

*Full Length Research Paper*

# Dynamical analysis of clustering-based wireless sensor networks

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**In wireless sensor networks (WSNs), clustering-based algorithms have been proved to be effective techniques to save energy and improve fault tolerance. In this paper, a scale-free evolving model for clustering-based WSNs with exponential growth behavior is presented, in which the nodes arrive as a Poisson process with rate  $\lambda$ . This model takes into account four types of evolving events: selection of cluster-head nodes, preferential attachment of non cluster-head nodes, failure of non cluster-head nodes, transition of non cluster-head nodes. By using continuum theory, the theoretical computing and simulated results show that the size distribution of cluster in this model follows a power law. The scale-free properties revealed in this model display a tempting application foreground.**

**Key words:** Wireless sensor networks, continuum theory, scale-free, exponential growth behavior.

## INTRODUCTION

Recent development in wireless communication networks has enabled the large-scale deployment of low cost, energy efficient and multi-purpose wireless sensor networks. A lot of real world applications have been already deployed and many of them will be based on wireless sensor networks. These applications include geographical monitoring, medical care, manufacturing, transportation, military and surveillance systems. Due to the constrained energy and unreliable topology in wireless sensor networks, many researchers have witnessed clustering based algorithms, to maintain network connectivity, prolong network lifetime and optimize the overall network performance. A wide range of algorithms aimed at clustering have been proposed, such as Leach (Heinzelman, 2002), Heed (Younis and Fahmy, 2004), Pegasus (Lindsey and Raghavendra, 2002), Teen (Manjeshwar and Agrawal, 2001), etc. Clustering is particularly useful for application that requires scalability to large-scale wireless sensor networks and clustering can be effective in routing protocol and broadcast communications (Younis and Fahmy, 2004). In general, clustering can be divided into two phases: cluster formation and steady data transmission, and cluster formation can be divided into four phases: selection of cluster-head nodes, broadcasting of cluster-head nodes, attachment of ordinary nodes and creation of schedule

mechanism.

The remainder of this paper is organized as follows. Subsequently, we basically describe the study's background and our contribution, followed by the exponential growth model, after which the design of the study's model is then discussed in details. Following the study's design, theoretical analysis was done using continuum theory, as well as the simulation, before the conclusion was finally given.

## Background and our contribution

In the past few years, various natural networks, such as Internet (Faloutsos et al., 1999; Guanrong et al., 2005), the World Wide Web (WWW) (Albert et al., 1999), traffic networks (Barrat et al., 2004), food webs (Drossel and McKane, 2003), Quantum Neural Network (Rigui and Quilin, 2008; RiguiZhou, 2010) etc., have been found that they generally exhibit universal properties of complex networks, including small world characteristic (Watts et al., 1998) and scale-free characteristic (Barabási and Albert, 1999) and more and more physicists devote their enthusiastic energy to explore and model the evolving mechanisms by using statistical approach. Scale-free networks are robust against random nodes failure or attack; they always exhibit low path length and high

clustering, and they have been proved to be efficient in routing or topology control of Wireless Sensor Networks (Garbinato et al., 2007; Wang et al., 2007).

A series of evolving models are presented, such as BA model (Barabási and Albert, 1999), Local-World model (Li and Chen, 2003), Fitness model (Bianconi and Barabasi, 2001), BBV model (Barrat et al., 2004) etc. But these prominent models have two shortcomings. Firstly, they always supposed that the time interval of nodes arrival is equivalent. That is to say, at time  $t$ , the number of nodes  $N(t)$  is proportional to  $t$ . Actually, in natural networks,  $N(t)$  is not always linear growth process, such as the consumers in barber shop and the packets appeared in switches and routers, they are stochastic Poisson process. Secondly, they did not consider the exit of nodes. For example, in WSNs, the nodes probably exit from the network due to exhaustibility of power or intended attack. Several recently proposed models have addressed these shortcomings. Shou-we and Xing-san (2005) proposed a modified BA model and in which the time interval followed exponential distribution. Guo and Wang (2007) proposes a Poisson network model with node batch arrival, which was based on the batch arrival concept in the queue theory. Sarshar and Roychowdhury, (2004) first showed that only relying on the classical BA model can not produce heavy-tailed degree distribution in complex ad hoc networks and P2P networks. The SR evolving model introduced the mechanisms of nodes deletion and links compensatory rewiring.

