, & \alpha=m\in N;\\ \dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_{a}^{t}\dfrac{f^{(m)}(T)}{(t-T)^{\alpha-m+1}}dT, & 0\le m-1<\alpha<m.\end{cases} \quad (6)$$

BASIC TOOLS

Proposed method

Haar wavelet was a system of square wave; the first curve was marked up as $h_0(t)$, the second curve marked up as $h_1(t)$ that is:

$$h_0(t)=\begin{cases}1, & 0\le x<1\\ 0, & otherwise\end{cases}$$

$$h_1(t)=\begin{cases}1, & 0\le x<1/2,\\ -1, & 1/2\le x<1,\\ 0, & otherwise,\end{cases}$$

where $h_0(t)$ is scaling function, $h_1(t)$ is mother wavelet. In order to perform wavelet transform, Haar wavelet uses dilations and translations of function, that is, the transform make the following function.

$$h_n(t)=h_1\left(2^{j}t-k\right), \quad n=2^{j}+k, \quad j\ge 0, \quad 0\le k<2^{j}.$$

Chen and Hsiao (1997) raised the ideology of operational matrix in 1975, and Kilicman and Al Zhour (2007) investigated the generalized integral operational matrix, that is, the integral of matrix $\phi(t)$ can be approximated as follows:

$$\int_{0}^{t}\phi(t)\,dt\cong Q_{\phi}\phi(t) \quad (7)$$

where Q_{ϕ} is an operational matrix of one-time integral matrix $\phi(t)$, similarly, we can get operational matrix Q_{ϕ}^{n} of n-time integral of $\phi(t)$. Wu and Hsiao (1997) proposed a uniform method to obtain the corresponding integral operational matrix of different basis. For example, the operational matrix of $\Phi(t)$ can be expressed by following:

$$Q_{\Phi}=\Phi Q_{B}\Phi^{-1} \quad (8)$$

Here Q_B is the operational matrix of the block pulse function.

$$Q_{B_m}=\frac{1}{2m}\begin{bmatrix}1 & 2 & 2 & \cdots & 2\\ 0 & 1 & 2 & \cdots & 2\\ 0 & 0 & 1 & \cdots & 2\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & 1\end{bmatrix} \quad (9)$$

where m is the dimension of matrix $\Phi(t)$, and usually $m=2^{\alpha}$, α is positive integer.

If $\Phi(t)$ is a unitary matrix, then $Q_{\Phi}=\Phi Q_{B}\Phi^{T}$, Q_{Φ} is a matrix with characteristic of briefness and profound utility. For $t\in[0,1]$, Haar wavelet function is defined as follows:

$$h_0(x)=\frac{1}{\sqrt{m}}$$

$$h_i(x)=\frac{1}{\sqrt{m}}\begin{cases}2^{\frac{j}{2}}, & \dfrac{k-1}{2^{j}}\le x<\dfrac{k-(1/2)}{2^{j}}\\ -2^{\frac{j}{2}}, & \dfrac{k-(1/2)}{2^{j}}\le x<\dfrac{k}{2^{j}}\\ 0, & otherwise\end{cases}$$

Integer $m = 2^j$ $(j = 0,1,2...J)$ indicates the level of the wavelet; $k = 0,1,2,.....m-1$ is the translation parameter. Maximal level of resolution is J. The index i is calculated according to the formula $i = m + k + 1$; in the case of minimal values $m = 1, k = 0$ we have $i = 2$, the maximal value of i is $i = 2M = 2^{J+1}$. It is assumed that the value $i = 1$ corresponds to the scaling function for which $h_1 \equiv 1 \ in \left[0,1\right]$. Let us define the collocation points $t_l = (l - 0.5)/2M,\ (l = 1,2....2M)$ and discretise the Haar function $h_i(x)$; in this way we get the coefficient matrix $H(i,l) = (h_i(x_l))$, which has the dimension $2M \times 2M$.

