Advances in Difference Equations
a SpringerOpen Journal

RESEARCH Open Access

Some properties of certain subclasses of analytic functions involving a differential operator

Loriana Andrei*

*Correspondence: lori_andrei@yahoo.com
Department of Mathematics and Computer Science, University of Oradea, 1 Universitatii street, Oradea, 410087, Romania

Abstract

In the present paper, we introduce and study certain subclasses of analytic functions in the open unit disk U which is defined by the differential operator $DR_{\lambda}^{m,n}$. We study and investigate some inclusion properties of these classes. Furthermore, a generalized Bernardi-Libera-Livington integral operator is shown to be preserved for these classes.

MSC: 30C45

Keywords: analytic functions; differential operator; differential subordination; differential superordination

1 Introduction

Let $\mathcal{A}$ be a class of functions f in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$. Thus each $f \in \mathcal{A}$ has a Taylor series representation

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j. \tag{1.1}$$

We denote by $\mathcal{S}(\xi)$ the well-known subclass of $\mathcal{A}$ consisting of all analytic functions which are, respectively, starlike of order ξ [1, 2]

$$\mathcal{S}(\xi) = \left\{ f \in \mathcal{A} : \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \xi, z \in U \right\}, \quad 0 \le \xi < 1.$$

Let $\mathcal{R}$ be a class of all functions ϕ which are analytic and univalent in U and for which $\phi(U)$ is convex with $\phi(0) = 1$ and $\operatorname{Re}\phi(z) > 0$, $z \in U$.

For two functions f and g analytic in U, we say that the function f is subordinate to g in U and write $f(z) \prec g(z)$, $z \in U$, if there exists a Schwarz function $w(z)$ which is analytic in U with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$, $z \in U$.

Making use of the principle of subordination between analytic functions, denote by $\mathcal{S}(\xi, \phi)$ [3] a subclass of the class $\mathcal{A}$ for $0 \le \xi < 1$ and $\phi \in \mathcal{R}$ which are defined by

$$\mathcal{S}(\xi, \phi) = \left\{ f \in \mathcal{A} : \frac{1}{1-\xi}\left(\frac{zf'(z)}{f(z)} - \zeta\right) \prec \phi(z), z \in U \right\}.$$

Let $f, g \in \mathcal{A}$, where f and g are defined by $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$ and $g(z) = z + \sum_{j=2}^{\infty} b_j z^j$. Then the Hadamard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) = z + \sum_{j=2}^{\infty} a_j b_j z^j.$$

Definition 1.1 (Al-Oboudi [4]) For $f \in \mathcal{A}$, $\lambda \geq 0$ and $m \in \mathbb{N}$, the operator D_λ^m is defined by $D_\lambda^m : \mathcal{A} \to \mathcal{A}$,

$$\begin{aligned}
&D_\lambda^0 f(z) = f(z),\\
&D_\lambda^1 f(z) = (1-\lambda) f(z) + \lambda z f'(z) = D_\lambda f(z),\\
&\ldots,\\
&D_\lambda^m f(z) = (1-\lambda) D_\lambda^{m-1} f(z) + \lambda z \bigl(D_\lambda^m f(z)\bigr)' = D_\lambda \bigl(D_\lambda^{m-1} f(z)\bigr), \quad z \in U.
\end{aligned}$$

Remark 1.1 If $f \in \mathcal{A}$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $D_\lambda^m f(z) = z + \sum_{j=2}^{\infty} [1 + (j-1)\lambda]^m a_j z^j$, $z \in U$.

Remark 1.2 For $\lambda = 1$ in the above definition, we obtain the Sălăgean differential operator [5].

Definition 1.2 (Ruscheweyh [6]) For $f \in \mathcal{A}$ and $n \in \mathbb{N}$, the operator R^n is defined by $R^n : \mathcal{A} \to \mathcal{A}$,

$$\begin{aligned}
&R^0 f(z) = f(z),\\
&R^1 f(z) = z f'(z),\\
&\ldots,\\
&(n+1) R^{n+1} f(z) = z\bigl(R^n f(z)\bigr)' + n R^n f(z), \quad z \in U.
\end{aligned}$$

Remark 1.3 If $f \in \mathcal{A}$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $R^n f(z) = z + \sum_{j=2}^{\infty} \frac{(n+j-1)!}{n!(j-1)!} a_j z^j$, $z \in U$.

