

APPLICATIONS OF FOURIER SERIES IN ELECTRIC CIRCUIT AND COMMUNICATION SYSTEM

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ABSTRACT

In this paper, we analysis of square wave in terms of Fourier component, may occur in electric circuits designed to handle sharply rising pulses and how to convert analog to digital system by using Fourier series and its application in electronics and communication system

KEYWORDS

Fourier Transform (FT), Electric circuit, Frequency, Digital signal process.

1. Introduction

Mathematics is everywhere in every phenomena, technology, observation, experiment etc. All we need to do is to understand the logic hidden behind. In this article we are focusing on application of Fourier series in electric circuit and communication system. FT is name in the *honour* of Joseph Fourier (1749-1829), one of the greatest name in the history of mathematics and physics. Mathematically speaking, the Fourier transform is a linear operator that maps a functional space to another function space and decomposes a function into another function of its frequency components. In particular, the fields of electronics, quantum mechanics, and electrodynamics all mark heavy use to Fourier series. Additionally, other methods based on the Fourier seers, such that as the FFT (Fast Fourier Transform a from Discrete Fourier Transform [DFT]), are particularly useful for the fields of Digital signal Processing (DSP) and Multi-channel sound analysis. In this Study, *Mathematica* 9.0 is use to sketch all figure

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2. Methodology

Let $f(x)$ is a real-valued function of period 2π and it's a finite sum of harmonically related sinusoids, mathematically the expression for a Fourier series is

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \quad \text{where, } -\pi \leq t \leq \pi \quad (1)$$

$$\text{Where } a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt$$

2.1 Definition of F.T

If $f(t)$ is piece-wise continuous and absolutely *integrable* in $(-\infty, \infty)$, then the F.T of $f(t)$ is denoted by $f(\phi)$ and is define by

$$f[f(t)] = f(\phi) = \int_{-\infty}^{\infty} f(t) e^{i\phi t} dt \quad (2)$$

And inverse Fourier transformation (IFT) is given by $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\phi) e^{-i\phi t} d\phi \quad (3)$

2.2 Digital Signal Processing

One of the most common applications of the IFFT in laboratory is to provide Digital signal processing (DSP). In general, the idea of DSP is to use configurable digital electronics to clean up, transform, or amplify a signal by *FFT'ing* the signal, removing, shifting or Damping the unwanted frequency components, and then transforming the signal back using the IFFT on the filtered signals.

There are many advantages to doing DSP as opposed to doing analog signal processing. To begin with, practically speaking, we can have a much more complicated filtering function (the function that transforms the coefficients of the DFT) with DSP than analog signal processing. While it is fairly easy to make a single band pass, low pass, or high pass filter with capacitors, resistors, and inductors, it is relatively difficult and time consuming to implement anything more complicated than these three simple filters. Furthermore, even if a more complicated filter was implemented with analog electronics, it is difficult to make even small modification to the filter (there are exceptions to this, such as FPGA's, but those are also more difficult to implement than sample software solution). DSP is not limited by either of those effects since the processing is (usually) done in software, which can be programmed to do whatever the user desires. Probably the most important advantage that DSP has over analog signal processing is the fact that the processing may be done after the signal has been taken.

In modern-day experiments, raw data is often recorded during the experiment and corrected for noise in software during the analysis step. If one filters the signal beforehand (with analog signal processing), it is possible that the later in the experiment, the experimenter could find that they filtered out good signals. The only option in this case is for experiment to be return. On the other hand, if the signal processing was done digitally, all that the experimenter has to do is edit their analysis code and rerun the analysis; this could save both time and money.

