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Comments on 'Sweep algorithm for solving optimal control problem with multi-point boundary conditions' by M Mutallimov, R Zulfuqarova, and L Amirova

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Abstract
A counter example is given for the solution of the linear-quadratic optimization problem with three-point boundary conditions. The example shows that the solution obtained in (Mutallimov *et al.* in Adv. Differ. Equ. 2015:233, 2015) by using a sweep method is not optimal.

Keywords: sweep algorithm; optimization; three-point boundary conditions

1 Introduction

In [1] the linear-quadratic optimization problem with multi-point boundary conditions, both in the continuous and the discrete cases, are considered. The sweep method [2, 3], which generalizes the results [4] for the two-point boundary conditions is given in [5]. However, the results obtained for the discrete case [1] are not optimal.

Not passing to the illustration of an example, we form the problem of discrete optimal control with multi-point boundary conditions [1, 4]. Let the motion of an object be described by the following linear system of finite-difference equations:

$$x(i+1) = \psi(i)x(i) + \Gamma(i)u(i) \quad (i = 0,1,\ldots,l-1), \tag{1}$$

with nonseparate boundary conditions

$$\Phi_1 x(0) + \Phi_2 x(s) + \Phi_3 x(l) = q. \tag{2}$$

Here $x(l)$ is an n-dimensional phase vector, $u(i)$ an m-dimensional vector of control influences, $\psi(i)$, $\Gamma(i)$ $(i = 0,1,\ldots,l-1)$ matrices of the corresponding dimensions, being a controllability pair [4, 6], Φ_1, Φ_2, Φ_3 are constant matrices, such that the system (2) satisfies the Kronecker-Capelli condition [3, 4], $0 < s < l$.

It is required to find such a control $u(i)$ as minimizes the following quadratic functional:

$$J = \sum_{i=0}^{l-1} \bigl(x'(i)Q(i)x(i) + u'(i)C(i)u(i) \bigr), \tag{3}$$

under the conditions (1), (2), where $Q(i) = Q'(i) \geq 0$, $C(i) = C'(i) \geq 0$ are the periodic matrices of the corresponding dimensions.

Let us illustrate this on the example from [4] in the one-dimensional case. Indeed, in the problem (23)-(25) from [1], let

$$\begin{aligned} & n = 1, \qquad m = 1, \qquad \psi(0) = \psi(1) = 1, \qquad \psi(2) = \psi(3) = 2, \\ & \Gamma(0) = \Gamma(1) = \Gamma(2) = \Gamma(3) = 1, \qquad \Phi_1 = \Phi_2 = \Phi_3 = 1, \qquad q = 1, \\ & Q(0) = Q(1) = Q(2) = Q(3) = 1, \qquad C(0) = C(1) = C(2) = C(3) = 1. \end{aligned} \tag{4}$$

Using the algorithm given in [1] we can see that the 'optimal' phase trajectory and control, respectively, have the form

$$\begin{aligned} & x(0) = \frac{6}{19}, \qquad x(1) = \frac{5}{19}, \qquad x(2) = \frac{4}{19}, \qquad x(3) = \frac{7}{19}, \qquad x(4) = \frac{9}{19}, \\ & u(0) = -\frac{1}{19}, \qquad u(1) = -\frac{1}{19}, \qquad u(2) = -\frac{1}{19}, \qquad u(3) = -\frac{5}{5}. \end{aligned}$$

Then it is easy to calculate [6, 7] that the 'optimal' value of the functional (25) of [1] will be $J \approx 0.8$.

However, the algorithm as given in [4, 6] gives other results, *i.e.*

$$\begin{aligned} & x(0) = \frac{5}{26}, \qquad x(1) = \frac{1}{26}, \qquad x(2) = \frac{1}{13}, \qquad x(3) = \frac{7}{26}, \qquad x(4) = \frac{23}{26}, \\ & u(0) = -\frac{2}{13}, \qquad u(1) = \frac{3}{26}, \qquad u(2) = \frac{11}{26}, \qquad u(3) = \frac{9}{26}, \end{aligned}$$

and the functional (25) of [1] takes the value

$$J \approx 0.5.$$

Thus, the above solution in [1] is not optimal.

Competing interests
The author declares that they have no competing interests.

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