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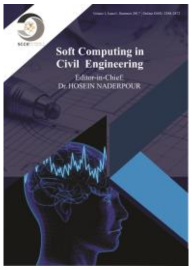

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Reliability Analysis of Structures Using Modified FA_PSO Algorithm

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ABSTRACT

Designing buildings with a very high safety factor is one of the main purposes of a civil engineer. Since in the structural design process, there are several no-confidence; we cannot achieve a perfect safe design. In these cases, we face amount of the probability of failure. So the theory of reliability used to assess the uncertainty. This theoretical for expression the safety of a system uses the reliability index, so it can be said that the calculation of reliability index is an important part of the theory. By the theory of structural reliability, uncertainties arising from the nature of the statistical parameters can be written mathematical equations and considerations of safety and performance of the structure into the design process. Since classical methods are not capable of solving complex functions, metaheuristic algorithms used. In fact, a metaheuristic algorithm is a set of concepts, which significantly able to solve many complex issues, which they can reach an optimal solution in a short time. In this paper, the particle swarm algorithm combined with Firefly and to assess the reliability theory has been used. Reliability index is calculated by searching the shortest distance between the origin and the closed point of Limit State Surface in the Standard normalized space.Mathematical and engineering studies on the issues indicated; Hybrid Firefly and particle swarm algorithm are with great accuracy and speed.

1. Introduction

Reliability theory is the science of estimating the probability of system failure due to uncertainty in design parameters. The definition of reliability varies with the context, but according to the most commonly used definition, reliability is the probability that a component will function properly for a specified period of time under specified conditions. The performance of a building can be described as a function of its safety, serviceability and cost-effectiveness. However, the information available about input variables such as these is rarely, accurate, complete and definite. With such uncertainties, it is impossible to acquire an absolute estimation about building safety. The structural systems reliability theory allow us to use statistical concepts to formulate the uncertainty of parameters and complete the design with safety and performance considerations accounted for quantitatively. According to this theory, one should carefully study not only the probability of failure in each single element, but also the probability of failure of the entire structural system as a whole. To do so, after ensuring the safety of every single element, one needs to examine the combined effect of their interactions on the system safety.

Engineering problems are typically associated with a wide range of uncertainties. The nature of these uncertainties varies with the nature of the problem and the type of analysis. The most common types of uncertainty that engineers encounter during structural design and construction include physical uncertainty, uncertainty in modeling, and uncertainty in statistics. These uncertainties are particularly important for metal structures such as bridges, tanks, and spaceframe structures, as uncertainties in the features of the site, nature of the applied loads, and durability of the materials are more likely to lead to their failure or unserviceability. Structural design should be able to account for a variety of uncertainties in -for example- the loads, strengths, and geometric properties. This issue can be viewed as a statistical problem subject to multiple uncertainties, which can be analyzed based on the principles of probability and reliability theories. Such reliability analysis assigns each structural member with a certain probability of failure, which allows us to evaluate its functionality or non-functionality and decide when it should be repaired or replaced accordingly [1].

A question that regularly arises in regard to existing structures -even those that are in operation for a long time- is that how safe this building is? Clearly, this question is completely different from the question that a designer must answer when planning a new building. Evidence also suggests that checking the safety uncertainties of an existing structure with the building codes by which it has been designed may not be helpful. This is because design codes and regulation have been devised such as to account for certain uncertainties in the design and construction, but for a built structure, these uncertainties are all identified (i.e. completely determined) and can no longer be considered uncertainty. Also, determining the actual value of structural parameters of an existing building is never easy and can have its own uncertainties [2].

The uncertainty in the safety of structures can be addressed more logically with the help of probabilistic methods. Naturally, the main objective of structural safety assessments and undertakings is to ensure the proper functionality of the building over its useful life. Some of the parameters involved in this assessments are not constant but rather a sample from a set of probable values. In other words, these parameters can be considered as stochastic variables. Basically, the variable nature of these parameters is not completely known, and it is suggested that safety factors should be employed to account for their variations. Structural reliability analysis is a method for achieving this objective [3].

Progress and development of the reliability theory has about 80 years of history consisting of multiple periods. The first period from 1920 to 1960 was the era of gradual emergence of the structural reliability theory. In 1924, Forsell described the optimal design as a design that minimizes the sum of expected costs, including the cost of initial construction and the expected cost of failure [4].

In 1947, the principles of structural safety of members under random loads were introduced, and this provided a foundation for future works of structural engineers [5]. In 1953, Johnson published the first comprehensive formulation of structural reliability and economic design theory, including the statistical theories of strength [6]. In 1974, Hasofer & Lind introduced an efficient method for determination of reliability index through a search for the design point, that is, the point of minimum distance from the limit state function to the origin of the standard normal coordinate system [7]. In 1991, reliability problems were analyzed with the help of gradient method [8]. Then, Melchers suggested a probabilistic model for assessing the reliability of steel structures against marine corrosion [9]. In 2007, a series of studies investigated the utility of genetic algorithms for structural reliability analysis by solving complex reliability functions [10]. In 2011, a study on the reliability of wooden structures with ductile behavior introduced a new evaluation method for this purpose [11]. In 2013, Jahani et al. developed an extremely effective sampling methodology for solving the reliability problems [12]. In the same year, a method was proposed for estimating the reliability of RC beams against fire [13]. Then, in a study by Xiao et al., they proposed an efficient and effective method based on truncated random variables for estimating the reliability of structural systems [14]. In 2015, several studies investigated the utility of artificial neural network modeling in the reliability analysis of steel structures [15]. Recently, a new generation of the reliability index has been defined by Ghasemi and Nowak (2016-2017). The advantage of Ghasemi-Nowak formula is to measure the safety level of a non-normal limit state function using the simplified convolution concepts [16, 17]. For evaluation of the required reliability index of a structures, there is a need for expression of the optimum safety level, which is known as a target reliability. Ghasemi and Nowak (2017) and Yanaka et al. (2016) proposed several approaches to estimate the target reliability for bridges with respect to the minimization of the cost [18, 19].

