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A very general covering property

Comment.Math.Univ.Carolin. 53,2 (2012) 281 –306.

Abstract: We introduce a general notion of covering property, of which many classical definitions are particular instances. Notions of closure under various sorts of convergence, or, more generally, under taking kinds of accumulation points, are shown to be equivalent to a covering property in the sense considered here (Corollary 3.10). Conversely, every covering property is equivalent to the existence of appropriate kinds of accumulation points for arbitrary sequences on some fixed index set (Corollary 3.5). We discuss corresponding notions related to sequential compactness, and to pseudocompactness, or, more generally, properties connected with the existence of limit points of sequences of subsets. In spite of the great generality of our treatment, many results here appear to be new even in very special cases, such as D-compactness and D-pseudocompactness, for D an ultrafilter, and weak (quasi) M-(pseudo)-compactness, for M a set of ultrafilters, as well as for $[\beta, \alpha]$-compactness, with β and α ordinals.

Keywords: covering property, subcover, compactness, accumulation point, convergence, pseudocompactness, limit point

AMS Subject Classification: Primary 54D20; Secondary 54A20

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