Whether WSNs have the same mechanisms like the evolving model which we have mentioned? How to model the dynamics of clustering-based WSNs? Jin and Papavassiliou (2002) modeled mobile wireless sensor networks, which described the dynamics of the network and facilitated the understanding of the effect of the various events, this is the originate work, to the best of our knowledge. Iyer et al. (2003) showed the dynamics of self-organizing ad hoc networks, and the presented evolving model could well illuminate clustering phenomena; Helmy et al. (2003) investigated the small-world effects in wireless ad hoc or sensor networks by randomly link re-wiring and link addition; Ishizuka and Aida (2004) probed the scale-free properties of wireless sensor networks by using probability density function. Hui and Chaintreau (2005) measured human mobility at academic working environments and the Infocom (2005) conference, and found that the time between pair wise node contacts followed power law distribution. Chen et al. (2007) presented an evolving WSNs model with equivalent growth of the nodes, which was based on BA evolving model (Barabási and Albert, 1999) and Local-World evolving model (Li and Chen, 2003), and this model can improve the uniformity of the degree distribution.

In this paper, our contributions include; an evolving model for clustering-based wireless sensor networks is proposed, which is based on (Iyer et al., 2003); and four types of evolving events are introduced into it, which

separately are: (1) Selection of cluster-head nodes, (2) Local preferential attachment of non cluster-head nodes, (3) Failure of non cluster-head nodes and (4) Transition of non cluster-head nodes. We restrict our attention to the events of nodes failure/deletion, and the time interval of nodes arrival follows exponential distribution in particular. With the exponential growth behavior of nodes and using the continuum theory, we get the size distribution of clusters; the numerical simulation is conducted consequently.

### EXPONENTIAL GROWTH MODEL (SHOU-WE and Xing-San, 2005; GUO and WANG, 2007)

The evolving process of exponential growth model includes two ingredients as follows: exponential growth of the nodes and degree preferential attachment. The exponential growth implies that the dimension of entire network grows exponentially, is different from the small-world network and random network where the number of nodes is static; degree preferential attachment means new adding nodes incline to be attached with high-degree clusters. Based on two aspects which have been mentioned, the evolving exponential model is carried out as follows:

#### Exponential growth

Starting with  $m_0$  nodes, at every time interval  $t_n$ , we add a new node with  $m$  links that connect the new node to  $m$  different nodes already present in the network ( $m < m_0$ ); the time interval  $t_n$  follows exponential growth:

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \lambda > 0, t > 0 \\ 0 & t \leq 0 \end{cases}$$

#### Preferential attachment

The probability that a new node will be connected to node  $i$  depends on the degree  $k_i$  of that node, so that

$$\prod_i = k_i / \sum_j k_j$$

After time  $t$ , the model leads to a random network with  $\lambda t$  vertices

$$\sum_j k_j = 2m\lambda t$$

and  $m\lambda t$  edges, then

We use continuum theory to analyze the degree distribution of the BA model (Barabási and Albert, 1999). The rate at which a node acquires links is:  $\partial k_i / \partial t = m k_i / \sum_j k_j$  and  $\sum_j k_j = 2m\lambda t$ , so

we can get  $\partial k_i / \partial t = k_i / 2t$ , it gives,  $k_i(t) = m(t/t_i)^\beta$ ,

$\beta = 1/2$ . The probability that a node  $i$  has a degree smaller than  $k$   $P(k_i(t) < k) = P[t_i > m^{1/\beta} t / k^{1/\beta}]$ . Assuming that we add

the nodes to the network at time intervals with the probability  $P_i(t_i) = \lambda e^{-\lambda t_i}$ , so  $P[t_i > m^{1/\beta} t / k^{1/\beta}] = \exp[-\lambda t(m/k)^{2\lambda}]$

We get the degree distribution:

$$P(k) = \partial P[k(i,t) < k] / \partial k = \exp[-\lambda t(m/k)^{2\lambda}] \cdot 2m^{2\lambda} \lambda^2 k^{-(2\lambda+1)} t,$$

and

$$\exp[-\lambda t(m/k)^{2\lambda}] = 1 - \lambda t(m/k)^{2\lambda} + (1/2!)\lambda^2 t^2(m/k)^{4\lambda} - \dots$$

With the condition of  $k \ll m$ , we can get

$$P(k) = 2m^{2\lambda} \lambda^2 k^{-(2\lambda+1)} t$$

And the exponent of degree distribution

$$\gamma = 2\lambda + 1.$$

### Clustering evolving model in wireless sensor network

We will start with a definition of network model in a quite general form. This will later enable us to introduce our model.