2.3 Preliminary

- a) Communication: Fourier transform is essential to understand how signals behave when they pass through filters, amplifiers and communication multi-channel.
- b) Data analysis: Fourier transform can be used as high-pass, low-pass, and band-pass filters and it can also be applied to signals and noise estimation by encoding the time series.
- c) Cell phone: Every mobile device net-book, notebook, tablet and phone have been built in high speed cellular data connection, just like Fourier transform because sound may be represented by a complex combination of their waves. Human very easily perform Fourier transform mechanically every day. For example when we are in room with great deal of noise and you selectively hear our name above a noise, we have just performed Fourier series.
- d) Input transducer: The device that converts a physical signal from source to an electrical, mechanical, electronic signal more suitable for communicating.
- e) Transmitter: The device that sends the transducer signal
- f) Transmission channel: The physical medium on which the signals is carried.
- g) Receiver: The device that recover the transmitted signal from the channel.
- h) Output transducer: The device that converts the received signal back into useful quantity.

2.4 Pairs of frequency used in dual-tone multi-frequency Digital signal.

In above discussion, let us consider a list of set 12 channel frequency is
Frequency={ {697,1209},{697,1336},{697,1477},{770,1209},{770,1336},{770,1477},{852,1209},{852,1336},{852,1477},{941,1209},{941,1336},{941,1477}}. Now we calculate this data by *mathematica* 9.0 we have the following Digital signal processing figure .

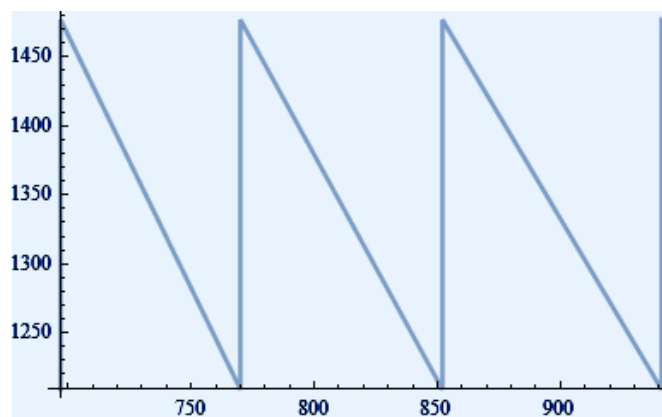


Figure 2.2.1: In a frame of frequency set of data.

	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

Figure 2.2.2: Create pairs of low and high frequencies for each key

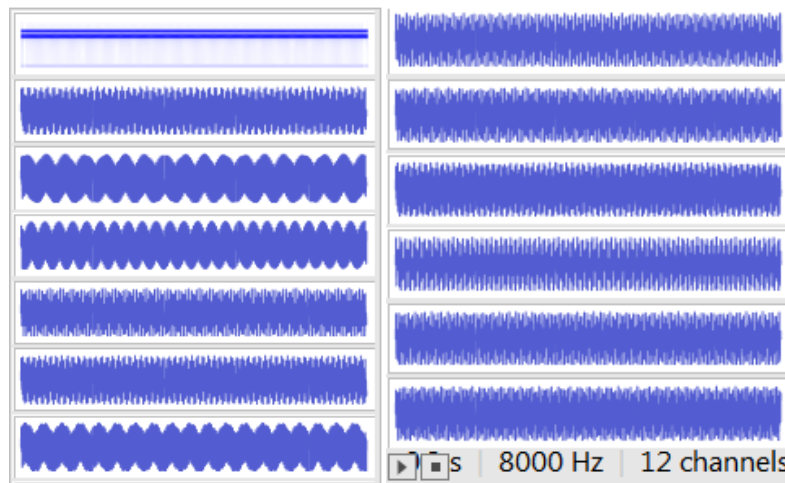


Figure 2.2.3: The physical medium on which the 12 channels digital signals.