Definition 1.3 ([7]) Let $\lambda \geq 0$ and $n, m \in \mathbb{N}$. Denote by $DR_\lambda^{m,n} : \mathcal{A} \to \mathcal{A}$ the operator given by the Hadamard product of the generalized Sălăgean operator D_λ^m and the Ruscheweyh operator R^n,

$$DR_\lambda^{m,n} f(z) = \bigl(D_\lambda^m * R^n\bigr) f(z),$$

for any $z \in U$ and each nonnegative integer m, n.

Remark 1.4 If $f \in \mathcal{A}$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $DR_\lambda^{m,n} f(z) = z + \sum_{j=2}^{\infty} [1 + (j-1)\lambda]^m \frac{(n+j-1)!}{n!(j-1)!} \times a_j^2 z^j$, $z \in U$.

Remark 1.5 The operator $DR_\lambda^{m,n}$ was studied also in [8–10].

For $\lambda = 1$, $m = n$, we obtain the Hadamard product SR^n [11] of the Sălăgean operator S^n and the Ruscheweyh derivative R^n, which was studied in [12, 13].

For $m = n$, we obtain the Hadamard product DR_λ^n [14] of the generalized Sălăgean operator D_λ^n and the Ruscheweyh derivative R^n, which was studied in [15–20].

Using a simple computation, one obtains the next result.

Proposition 1.1 ([7]) *For $m, n \in \mathbb{N}$ and $\lambda \geq 0$, we have*

$$DR_\lambda^{m+1,n} f(z) = (1-\lambda) DR_\lambda^{m,n} f(z) + \lambda z \left(DR_\lambda^{m,n} f(z)\right)' \tag{1.2}$$

and

$$z\left(DR_\lambda^{m,n} f(z)\right)' = (n+1) DR_\lambda^{m,n+1} f(z) - n DR_\lambda^{m,n} f(z). \tag{1.3}$$

By using the operator $DR_\lambda^{m,n} f(z)$, we define the following subclasses of analytic functions for $0 \leq \zeta < 1$ and $\phi \in \mathcal{R}$:

$$\mathcal{S}_\lambda^{m,n}(\xi) = \left\{ f \in \mathcal{A} : DR_\lambda^{m,n} f \in \mathcal{S}(\xi) \right\},$$
$$\mathcal{S}_\lambda^{m,n}(\xi, \phi) = \left\{ f \in \mathcal{A} : DR_\lambda^{m,n} f \in \mathcal{S}(\xi, \phi) \right\}.$$

In particular, we set

$$\mathcal{S}_\lambda^{m,n}\left(\xi, \frac{1+Az}{1+Bz}\right) = \mathcal{S}_\lambda^{m,n}(\xi, A, B), \quad -1 < B < A \leq 1.$$

Next, we will investigate various inclusion relationships for the subclasses of analytic functions introduced above. Furthermore, we study the results of Faisal *et al.* [21], Darus and Faisal [3].

2 Inclusion relationship associated with the operator $DR_\lambda^{m,n}$

First, we start with the following lemmas which we need for our main results.

Lemma 2.1 ([22, 23]) *Let $\varphi(\mu, \nu)$ be a complex function such that $\varphi : D \to \mathbb{C}$, $D \subseteq \mathbb{C} \times \mathbb{C}$, and let $\mu = \mu_1 + i\mu_2$, $\nu = \nu_1 + i\nu_2$. Suppose that $\varphi(\mu, \nu)$ satisfies the following conditions*:

1. *$\varphi(\mu, \nu)$ is continuous in D,*
2. *$(1, 0) \in D$ and $\operatorname{Re} \varphi(1, 0) > 0$,*
3. *$\operatorname{Re} \varphi(i\mu_2, \nu_1) \leq 0$ for all $(i\mu_2, \nu_1) \in D$ such that $\nu_1 \leq -\frac{1}{2}(1 + \mu_2^2)$.*

Let $h(z) = 1 + c_1 z + c_2 z^2 + \cdots$ be analytic in U, such that $(h(z), zh'(z)) \in D$ for all $z \in U$. If $\operatorname{Re}\{\varphi h(z), zh'(z)\} > 0$, $z \in U$, then $\operatorname{Re}\{h(z)\} > 0$.