#### Definition 1

The entire sensor network is formalized as a undirected graph:  $G = \langle V(G), E(G) \rangle$ , where  $V(G) = \{v_1, v_2, \dots, v_3\}$  is the set of nodes and  $E(G)$  the set of wireless links;  $range_{v_i}$  is the max transmission range of sensor node  $v_i$ . If the distance between  $v_i$  and  $v_j$  is less than  $range_{v_i}$ ,  $link(v_i, v_j)$  will exist. Thus  $E(G)$  can be expressed as:

$$E(G) = \{(v_i, v_j) : d(v_i, v_j) \leq range_{v_i}, v_i, v_j \in V(G)\}, \text{ here we assume } |V(G)|=n, |E(G)|=m.$$

#### Definition 2

At time  $t$ , the number of arrival sensor nodes is  $N(t)$ , and  $N(t)$  follows Poisson distribution, the arrival rate is  $\lambda$ . Then we can get:

$$(1) \quad \forall t \geq 0,$$

$$P\{N(t) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, \quad k = 0, 1, 2, \dots \quad (1)$$

(2)  $\forall 0 \leq t_1 \leq t_2$ ,  $N(t_1, t_2)$  is the increment of  $N(t)$ , it follows Poisson distribution, and

$$P\{N(t_1, t_2) = k\} = \frac{(\lambda(t_2 - t_1))^k}{k!} e^{-\lambda(t_2 - t_1)}, \quad k = 0, 1, 2, \dots \quad (2)$$

(3) We assume  $t_n$  is the arrival time of node  $n$ , Then  $t_n \ll \Gamma(n, \lambda)$ ,

$$f_{t_n}(t) = \begin{cases} \lambda e^{-\lambda t} \frac{(\lambda t)^{n-1}}{(n-1)!} & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (3)$$

(4) We assume  $\tau_n = t_n - t_{n-1}$ , that is to say,  $\tau_n$  is the time interval of nodes  $n$  and  $n-1$ , then  $\tau_n$  follows exponential distribution,

$$f_{\tau_n}(t) = \begin{cases} \lambda e^{-\lambda t} & t > 0 \\ 0 & t \leq 0 \end{cases} \quad (4)$$

The initial network has  $c_0$  initial clusters and  $e_0$  edges with the same condition of BA model. Nodes will be placed in  $L \times L$  square area and the coordinate of nodes will be decided by probability density function (p.d.f.). At time interval  $t$ , we add a new node into the network and denote  $c(i, t)$  is the size of cluster  $i$  at time  $t$ . Then we perform the following four phases at random:

1. Selection of cluster-head nodes: With probability  $p$ , the new node forms a new cluster at a random location. With this evolving rule, the dynamic equations of  $c(i, t)$  is:

$$\left( \frac{\partial c(i, t)}{\partial t} \right)_{(I)} = 0 \quad (5)$$

2. Local preferential attachment of non cluster-head nodes: With probability  $1-p$ , the new node randomly selects  $M$  clusters from the existing clusters, which are referred to "Local-World" of the new node. Then it joins one of the "Local-World" clusters with preferential attachment probability.

$$\Pi_{Local} = c(i, t) / \sum_{j \in local} c(j, t) \quad (6)$$

We can get the dynamic equations of  $c(i, t)$

$$\left( \frac{\partial c(i, t)}{\partial t} \right)_{(II)} = (1-p) \frac{M}{C(t)} \frac{c(i, t)}{\sum_{j \in local} c(j, t)} \quad (7)$$

3. Failure of non cluster-head nodes. With probability  $q$ , the event of one ordinary node failure will happen. We can get the dynamic equations of  $c(i, t)$

$$\left( \frac{\partial c(i, t)}{\partial t} \right)_{(III)} = -q \frac{c(i, t)}{C(t)} \quad (8)$$