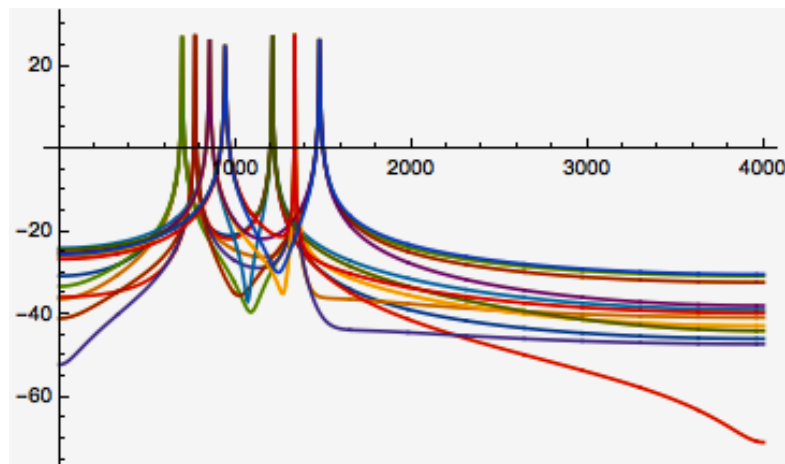


Figure 2.2.4: Visualize the periodogram of the create sound for 12 channel

3. 1 Example of square wave rectifier

An example of the above discussion, we consider a square wave by the function $f(x)$ such that

$$f(x) = \begin{cases} -0, & -\pi < t < 0 \\ h, & 0 < t < \pi \end{cases} \quad (4)$$

Comparing equation (1) with equation (4), we obtain

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_0^\pi h \, dt \\ &= \frac{h}{2} [t]_0^\pi \\ &= \frac{h}{2} \end{aligned}$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_0^\pi h \cos nt \, dt \\ &= \frac{h}{n\pi} [\sin nt]_0^\pi \\ &= \frac{h}{n\pi} [\sin n\pi - \sin 0] \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{And } b_n &= \frac{1}{\pi} \int_0^\pi h \sin nt \, dt \\ &= -\frac{h}{n\pi} [\cos nt]_0^\pi \\ &= -\frac{h}{n\pi} [\cos n\pi - \cos 0] \\ &= \frac{h}{n\pi} [1 - \cos n\pi] \\ &= \frac{h}{n\pi} [1 - (-1)^n] \end{aligned}$$

Now putting the value of a_0 , a_n and b_n in (1), we get

$$\begin{aligned} f(t) &= \frac{h}{2} + \sum_{n=1}^{\infty} \left[0 + \frac{h}{n\pi} \{1 - (-1)^n\} \sin nt \right] \\ &= \left[\frac{h}{2} + \frac{2h}{\pi} \left[\frac{\sin t}{1} + \frac{\sin 3t}{3} + \frac{\sin 5t}{5} + \dots \right] \right] \end{aligned}$$

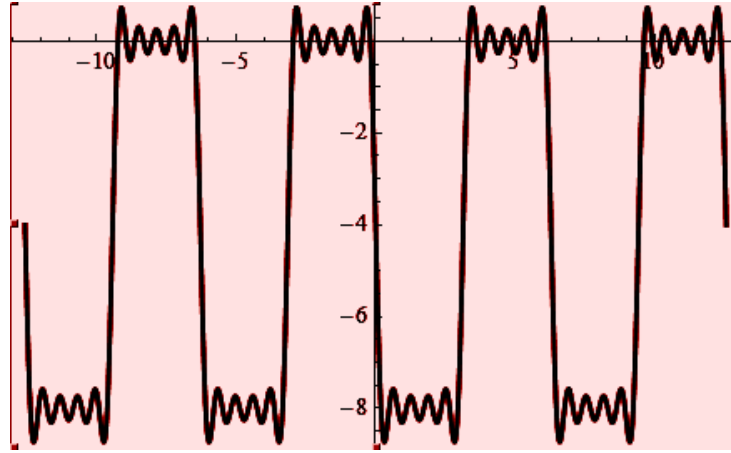


Figure3.1.1: upper part of square wave rectifier.