Lemma 2.2 ([24]) *Let ϕ be convex univalent in U with $\phi(0) = 1$ and $\operatorname{Re}\{k\phi(z) + \nu\} > 0$, $k, \nu \in \mathbb{C}$. If p is analytic in U with $p(0) = 1$, then*

$$p(z) + \frac{zp'(z)}{kp(z) + \nu} \prec \phi(z), \quad z \in U,$$

implies $p(z) \prec \phi(z)$, $z \in U$.

Theorem 2.1 *Let $f \in \mathcal{A}$, $0 \leq \xi < 1$, $m, n \in \mathbb{N}$, $\lambda > 0$, then*

$$\mathcal{S}_{\lambda}^{m,n+1}(\xi) \subseteq \mathcal{S}_{\lambda}^{m,n}(\xi) \subseteq \mathcal{S}_{\lambda}^{m,n-1}(\xi).$$

Proof Let $f \in \mathcal{S}_{\lambda}^{m,n+1}(\xi)$ and suppose that

$$\frac{z(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)} = \xi + (1-\xi)h(z). \tag{2.1}$$

Since from (1.3)

$$(n+1)\frac{DR_{\lambda}^{m,n+1}f(z)}{DR_{\lambda}^{m,n}f(z)} = n + \xi + (1-\xi)h(z),$$

we obtain

$$\begin{aligned}
(1-\xi)h'(z) &= (n+1)\left[\frac{(DR_{\lambda}^{m,n+1}f(z))'}{DR_{\lambda}^{m,n}f(z)} - \frac{DR_{\lambda}^{m,n+1}f(z)}{DR_{\lambda}^{m,n}f(z)} \cdot \frac{(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)}\right], \\
(1-\xi)zh'(z) &= (n+1)\frac{DR_{\lambda}^{m,n+1}f(z)}{DR_{\lambda}^{m,n}f(z)}\left[\frac{z(DR_{\lambda}^{m,n+1}f(z))'}{DR_{\lambda}^{m,n+1}f(z)} - \xi - (1-\xi)h(z)\right], \\
\frac{(1-\xi)h'(z)z}{n+\xi+(1-\xi)h(z)} &= \frac{z(DR_{\lambda}^{m,n+1}f(z))'}{DR_{\lambda}^{m,n+1}f(z)} - \xi - (1-\xi)h(z), \\
\frac{z(DR_{\lambda}^{m,n+1}f(z))'}{DR_{\lambda}^{m,n+1}f(z)} - \xi &= (1-\xi)h(z) + \frac{(1-\xi)h'(z)z}{n+\xi+(1-\xi)h(z)}.
\end{aligned}$$

Taking $h(z) = \mu = \mu_1 + i\mu_2$ and $zh'(z) = \nu = \nu_1 + i\nu_2$, we define $\varphi(\mu,\nu)$ by

$$\varphi(\mu,\nu) = (1-\xi)\mu + \frac{(1-\xi)\nu}{n+\xi+(1-\xi)\mu}$$

and

$$\begin{aligned}
\operatorname{Re}\{\varphi(i\mu_2,\nu_1)\} &= \frac{(1-\xi)(n+\xi)\nu_1}{(n+\xi)^2 + (1-\xi)^2\mu_2^2}, \\
\operatorname{Re}\{\varphi(i\mu_2,\nu_1)\} &\leq -\frac{(1-\xi)(n+\xi)(1+\mu_2^2)}{2[(n+\xi)^2 + (1-\xi)^2\mu_2^2]} < 0.
\end{aligned}$$

Clearly, $\varphi(\mu,\nu)$ satisfies the conditions of Lemma 2.1. Hence $\operatorname{Re}\{h(z)\} > 0$, $z \in U$, implies $f \in \mathcal{S}_{\lambda}^{m,n}(\xi)$. □

Remark 2.1 Using relation (1.2) and the same techniques as to prove the earlier results, we can obtain a new similar result.

Theorem 2.2 *Let $f \in \mathcal{A}$ and $\phi \in \mathcal{R}$ with*

$$\operatorname{Re}\{\phi(z)\} < \frac{\xi - 1 + \frac{1}{\lambda}}{1-\xi}.$$

Then

$$\mathcal{S}_{\lambda}^{m+1,n}(\xi,\phi) \subset \mathcal{S}_{\lambda}^{m,n}(\xi,\phi) \subset \mathcal{S}_{\lambda}^{m-1,n}(\xi,\phi).$$

Proof Let $f(z) \in \mathcal{S}_{\lambda}^{m+1,n}(\xi,\phi)$ and set

$$p(z) = \frac{1}{1-\xi}\left(\frac{z(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)} - \xi\right), \tag{2.2}$$

where p is analytic in U with $p(0) = 1$.