4. Transition of non cluster-head nodes. With probability  $r$ , a randomly selected node from a cluster moves from its existing cluster to a new cluster, the target cluster is chosen preferentially. Different from Iyer et al. (2003), the evolving rule 4 shows the equilibrium of clusters, which can avoid the excess energy consumption in some clusters. We can get the dynamic equations of  $c(i, t)$

$$\left( \frac{\partial c(i, t)}{\partial t} \right)_{(IV)} = -r \frac{1}{C(t)} + r \frac{M}{C(t)-1} \frac{c(i, t)}{\sum_{j \in local} c(j, t)} \quad (9)$$

### Dynamical analysis

The cumulative cluster size of the Local-World (Li and Chen, 2003) is introduced into our analysis, which is showed as follows:

$$\sum_{j \in local} c(j, t) = M \langle c(t) \rangle \tag{10}$$

Where  $\langle c(t) \rangle$  is the average cluster size. It can be expressed as:

$$\langle c(t) \rangle = \sum_j c(j, t) / C(t) \tag{11}$$

Where  $\sum_j c(j, t)$  the total is number of nodes in all the clusters of the networks and  $C(t)$  is the total number of clusters (Iyer et al., 2003). With the stochastic process theory, we can get:

$$\begin{aligned} \left( \frac{\partial c(i, t)}{\partial t} \right) &= (1-p) \frac{M}{C(t)} \frac{c(i, t)}{\sum_{j \in local} c(j, t)} - q \frac{c(i, t)}{C(t)} - r \frac{1}{C(t)} + r \frac{M}{C(t) - 1} \frac{c(i, t)}{\sum_{j \in local} c(j, t)} \\ &\approx (1-p+r) \frac{M}{C(t)} \frac{c(i, t)}{\sum_{j \in local} c(j, t)} - q \frac{c(i, t)}{C(t)} - r \frac{1}{C(t)} \\ &= (1-p+r) \frac{c(i, t)}{\sum_j c(j, t)} - q \frac{c(i, t)}{C(t)} - r \frac{1}{C(t)} \end{aligned} \tag{14}$$

Substituting Equation 12 and 13 into 14 leads to,

$$\begin{aligned} \left( \frac{\partial c(i, t)}{\partial t} \right) &= (1-p+r) \frac{c(i, t)}{\sum_j c(j, t)} - q \frac{c(i, t)}{C(t)} - r \frac{1}{C(t)} \\ &= \frac{1}{\lambda} \left( \frac{1-p+r}{1-q} - \frac{q}{p} \right) \frac{c(i, t)}{t} - \frac{r}{\lambda p t} \end{aligned} \tag{15}$$

Let,

$$A(\lambda, p, q, r) = \frac{1}{\lambda} \left( \frac{1-p+r}{1-q} - \frac{q}{p} \right) \quad \text{and} \quad B(\lambda, p, q, r) = \frac{r}{\lambda p}$$

then

$$\left( \frac{\partial c(i, t)}{\partial t} \right) = A(\lambda, p, q, r) \frac{c(i, t)}{t} - B(\lambda, p, q, r) \frac{1}{t} \tag{16}$$

With the initial condition  $c(i, t_i) = 1$ , we can get the solution of (16):

$$c(i, t) = \left( 1 - \frac{B(\lambda, p, q, r)}{A(\lambda, p, q, r)} \right) \left( \frac{t}{t_i} \right)^{A(\lambda, p, q, r)} + \frac{B(\lambda, p, q, r)}{A(\lambda, p, q, r)} \tag{17}$$

Since  $t_i$  follows  $\Gamma(n, \lambda)$  distribution:

$$\sum_j c(j, t) = \lambda[t - qt] \tag{12}$$

$$C(t) = \lambda pt \tag{13}$$

And we can easily prove that  $0 < \lambda \leq 1$  (Shou-we and Xing-San, 2005). By combining Equation 6 and 9, the total dynamic equations of  $c(i, t)$  is as follows:

$$F_{t_i}(t) = P[t_i \leq t] = \begin{cases} 1 - e^{-\lambda t} \sum_{j=0}^{n-1} \frac{(\lambda t)^{j-1}}{j!} & t > 0 \\ 0 & t \leq 0 \end{cases} \tag{18}$$