Function approximation

Any square integrable function $y(x) \in L^2\left[0,1\right)$ can be expanded by a Haar series of infinite terms:

$$y(x,t) \approx \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} c_{i,j} h_i(x) h_j(t) \tag{10}$$

where the Haar coefficients c_{ij} are determined as,

$$c_{i,j} = \int_0^1 y\left(x,t\right) h_i\left(x\right) dx. \int_0^1 y\left(x,t\right) h_j\left(t\right) dt,\ \left(i,j = 0,1,2,...,m-1\right)$$

coefficients, discrete $y\left(x,t\right)$ by choosing the same step of x and t, we obtain:

$$Y\left(x,t\right) = H^T\left(x\right) C H\left(t\right) \tag{11}$$

where $Y\left(x,t\right)$ is the discrete form of $y\left(x,t\right)$, and

$$H = \begin{bmatrix} h_{0,0} & h_{0,1} & \cdots & h_{0,m-1} \\ h_{1,0} & h_{1,1} & \cdots & h_{1,m-1} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m-1,0} & h_{m-1,1} & \cdots & h_{m-1,m-1} \end{bmatrix}$$

$$C = \begin{bmatrix} c_{0,0} & c_{0,1} & \cdots & c_{0,m-1} \\ c_{1,0} & c_{1,1} & \cdots & c_{1,m-1} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m-1,0} & c_{m-1,1} & \cdots & c_{m-1,m-1} \end{bmatrix}$$

C is the coefficient matrix of Y, and it can be obtained by formula:

$$C = \left(H^T\right)^{-1} Y H^{-1}. \tag{12}$$

H is an orthogonal matrix, then,

$$C = H.Y.H^{-1}. \tag{13}$$

Solving space-fractional telegraph equations by the Haar wavelet method

Consider the space-fractional telegraph equation,

$$\frac{\partial^\alpha u}{\partial x^\alpha} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + g\left(t\right),\ \ t \ge 0,\ 0 < \alpha \le 2. \tag{14}$$

subject to the initial and boundary conditions:

$$\begin{aligned} &u\left(0,t\right) = f_1\left(t\right), \quad t \ge 0 \\ &\frac{\partial u\left(0,t\right)}{\partial x} = f_2\left(t\right),\ t \ge 0 \\ &u\left(x,0\right) = S\left(x\right),\ 0 < x < 1. \end{aligned} \tag{15}$$

Since $u\left(x,t\right) \in L^2\left(R\right)$, we suppose:

$$u\left(x,t\right) \approx \sum_{i=0}^{m-1}\sum_{j=0}^{m-1} C_{ij} h_i\left(x\right) h_j\left(t\right) \tag{16}$$

Then, we can obtain the discrete form of Equation 16 by taking step $\Delta = \frac{1}{m}$ of $\left(x,t\right)$, there is

$$U = H^T\left(x\right) C H\left(t\right) \tag{17}$$

Then, combining Equation 16 with Equation 8, we get:

Table 1. The absolute error for different values of (x, t).

x	*t=0.1*		*t=0.3*		*t=0.5*	
	ADM method	**Haar method**	**ADM method**	**Haar method**	**ADM method**	**Haar method**
1.0	5.201×10^{-11}	3.201×10^{-10}	4.427×10^{-8}	3.121×10^{-7}	2.325×10^{-5}	1.907×10^{-3}
2.0	3.362×10^{-11}	4.809×10^{-10}	6.142×10^{-9}	4.209×10^{-7}	7.570×10^{-6}	9.218×10^{-5}
3.0	2.379×10^{-11}	4.982×10^{-10}	3.528×10^{-10}	8.382×10^{-9}	4.958×10^{-7}	3.214×10^{-6}
4.0	1.509×10^{-11}	2.774×10^{-10}	2.774×10^{-11}	9.234×10^{-10}	1.545×10^{-8}	1.263×10^{-7}
5.0	1.496×0^{-11}	1.292×10^{-10}	8.682×10^{-12}	4.282×10^{-11}	3.039×10^{-9}	4.382×10^{-8}
6.0	2.471×10^{-11}	2.398×10^{-10}	1.430×10^{-13}	2.398×10^{-12}	1.554×10^{-9}	5.127×10^{-8}
7.0	2.250×10^{-11}	4.825×10^{-10}	5.498×10^{-13}	6.475×10^{-12}	5.924×10^{-10}	5.524×10^{-9}
8.0	1.613×10^{-11}	6.281×10^{-10}	1.514×10^{-13}	3.455×10^{-12}	2.194×10^{-10}	3.342×10^{-9}
9.0	1.541×10^{-11}	2.362×10^{-10}	4.975×10^{-14}	2.362×10^{-10}	8.073×10^{-11}	2.362×10^{-10}
10.0	1.108×10^{-11}	8.745×10^{-10}	4.353×10^{-11}	8.745×10^{-10}	2.968×10^{-11}	3.342×10^{-10}