By using (1.2) we have

$$\frac{z(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)} = \frac{1}{\lambda}\frac{DR_{\lambda}^{m+1,n}f(z)}{DR_{\lambda}^{m,n}f(z)} - \frac{1-\lambda}{\lambda}.$$

Now, by using (2.2) we get

$$p'(z) = \frac{1}{1-\xi}\left(\frac{1}{\lambda}\frac{DR_{\lambda}^{m+1,n}f(z)}{DR_{\lambda}^{m,n}f(z)} - \frac{1-\lambda}{\lambda} - \xi\right),$$

$$\frac{1}{\lambda}\frac{DR_{\lambda}^{m+1,n}f(z)}{DR_{\lambda}^{m,n}f(z)} = \xi + \frac{1-\lambda}{\lambda} + (1-\xi)p(z). \tag{2.3}$$

By using (2.2) and (2.3), we obtain

$$zp'(z) = \frac{1}{1-\xi}\cdot\frac{1}{\lambda}\left[\frac{z(DR_{\lambda}^{m+1,n}f(z))'}{DR_{\lambda}^{m,n}f(z)} - \frac{DR_{\lambda}^{m+1,n}f(z)}{DR_{\lambda}^{m,n}f(z)}\cdot\frac{z(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)}\right],$$

$$(1-\xi)zp'(z) = \frac{1}{\lambda}\cdot\frac{DR_{\lambda}^{m+1,n}f(z)}{DR_{\lambda}^{m,n}f(z)}\left[\frac{z(DR_{\lambda}^{m+1,n}f(z))'}{DR_{\lambda}^{m+1,n}f(z)} - \frac{z(DR_{\lambda}^{m,n}f(z))'}{DR_{\lambda}^{m,n}f(z)}\right],$$

$$(1-\xi)zp'(z) = \left[\zeta - 1 + \frac{1}{\lambda} + (1-\xi)p(z)\right]\left[\frac{z(DR_{\lambda}^{m+1,n}f(z))'}{DR_{\lambda}^{m+1,n}f(z)} - (1-\xi)p(z) - \xi\right],$$

$$\frac{(1-\xi)zp'(z)}{(1-\xi)p(z) + \zeta - 1 + \frac{1}{\lambda}} = \frac{z(DR_{\lambda}^{m+1,n}f(z))'}{DR_{\lambda}^{m+1,n}f(z)} - \xi - (1-\xi)p(z).$$

Hence,

$$\frac{1}{1-\xi}\left[\frac{z(DR_{\lambda}^{m+1,n}f(z))'}{DR_{\lambda}^{m+1,n}f(z)} - \xi\right] = p(z) + \frac{zp'(z)}{(1-\zeta)p(z) + \zeta - 1 + \frac{1}{\lambda}}. \tag{2.4}$$

Since $\mathrm{Re}\{\phi(z)\} < \frac{\xi-1+\frac{1}{\lambda}}{1-\xi}$ implies $\mathrm{Re}\{(1-\xi)p(z) + \xi - 1 + \frac{1}{\lambda}\} > 0$, applying Lemma 2.2 to (2.4) we have that $f(z) \in \mathcal{S}_{\lambda}^{m,n}(\xi,\phi)$, as required. □

Remark 2.2 By using relation (1.3) and the same techniques as to prove the earlier results, we can obtain a new similar result.

Corollary 2.3 *Let* $\frac{1+A}{1+B} < \frac{\xi-1+\frac{1}{\lambda}}{1-\xi}$ *for* $-1 < B < A \leq 1$, *then*

$$\mathcal{S}_{\lambda}^{m+1,n}(\xi,A,B) \subset \mathcal{S}_{\lambda}^{m,n}(\xi,A,B) \subset \mathcal{S}_{\lambda}^{m-1,n}(\xi,A,B).$$

Proof Taking $\phi(z) = \frac{1+Az}{1+Bz}$, $-1 < B < A \leq 1$ in Theorem 2.2, we get the corollary. □