Thus the probability that a cluster has a size  $c(i, t)$  smaller than  $c$   $P[c(i, t) < c]$  can be written as:

$$\begin{aligned} P[c(i, t) < c] &= P \left[ t_i > \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} t \right] \\ &= 1 - P \left[ t_i \leq \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} t \right] \\ &= e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \\ &\quad \times \sum_{j=0}^{i-1} \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^j}{j!} \end{aligned} \tag{19}$$

Then we can get the moment size distribution of cluster  $i$ :

$$\begin{aligned}
 P\{c(i,t) = c\} &= \frac{\partial P[c(i,t) < c]}{\partial c} \\
 &= \frac{\lambda t \left[ (1 - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1/A(\lambda, p, q, r)} \right]}{A(\lambda, p, q, r) \left[ (c - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1+1/A(\lambda, p, q, r)} \right]} \\
 &\times e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \times \sum_{j=0}^{i-1} \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^j}{j!} \\
 &- \frac{1}{A(\lambda, p, q, r) \left[ (c - A(\lambda, p, q, r) / B(\lambda, p, q, r)) \right]} \times e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \\
 &\times \sum_{j=0}^{i-2} \frac{j \left[ \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right]^j}{j!} \\
 &= \frac{\lambda t \left[ (1 - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1/A(\lambda, p, q, r)} \right]}{A(\lambda, p, q, r) \left[ (c - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1+1/A(\lambda, p, q, r)} \right]} \\
 &\times e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \times \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^{i-1}}{(i-1)!} \\
 &+ \frac{1}{A(\lambda, p, q, r) \left[ (c - A(\lambda, p, q, r) / B(\lambda, p, q, r)) \right]} \times e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \\
 &\times \left( \sum_{j=0}^{i-2} \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^{j+1}}{j!} \right. \\
 &\left. - \sum_{j=1}^{i-1} \frac{j \left[ \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right]^j}{j!} \right) \\
 &= \frac{\lambda t \left[ (1 - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1/A(\lambda, p, q, r)} \right]}{A(\lambda, p, q, r) \left[ (c - A(\lambda, p, q, r) / B(\lambda, p, q, r))^{1+1/A(\lambda, p, q, r)} \right]} \\
 &\times e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \times \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^{i-1}}{(i-1)!} \tag{20}
 \end{aligned}$$