$$\frac{\partial^\alpha u}{\partial x^\alpha} \approx \frac{\partial^\alpha U}{\partial x^\alpha} = \left(\frac{\partial^\alpha}{\partial x^\alpha} H^T(x)\right) CH(t) = \left(\frac{\partial^\alpha}{\partial x^\alpha} H(x)\right)^T CH(t) = H^T(x)\left(Q_H^{-\alpha}\right)^T CH(t) \tag{18}$$

Substituting Equation 18 into 14, there is,

$$\left(\frac{\partial^\alpha}{\partial x^\alpha} H(x)\right)^T CH(t) = H^T(x) C Q_H^{-2} + \left(\frac{\partial}{\partial t} H(t)\right)^T CH(t) + g(t) \tag{19}$$

Then, we have:

$$H\left[\left(\frac{\partial^\alpha}{\partial x^\alpha} H(x)\right)^T - \left(\frac{\partial}{\partial t} H(t)\right)^T\right] C + CQ_H^2 = HDH^{-1} \tag{20}$$

From Equation 20, the wavelet coefficients C can be calculated successively.

NUMERICAL EXAMPLES

Example 1

Consider the following space-fractional telegraph equation,

$$\frac{\partial^\alpha u}{\partial x^\alpha} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u, \quad t \geq 0,\ 0 < \alpha \leq 2. \tag{21}$$

subject to the initial and boundary conditions:

$$\begin{aligned} &u(0,t) = e^{-t}, \quad t \geq 0 \\ &\frac{\partial u(0,t)}{\partial x} = e^{-t},\ t \geq 0 \\ &u(x,0) = e^{-x},\ 0 < x < 1. \end{aligned} \tag{22}$$

where $0 < \alpha \leq 2.$ For special case when $\alpha = 2$, this system represents a homogeneous telegraph and was solved in the work of Kaya (2000).

Using ADM, the solution in series form is given by:

$$u(x,t) = e^{-t}\left[\frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{x^{\alpha+1}}{\Gamma(\alpha+2)} + \frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{x^{2\alpha+1}}{\Gamma(2\alpha+2)} + \ldots\right] \tag{23}$$

Setting $\alpha = 2$ in Equation 23, we produce the solution as follows:

$$u(x,t) = e^{-t}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right] \tag{24}$$

The solution is equivalent to the exact solution in a closed form:

$$u(x,t) = e^{x-t}. \tag{25}$$

Taking $m = 16$ and making use of MATLAB (Table 1), we obtain the results of computations of $u(x,t)$ for $\alpha = 0.5$ (Figure 1).

From Figure 1, we can see that with m increasing, the numerical solution is closer to the exact solution. From Table 2, we can find the numerical solutions which is good with exact solution that it is not only effective to get numerical solutions with good agreement, and by Table 1

