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Some regular quasivarieties of commutative binary modes

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Abstract: Irregular (quasi)varieties of groupoids are (quasi)varieties that do not contain semilattices. The regularization of a (strongly) irregular variety $\mathcal{V}$ of groupoids is the smallest variety containing $\mathcal{V}$ and the variety $\mathcal{S}$ of semilattices. Its quasiregularization is the smallest quasivariety containing $\mathcal{V}$ and $\mathcal{S}$. In an earlier paper the authors described the lattice of quasivarieties of cancellative commutative binary modes, i.e. idempotent commutative and entropic (or medial) groupoids. They are all irregular and the lattice contains all irregular varieties of such groupoids. This paper extends the earlier result, by investigating some regular quasivarieties. It provides a full description of the lattice of subquasivarieties of the regularization of any irregular variety of commutative binary modes.

Keywords: regular quasivarieties; regular quasi-identity; modes; affine spaces; commutative binary modes
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