3 Integral-preserving properties

In this section, we present several integral-preserving properties for the subclasses of analytic functions defined above. We recall the generalized Bernardi-Libera-Livington integral operator [25] defined by

$$F_c\big[f(z)\big] = \frac{c+1}{z^c}\int_0^z t^{c-1}f(t)\,dt = z + \sum_{j=2}^{\infty}\frac{c+1}{j+c}a_j z^c, \quad f \in \mathcal{A}, c > -1, \tag{3.1}$$

which satisfies the following equality:

$$cDR_\lambda^{m,n}F_c\big[f(z)\big] + z\big[DR_\lambda^{m,n}F_c\big(f(z)\big)\big]' = (c+1)DR_\lambda^{m,n}f(z). \tag{3.2}$$

Theorem 3.1 *Let $c > -1$, $0 \leq \xi < 1$. If $f \in \mathcal{S}_\lambda^{m,n}(\xi)$, then $F_c f \in \mathcal{S}_\lambda^{m,n}(\xi)$.*

Proof Let $f \in \mathcal{S}_\lambda^{m,n}(\xi)$. By using (3.2), we get

$$\frac{z[DR_\lambda^{m,n}F_c[f(z)]]'}{DR_\lambda^{m,n}F_c[f(z)]} = (c+1)\frac{DR_\lambda^{m,n}f(z)}{DR_\lambda^{m,n}F_c[f(z)]} - c.$$

Let

$$\frac{z[DR_\lambda^{m,n}F_c[f(z)]]'}{DR_\lambda^{m,n}F_c[f(z)]} = \xi + (1-\xi)h(z), \quad h(z) = 1 + c_1 z + c_2 z^2 + \cdots.$$

We obtain

$$\frac{z[DR_\lambda^{m,n}f(z)]'}{DR_\lambda^{m,n}f(z)} - \xi = (1-\xi)h(z) + \frac{(1-\xi)zh'(z)}{\xi + (1-\xi)h(z) + c}.$$

This implies

$$\varphi(\mu,\nu) = (1-\xi)\mu + \frac{(1-\xi)\nu}{c + \xi + (1-\xi)\mu}$$

(same as Theorem 2.1) and

$$\operatorname{Re}\big\{\varphi(i\mu_2,\nu_1)\big\} = \frac{(1-\xi)(c+\xi)\nu_1}{(c+\xi)^2 + (1-\xi)^2\mu_2^2},$$

$$\operatorname{Re}\big\{\varphi(i\mu_2,\nu_1)\big\} \leq -\frac{(1-\xi)(c+\xi)(1+\mu_2)^2}{2[(c+\xi)^2 + (1-\xi)^2\mu_2^2]} < 0.$$

After using Lemma 2.1 and Theorem 2.1, we have

$$F_c f \in \mathcal{S}_\lambda^{m,n}(\xi).$$ □

Theorem 3.2 *Let $c > -1$ and $\phi \in \mathcal{R}$ with*

$$\operatorname{Re}\big\{\phi(z)\big\} < \frac{c+\xi}{1-\xi}.$$

If $f \in \mathcal{S}_\lambda^{m,n}(\xi,\phi)$, then $F_c f \in \mathcal{S}_\lambda^{m,n}(\xi,\phi)$.

Proof Let $f(z) \in \mathcal{S}_{\lambda}^{m,n}(\xi,\phi)$ and set

$$p(z) = \frac{1}{1-\xi}\left(\frac{z[DR_{\lambda}^{m,n}F_c[f(z)]]'}{DR_{\lambda}^{m,n}F_c[f(z)]} - \xi\right), \tag{3.3}$$

where p is analytic in U with $p(0) = 1$.

Using (3.2) and (3.3), we have

$$(c+1)\frac{z[DR_{\lambda}^{m,n}f(z)]}{DR_{\lambda}^{m,n}F_c[f(z)]} = c + \xi + (1-\xi)p(z). \tag{3.4}$$

Then, using (3.2), (3.3) and (3.4), we obtain

$$\frac{1}{1-\xi}\left(\frac{z[DR_{\lambda}^{m,n}f(z)]'}{DR_{\lambda}^{m,n}f(z)} - \xi\right) = p(z) + \frac{zp'(z)}{(1-\xi)p(z)+c+\xi}. \tag{3.5}$$

Applying Lemma 2.2 to (3.5), we conclude that

$$F_c f \in \mathcal{S}_{\lambda}^{m,n}(\xi,\phi). \qquad \square$$

Competing interests
The author declares that she has no competing interests.

Author's contributions
The author drafted the manuscript, read and approved the final manuscript.

Acknowledgements
The author thanks the referee for his/her valuable suggestions to improve the present article.