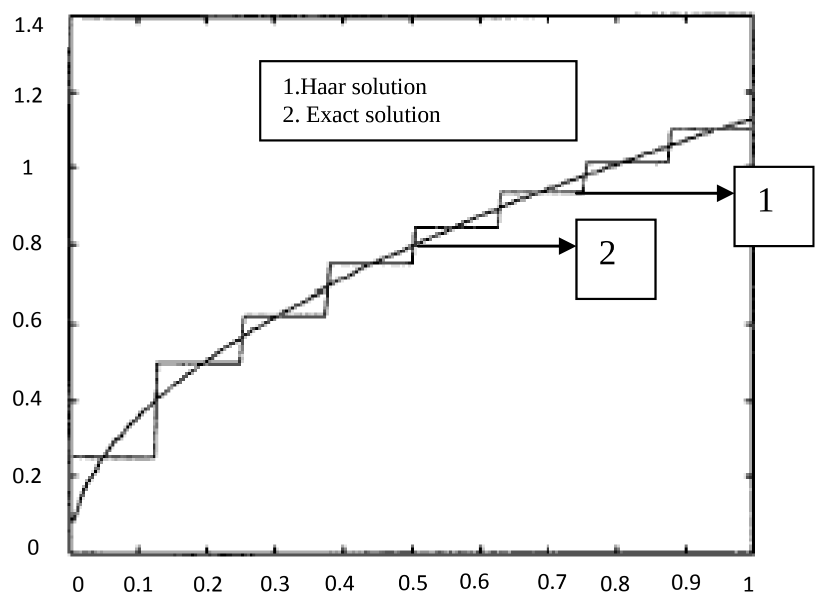

Figure 1. Comparison between exact and Haar solution of Example 1 for *j=4*.

Table 2. Comparison of the exact solution and the Haar wavelet (numerical) solution for example 5.3.

x	Wavelet solution (*j=4*)	Exact solution
0.015	2.133e-005	2.453e-005
0.046	9.385e-005	9.772e-005
0.078	2.549e-004	2.525e-004
0.109	1.118e-004	1.171e-004
0.453	4.556e-004	4.541e-004
0.484	2.153e-004	2.112e-004
0.515	3.342e-005	3.322e-005

we can see that with m increasing, the absolute error became more and smaller. The calculating results show that combining with wavelet matrix, the method in this paper can be effectively used in numerical calculus for constant coefficient fractional differential equations, and that the method is feasible. The power of the manageable method is confirmed.

Example 2

Consider the following non-homogeneous space fractional telegraph equation,

$$\frac{\partial^\alpha u}{\partial x^\alpha}=\frac{\partial^2 u}{\partial t^2}+\frac{\partial u}{\partial t}-x^2-t+1,\quad t\geq 0,\ 0<\alpha\leq 2. \qquad (26)$$

subject to the initial and boundary conditions:

$$\begin{aligned} &u(0,t)=t, \quad t\geq 0\\ &\frac{\partial u(0,t)}{\partial x}=0,\ t\geq 0 \qquad (27)\\ &u(x,0)=x^2,\ 0<x<1. \end{aligned}$$

The solution in series form is given by

$$u(x,t)=t-\frac{2x^{2+\alpha}}{\Gamma(\alpha+3)}+(1-t)\frac{x^\alpha}{\Gamma(\alpha+1)}+(1+t)\frac{x^\alpha}{\Gamma(\alpha+1)}-t\frac{x^{2\alpha}}{\Gamma(2\alpha+1)}-\ldots \qquad (28)$$

Figure 2 shows the exact solution of Equation 24 and Equation 25 using ADM. It is obvious that the self-cancelling “noise” terms appear between various components. Setting $\alpha=2$ and canceling the “noise” terms yield the exact solution for this special case, given by:

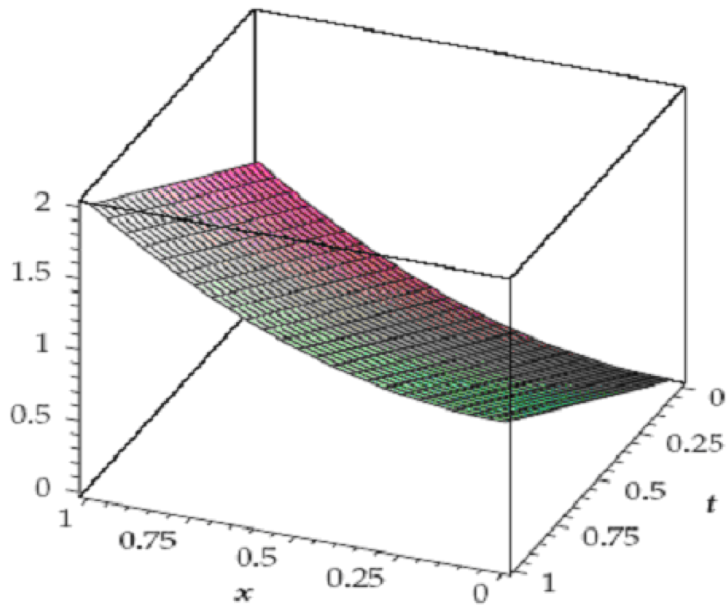

Figure 2. Exact solution of example 2.

$$u(x,t) = x^2 + t \tag{29}$$

Example 3

Consider the time-fractional telegraph equation,

$$\frac{\partial^{2\alpha} u}{\partial t^{2\alpha}} + \lambda \frac{\partial^{\alpha} u}{\partial t^{\alpha}} = v \frac{\partial^2 u}{\partial x^2}, \quad t \geq 0,\ 0 < \alpha \leq 1. \tag{30}$$

subject to the initial and boundary conditions:

$$u(0,t) = s(t), \quad t \geq 0$$
$$\frac{\partial u(x,0)}{\partial t} = h_2(x), \ t \geq 0$$
$$u(x,0) = h_1(x).$$

Using ADM, the solution in series form is given by:

$$\begin{aligned} u(x,t) &= h_1 + th_2 + v\left(h_1'' \frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + h_2'' \frac{t^{2\alpha+1}}{\Gamma(2\alpha+2)}\right) \\ &\quad - \lambda\left(h_1 \frac{t^{\alpha}}{\Gamma(\alpha+1)} + h_2 \frac{t^{\alpha+1}}{\Gamma(\alpha+2)}\right) + v^2\left(h_1^{(4)} \frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + h_2^{(4)} \frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}\right) \\ &\quad - 2v\lambda\left(h_1'' \frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + h_2'' \frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}\right) + \ldots \end{aligned} \tag{31}$$

Setting $\alpha = 1$ in Equation 31, we obtain the solution of classical telegraph equation given by:

$$\begin{aligned} u(x,t) &= h_1 + th_2 + v\left(h_1'' \frac{t^2}{2!} + h_2'' \frac{t^3}{3!}\right) \\ &\quad - \lambda\left(h_1 t + h_2 \frac{t^2}{3!}\right) + v^2\left(h_1^{(4)} \frac{t^4}{4!} + h_2^{(4)} \frac{t^5}{5!}\right) + \ldots \end{aligned}$$

Conclusion

In this study, solving space and time fractional telegraph equations using Haar wavelet method was discussed. The space and time fractional are considered in the Caputo sense. It has also been shown that the key idea is to perform the partial differential equation into a group of algebraic equations. The main advantage of this method is its simplicity and small computation costs, which is due to the sparcity of the transform matrices and to the small number of significant wavelet coefficients. In comparison with existing numerical schemes used to solve the space and time fractional telegraph equations, the scheme in this paper is an improvement over other methods in terms of accuracy. It is concluded that this technique is very powerful and efficient in finding the analytical solutions for a large class of linear partial differential equations of fractional order. This technique provides more realistic series solutions when compared with the Adomian decomposition technique. It is worth mentioning that Haar solution provides excellent results even for small values of m $(m=16)$. For larger values of m, that is, $(m=32,\ m=64,\ m=128,\ m=256)$, we can obtain the results closer to the real values. The method with far less degrees of freedom and with smaller CPU time provides better solutions than classical ones.

REFERENCES

Agrawal P (2002). Solution for a fractional diffusive-wave equation defined in a bounded domain. Nonlin. Dyn., 29: 145-155.

Ali S (2010). An Approximation to Solution of Space and Time Fractional Telegraph Equations by He's Variational Iteration Method. Mathematical Problems Eng. Article ID 290631, doi:10.1155/2010/290631.

Angulo JM, Medina MD, Anh VV, Grecksch W (2000). Fractional diffusion and fractional heat equation. Adv. Appl. Prob., 32(4): 1077-1099.

