Uzma Ahmad, Syed Husnine

Characterization of power digraphs modulo n

Comment.Math.Univ.Carolin. 52,3 (2011) 359 –367.

Abstract: A power digraph modulo n, denoted by $G(n,k)$, is a directed graph with $Z_n = \{0, 1, \ldots, n-1\}$ as the set of vertices and $E = \{(a,b) : a^k \equiv b \pmod{n}\}$ as the edge set, where n and k are any positive integers. In this paper we find necessary and sufficient conditions on n and k such that the digraph $G(n,k)$ has at least one isolated fixed point. We also establish necessary and sufficient conditions on n and k such that the digraph $G(n,k)$ contains exactly two components. The primality of Fermat number is also discussed.

Keywords: iteration digraph, isolated fixed points, Charmichael lambda function, Fermat numbers, Regular digraphs
AMS Subject Classification: 11A07, 11A15, 20K01, 05C20, 11A51

References

[1] Wilson B., *Power digraphs modulo n*, Fibonacci Quart. **36** (1998), 229–239.
[2] Lucheta C., Miller E., Reiter C., *Digraphs from powers modulo p*, Fibonacc Quart. **34** (1996), 226–239.
[3] Burton D.M., *Elementary Number Theory*, McGraw-Hill, 2007.
[4] Chartrand G., Oellermann O.R., *Applied and Algorithmic Graph Theory*, McGraw-Hill, New York, 1993.
[5] Kramer-Miller J. *Structural properties of power digraphs modulo n*, in Proceedings of the 2009 Midstates Conference for Undergraduate Research in Computer Science and Mathematics, Oberlin, Ohio, 2009, pp. 40–49.
[6] Ellson J., Gansner E., Koutsofios L., North S.C., Woodhull G., *Graphviz - open source graph drawing tools*, version 2.26.3, `http://www.graphviz.org`
[7] Somer L., Křížek M., *On a connection of number theory with graph theory*, Czechoslovak Math. J. **54** (2004), 465–485.
[8] Somer L., Křížek M., *Structure of digraphs associated with quadratic congruences with composite moduli*, Discrete Math. **306** (2006), 2174–2185.
[9] Somer L., Křížek M., *On semi-regular digraphs of the congruence* $x^k \equiv y \pmod{n}$, Comment. Math. Univ. Carolin. **48** (2007), no. 1, 41–58.
[10] Somer L., Křížek M., *On symmetric digraphs of the congruence* $x^k \equiv y \pmod{n}$, Discrete Math. **309** (2009), 1999–2009.
[11] Szalay L., *A discrete iteration in number theory*, BDTF Tud. Közl. **8** (1992), 71–91.
[12] MATLAB, *The language of technical computing*, version 7.0.0.19920 (R14).
[13] Deo N. *Graph theory with Application to Engineering and Computer Sciences*, Prentice-Hall of India private Limited, 1990.
[14] Carmichael R.D., *Note on a new number theory function*, Bull. Amer. Math. Soc. **16** (1910), 232–238.
[15] Husnine S.M., Ahmad U., Somer L., *On symmetries of power digraphs*, Utilitas Mathematica, to appear.
[16] Skowronek-Kaziów J., *Properties of digraphs connected with some congruences relations*, Czechoslovak Math. J. **59** (2009), 39–49.