Received: 16 March 2014 Accepted: 9 April 2014 Published: #PUBLICATION_DATE

References
1. Kumar, V, Shukla, SL: Certain integrals for classes of *p*-valent meromorphic functions. Bull. Aust. Math. Soc. **25**, 85-97 (1982)
2. Miller, SS, Mocanu, PT: Differential Subordination. Dekker, New York (2000)
3. Darus, M, Faisal, I: Inclusion properties of certain subclasses of analytic functions. Rev. Notas Mat. **7(1)**(305), 66-75 (2011)
4. Al-Oboudi, FM: On univalent functions defined by a generalized Sălăgean operator. Int. J. Math. Math. Sci. **27**, 1429-1436 (2004)
5. Sălăgean, GS: Subclasses of univalent functions. In: Complex Analysis - Fifth Romanian-Finnish Seminar. Lecture Notes in Math., vol. 1013, pp. 362-372. Springer, Berlin (1983)
6. Ruscheweyh, S: New criteria for univalent functions. Proc. Am. Math. Soc. **49**, 109-115 (1975)
7. Andrei, L: Differential sandwich theorems using a generalized Sălăgean operator and Ruscheweyh operator. Didact. Math. (submitted)
8. Andrei, L: On some differential sandwich theorems using a generalized Sălăgean operator and Ruscheweyh operator. J. Comput. Anal. Appl. **18** (2015, to appear)
9. Andrei, L: Certain differential sandwich theorem using a generalized Sălăgean operator and Ruscheweyh operator. Adv. Appl. Math. Sci. (submitted)
10. Andrei, L: Differential subordinations, superordinations and sandwich theorems using a generalized Sălăgean operator and Ruscheweyh operator. Rev. Unión Mat. Argent. (submitted)
11. Alb Lupas, A: Certain differential subordinations using Sălăgean and Ruscheweyh operators. Acta Univ. Apulensis **29**, 125-129 (2012)
12. Alb Lupas, A: A note on differential subordinations using Sălăgean and Ruscheweyh operators. ROMAI J. **6**(1), 1-4 (2010)
13. Alb Lupas, A: Certain differential superordinations using Sălăgean and Ruscheweyh operators. An. Univ. Oradea, Fasc. Mat. **XVII**(2), 209-216 (2010)
14. Alb Lupas, A: Certain differential subordinations using a generalized Sălăgean operator and Ruscheweyh operator I. J. Math. Appl. **33**, 67-72 (2010)
15. Alb Lupas, A: Certain differential subordinations using a generalized Sălăgean operator and Ruscheweyh operator II. Fract. Calc. Appl. Anal. **13**(4), 355-360 (2010)
16. Alb Lupas, A: Certain differential superordinations using a generalized Sălăgean and Ruscheweyh operators. Acta Univ. Apulensis **25**, 31-40 (2011)

17. Andrei, L: Differential subordination results using a generalized Sălăgean operator and Ruscheweyh operator. Acta Univ. Apulensis **37**(2) (2014)
18. Andrei, L: Some differential subordination results using a generalized Sălăgean operator and Ruscheweyh operator. Jökull **64**(4) (2014)
19. Andrei, L: Differential superordination results using a generalized Sălăgean operator and Ruscheweyh operator. An. Univ. Oradea, Fasc. Mat. **XXI**(2) (2014, to appear)
20. Andrei, L: Some differential superordination results using a generalized Sălăgean operator and Ruscheweyh operator. Stud. Univ. Babeş-Bolyai, Math. (to appear)
21. Faisal, I, Shareef, Z, Darus, M: On certain subclasses of analytic functions. Stud. Univ. Babeş-Bolyai, Math. **58**(1), 9-14 (2013)
22. Miller, SS: Differential inequalities and Carathéordory function. Bull. Am. Math. Soc. **8**, 79-81 (1975)
23. Miller, SS, Mocanu, PT: Second order differential inequalities in the complex plane. J. Math. Anal. Appl. **65**, 289-305 (1978)
24. Eenigenberg, P, Miller, SS, Mocanu, PT, Reade, MO: On a Briot-Bouquet differential subordination. In: General Inequalities, vol. 3, pp. 339-348 (1983)
25. Bernardi, SD: Convex and starlike univalent functions. Trans. Am. Math. Soc. **135**, 429-446 (1969)

#DIGITAL_OBJECT_IDENTIFIER
Cite this article as: Andrei: Some properties of certain subclasses of analytic functions involving a differential operator. *Advances in Difference Equations* #CITATION