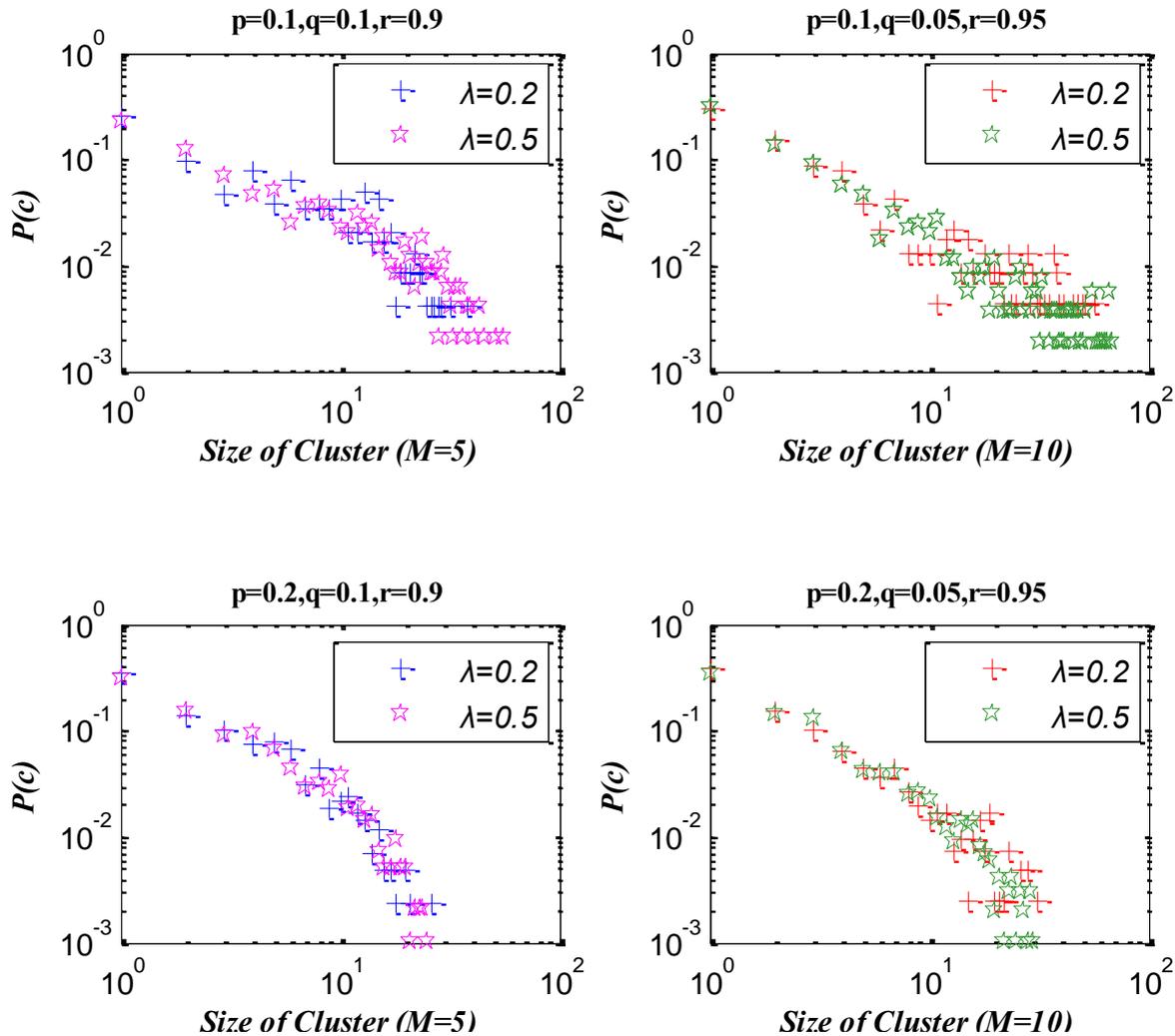


Figure 1. The cluster size distribution.

By using the similar method in (Guo and Wang 2007), we can get the stationary mean size distribution:

$$\begin{aligned}
 P(c) &= \lim_{t \rightarrow \infty} \left( \sum_{i=1}^{\infty} P\{c(i, t) = c\} \right) / \lambda t \\
 &= \lim_{t \rightarrow \infty} \left( \frac{\lambda t \left[ 1 - A(\lambda, p, q, r) / B(\lambda, p, q, r) \right]^{1/A(\lambda, p, q, r)}}{A(\lambda, p, q, r) \left[ c - A(\lambda, p, q, r) / B(\lambda, p, q, r) \right]^{1+1/A(\lambda, p, q, r)}} \right. \\
 &\quad \times \left. e^{-\lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)}} \right) / \lambda t \\
 &\quad \times \left( \sum_{i=1}^{\infty} \frac{\left( \lambda t \left( \frac{1 - A(\lambda, p, q, r) / B(\lambda, p, q, r)}{c - A(\lambda, p, q, r) / B(\lambda, p, q, r)} \right)^{1/A(\lambda, p, q, r)} \right)^{i-1}}{(i-1)!} \right) \\
 &= \frac{\left( 1 - A(\lambda, p, q, r) / B(\lambda, p, q, r) \right)^{1/A(\lambda, p, q, r)}}{A(\lambda, p, q, r) \left[ c - A(\lambda, p, q, r) / B(\lambda, p, q, r) \right]^{1+1/A(\lambda, p, q, r)}} \quad (21)
 \end{aligned}$$

Equation (21) exhibits scale-free properties of our model, and the exponent of size distribution  $\gamma$  is:

$$\gamma = 1 + 1 / A(\lambda, p, q, r) = 1 + \lambda / \left( \frac{1 - p + r}{1 - q} - \frac{q}{p} \right) \quad (22)$$

### SIMULATION RESULTS

We develop a MATLAB simulation toolbox for verifying the validity of this model. Randomly distributed nodes are used for two-dimensional network space which is set to  $1000 \times 1000$  m. The arrival rate is respectively set to 0.1 to 0.9. We start our simulations under four different scenarios:

- a.  $M=5, p=0.1, q=0.1, r=0.9$ ; b.  $M=5, p=0.2, q=0.1, r=0.9$ ;

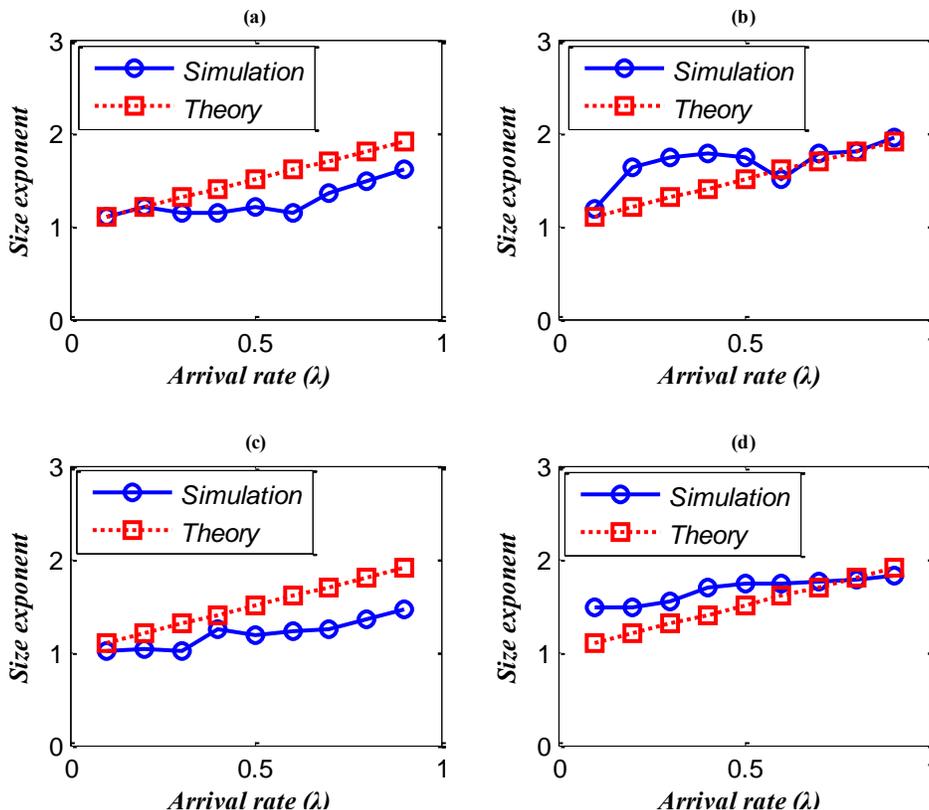


Figure 2. The size distribution with different scenarios.

c.  $M=10, p=0.1, q=0.05, r=0.95$ ;  
 d.  $M=10, p=0.2, q=0.05, r=0.95$ .

Figure 1 shows the size distribution of our model (log to log plot), which exhibits a power law. It can satisfy the Equation (21). Figure 2 shows the exponent of size distribution with different scenarios under arrival rate of 0.1 to 0.9, which verify Equation (22). Note that the size exponent of simulation is a little different than the predict value, that is caused by randomly nodes failure and local attachment rules. From Figures 1 and 2, we come to know that the probability of nodes failure can affect the network topology. However, this model is robust with randomly nodes failure and links failure. Overall, based on Figures 1 and 2, it has been seen that the simulation results are agreement with the theoretical analysis. On the evidence of computational results, the dynamic evolving model, described in section 4, provides better compensation for modeling real sensor networks.

**Conclusions**

In this paper, the dynamic evolving topology of clustering-based wireless sensor networks has been modeled. The following evolving events are introduced into the evolving model: selection of cluster-head nodes, preferential

attachment of non cluster-head nodes, failure of non cluster-head nodes, and transition of non cluster-head nodes. We assume that the time interval of adding new nodes follows exponential distribution and the number of arrival nodes follows Poisson distribution. We use continuum theory to predict theoretically the exponent of cluster size distribution and the analysis results shows that our model exhibits scale-free properties. In the situation of randomly nodes failure and links failure, this model has higher robustness, which provides a reference for constructing reliable topology of Wireless Sensor Networks. In future work, we will focus on improve robustness and introduce weighted networks to further enrich this model.

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