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Existence of a positive solution to a nonlocal semipositone boundary value problem on a time scale

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Abstract: We consider the existence of at least one positive solution to the dynamic boundary value problem

$$-y^{\Delta\Delta}(t) = \lambda f(t, y(t)),\ t \in [0, T]_{\mathbb{T}} y(0) \quad = \int_{\tau_1}^{\tau_2} F_1(s, y(s))\ \Delta s y\left(\sigma^2(T)\right) = \int_{\tau_3}^{\tau_4} F_2(s, y(s))\ \Delta s,$$

where $\mathbb{T}$ is an arbitrary time scale with $0 < \tau_1 < \tau_2 < \sigma^2(T)$ and $0 < \tau_3 < \tau_4 < \sigma^2(T)$ satisfying $\tau_1,\ \tau_2,\ \tau_3,\ \tau_4 \in \mathbb{T}$, and where the boundary conditions at $t = 0$ and $t = \sigma^2(T)$ can be both nonlinear and nonlocal. This extends some recent results on second-order semipositone dynamic boundary value problems, and we illustrate these extensions with some examples.

Keywords: time scales; integral boundary condition; second-order boundary value problem; cone; positive solution
AMS Subject Classification: Primary 34B10, 34B15, 34B18, 34N05, 39A10; Secondary 26E70, 47H07

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