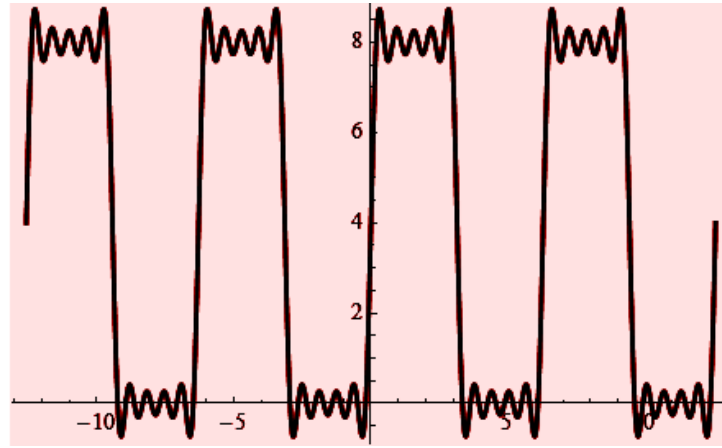


Figure3.1.1: lower part of square wave rectifier.

Showing that the square wave rectifier does a fairly good job of approximating the Alternative current (AC).

3. 2 Example of analog to digital conversion by using Fourier series

Let us consider the Fourier series of the following periodic function is

$$f(x) = \begin{cases} -\frac{h}{2}, & 0 < t < \pi \\ \frac{h}{2}, & \pi < t < 2\pi \end{cases} \quad (5)$$

Comparing equation (1) with equation (5), we obtain

$$\begin{aligned} a_0 &= \frac{1}{2\pi} \int_0^{2\pi} h \, dt \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} f(t) \, dt - \int_{\pi}^{2\pi} f(t) \, dt \right] \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} \frac{h}{2} \, dt - \int_{\pi}^{2\pi} \frac{h}{2} \, dt \right] \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2\pi} \left[\int_0^{\pi} \frac{h}{2} dt - \int_{\pi}^{2\pi} \frac{h}{2} dt \right] \\
 &= \frac{1}{2\pi} \left[\frac{\pi h}{2} - \pi h + \frac{\pi h}{2} \right] \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 a_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \cos nt \, dt \\
 &= \frac{1}{\pi} \left[\int_0^{\pi} f(t) dt - \int_{\pi}^{2\pi} f(t) dt \right] \\
 &= \frac{1}{\pi} \left[\int_0^{\pi} \frac{h}{2} \cos nt \, dt - \int_{\pi}^{2\pi} \frac{h}{2} \cos nt \, dt \right] \\
 &= \frac{1}{\pi} \left[\frac{h}{2} \frac{\sin nt}{n} \right]_0^{\pi} - \left[\frac{h}{2} \frac{\sin nt}{n} \right]_{\pi}^{2\pi} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin nt \, dt \\
 &= \frac{1}{\pi} \left[\int_0^{\pi} f(t) dt - \int_{\pi}^{2\pi} f(t) dt \right] \\
 &= \frac{1}{\pi} \left[\int_0^{\pi} \frac{h}{2} \sin nt \, dt - \int_{\pi}^{2\pi} \frac{h}{2} \sin nt \, dt \right] \\
 &= \frac{1}{\pi} \left[\frac{h}{2} \frac{\cos nt}{n} \right]_0^{\pi} - \left[\frac{h}{2} \frac{\cos nt}{n} \right]_{\pi}^{2\pi} \\
 &= \frac{-h(\cos[n\pi] - \cos[2n\pi] - 2\sin[\frac{n\pi}{2}]^2)}{2n\pi}
 \end{aligned}$$

Now putting the value of a_0 , a_n and b_n in (1), we get

$$f(t) = \left[\frac{2h \sin t}{\pi} + \frac{2h \sin 3t}{3\pi} + \frac{2h \sin 5t}{5\pi} + \frac{2h \sin 7t}{7\pi} + \dots \right]$$