Anh VV, Leonenko NN (2000). Scaling laws for fractional diffusion-wave equation with singular data. Stat. Prob. Lett., 48: 239-252.

Chen CF, Hsiao CH (1997). Haar wavelet method for solving lumped and distributed-parameter systems. IEEE Proc. Pt. D144(1): 87-94.

Chen Y, Wu Y, Cui Y, Wang Z, Jin D (2010). Wavelet method for a class of fractional convection-diffusion equation with variable coefficients. J. Comput. Sci., 1: 146–149.

Hariharan G (2010). Solving Finite Length Beam Equation by the Haar Wavelet Method. Int. J. Comput. Appl., 9(1): (0975–8887).

Hariharan G, Kannan K (2009). Haar wavelet method for solving Cahn-Allen equation. Appl. Math. Sci., 51(3): 2523–2533.

Hariharan G, Kannan K (2010). Haar wavelet method for solving some nonlinear parabolic equations. J. Math. Chem., 48: 1044-1061.

Hariharan G, Kannan K (2010). A Comparative Study of a Haar Wavelet Method and a Restrictive Taylor's Series Method for Solving Convection-diffusion Equations. Int. J. Comput. Methods Eng. Sci. Mech., 11(4): 173- 184.

Henry BI, Wearne SL (2000). Fractional reaction-diffusion. Phys. A., 276: 448- 455.

Kaya D (2000). A new approach to the telegraphic equation: An application of the decomposition method. Bull. Inst. Math. Acad. Sin., 28 (1): 51-57.

Kilicman A, AL Zhour (2007). Kronecker operational matrices for fractional calculus and some applications. Appl. Math. Comput., 187(1): 250–265.

Lepik U (2005). Numerical solution of differential equations using Haar wavelets. Math. Comput. Simul., 68: 127-143.

Lepik U (2007). Application of the Haar wavelet transform to solving integral and differential Equations. Appl. Math. Comput., 56(1): 28-46.

Lepik U (2007). Numerical solution of evolution equations by the Haar wavelet method. Appl. Math. Comput., 185: 695–704.

Lepik U (2010). Solving fractional integral equations by the Haar wavelet method. Appl. Math. Comput., DOI: 10.1016/j.amc.2009.04.015.

Momani S (2005). Analytic and approximate solutions of the space- and time-fractional telegraph equations. Appl. Math. Comput., 170(2): 1126–1134.

Momani S, Odibat Z (2007). Numerical comparison of methods for solving linear differential equations of fractional order. Chaos Solitons Fractals, 31(5): 1248-1255.

Nigmatullin RR (1986). The realization of the generalized transfer equation in a medium with fractional geometry. Phys.Status (B): Basic Res., 133(1): 425-430.

Odibata Z, Momanib S (2008). A generalized differential transform method for linear partial differential equations of fractional order. Appl. Math. Lett., 21: 194–199.

Orsingher E, Beghin L (2004). Time-fractional telegraph equation and telegraph processes with Brownian time. Prob. Theory Relat. Fields, 128: 141-160.

Orsingher E, Zhao X (2003). The space-fractional telegraph equation and the related fractional telegraph process. Chin. Ann. Math., 24B(1): 1-12.

Wu JL, Hsiao CH (1997). Haar wavelet method for solving lumped and distributed parameter systems. IEE Proceedings-Control Theory Appl., 144: 87–94.

Yıldırım A (2010). He's homtopy perturbation method for solving the space and time fractional telegraph equations. Int. J. Comput. Math., 87(13): 2998-3006.

Zhou Y, Jiao F, Li J (2009). Existence and uniqueness for p-type fractional neutral differential equations. Nonlin. Anal., 71(8): 2724–2733.

Zhou Y, Jiao F, Li J (2009). Existence and uniqueness for fractional neutral differential equations with infinite delay. Nonlin. Anal., 71(7–8): 3249- 3256.

Zhuang P, Liu F (2005). An explicit approximation for the space–time fractional diffusion equation. Numer. Math. A: J. Chin. Univ., 27: 223–228.