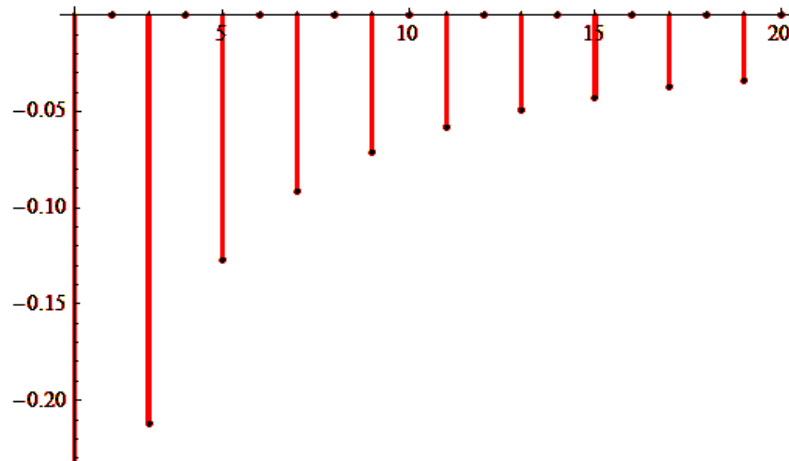


Figure 3.2.1: Discrete sketch of analog to Digital signal process.

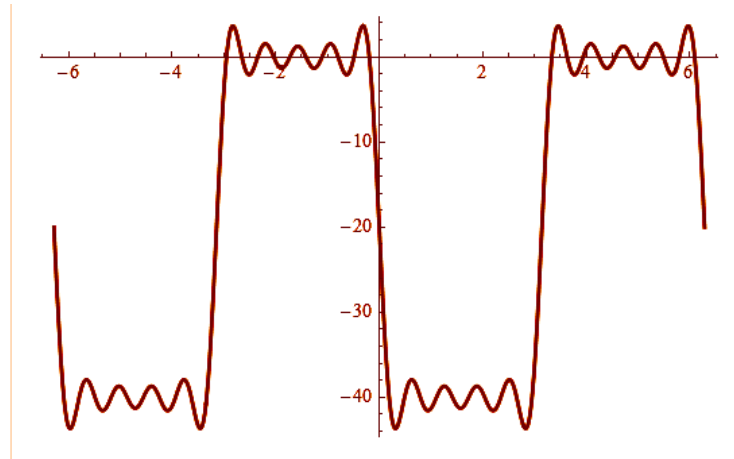


Figure3.2.1: upper part of square wave rectifier

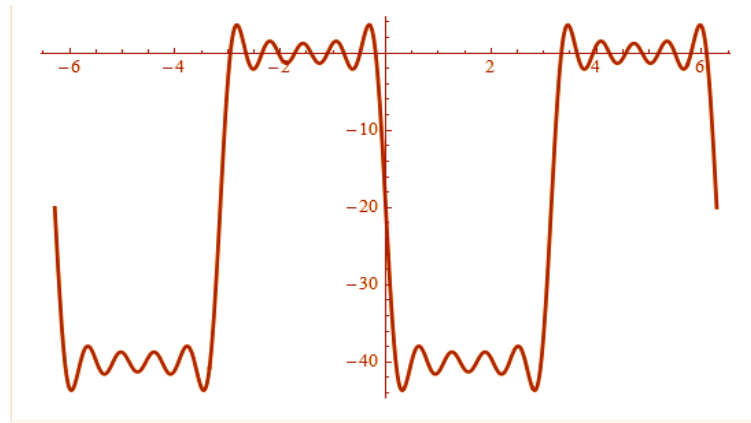


Figure3.2.1: lower part of square wave rectifier.

Thus the graph indicate that the series is convergent and has sum of $f(t)$.

4. Conclusion

Fourier series is useful in many applications ranging from experimental instruments to rigorous mathematical analysis techniques. Thanks to modern developments in digital electronics and telecommunication. In this paper a brief overview of Fourier transform. The primary use of Fourier transform of converting a time domain function into its frequency domain with digital communication signals. And we have seen Fourier converts signals from analog to digital signal, and how to square wave rectifier use in electricity. Fourier method are commonly used for signal analysis and system design in modern electronics, information and communication technology. Like as cell phone, radio, television and vibration analysis.

5. Reference

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