2. Structural reliability

Reliability relationship is defined based on two major parameters: structural strength and load. Accordingly, the limit state function is written as follows [3]:

$$G(R, Q) = R - Q \tag{1}$$

Where $G(R, Q)$ is the limit state function of structural strength and load. In the above equation, strength function R and load function Q both consists of several random variables with different probability distribution functions that depend on the structural dimensions, the type of materials, and the loads exerted on the structure.

In the reliability analysis, the failure is expressed by the concept of limit state function, which represents the extent of functionality. A limit state function that is equal to zero ($g(x) = 0$) represents a state that lies on the boundary of functionality (safety) and dysfunctionality (unsafety). Accordingly, $g(x) > 0$ has been defined as the safety region and $g(x) < 0$ as the unsafety region.

2.1. Classification of Reliability Methods

There are various methods for idealizing the models of structural reliability, and various ways to combine the idealized models for a design problem. Given this high degree of diversity, these methods have been classified as follows.

Level-I methods: In these methods, only one characteristic value (usually the mean value) of each uncertain parameter is used. The allowed stress methods are examples of this group.

Level-II methods: these methods rely on two characteristic values (usually the mean and variance) and the correlation between them (usually covariance). The reliability index methods are examples of this group.

Level-III methods: These methods use the probability of failure as a measure, and thus require the joint distribution of all uncertain parameters.

Level-IV methods: these methods analyze our expectations from the structure with a measure of acceptability defined based on the probabilistic conditions and in accordance with the principles of engineering economics analysis, i.e. the cost and benefit of construction, maintenance, repair, failure consequences, etc. This approach is suitable for structures of great economic importance such as highway bridges, power transmission towers, nuclear installations, offshore platforms and ship and airplanes structures, but is still in development.

2.2. Cornell reliability index

In 1969, Cornell defined the reliability index as follows:

$$\beta_c = \frac{E[M]}{D[M]} \tag{2}$$

Cornell's reliability index provides a reasonable estimation of reliability by measuring the distance from *E[M]* to the failure surface as a multiple of uncertainty parameter *D[M]* (*E[M]* and *D[M]* are the mean and standard deviation of the safety margin).

In Cornell's initial formulation, the failure function was expressed as the difference between strength R from the effect of load S.

$$G(R,\, S) = R - S \tag{3}$$

The safety margin corresponding to the above relationship is:

$$M = R - S \tag{4}$$

Assuming that R and S are independent, the reliability index can be calculated as follows:

$$\beta_c = \frac{E[R] - E[S]}{\sqrt{Var[R] + Var[S]}} \tag{5}$$

If the limit state function is a first-order function, we have:

$$G(x_i) = a_0 + \sum_{1}^{n} (a_i x_i) \tag{6}$$

Thus, the reliability index is given by:

$$\beta = \frac{a + a^T E[X]}{\sqrt{a^T c_x a}} \tag{7}$$

Where *E[16]* denotes the expected value of the main variables and the matrix C_x is their covariance. This reliability index will remain constant under any linear transformation. When the limit state function is not linear, it is impossible to formulate linear safety margin based on the main variables, because the mean and variance of the main variables are not enough to derive the expected value and variance of the safety margin.

2.3. Hasofer & Lind reliability index

As mentioned earlier, Hasofer & Lind [7] have introduced an efficient method for determination of reliability index. In this method, reliability index is obtained through a search for the point of minimum distance from the limit state function to the origin of the standard normal coordinate system. The distance of the failure surface from the origin is called the reliability index function and has been defined as follows.

$$\beta_c(U) = \min(U^T.U)^{0.5} \tag{8}$$

$$U \in G(U) = 0 \tag{9}$$

Where *G(U)* is the limit state function in the standard normal domain of the base random variables *(U)*. Given the above relation and the definition of *HL*, the reliability index for has been defined as follows.

$$\beta_c(U) = \min(U^T.U)^{0.5} \tag{10}$$

$$Subject\ to:\ G(U) = 0 \tag{11}$$

In the above equation, which is known as the first-order reliability equation, the point $U^*(u^*_1, u^*_2, \ldots, u^*_n)$ on the failure surface $G(U)=0$ is called the design point.

2.4. Reliability Index & Probability of Failure

The most important application of reliability index is the estimation of failure probability. The relationship between the two can be expressed as follows:

$$\beta = \phi^{-1}(1 - p_f) = -\phi^{-1}(p_f) \rightarrow p_f = \phi(-\beta) \tag{12}$$

Where P_f is the failure probability, β is the safety index, and Ø is the standard normal distribution function. As mentioned, β can be determined by various methods, and then, the probability of failure can be calculated by the above equation.

3. Proposed algorithm

Firefly algorithm is a meta-heuristic optimization algorithm inspired by the simple communication behavior of fireflies. A firefly can understand its environment through its sensors and comminute with the environment using the light produced from its lower abdomen. Studies have shown that fireflies are social insects living in colonies and that their behavior is governed by the desire for the survival of the self as well as the colony. One of the most interesting facts about fireflies is their behavior during search for food, and in particular, how they find the shortest route between food and the nest. For this purpose, each individual firefly moves to the point where it has had the best experience of finding food. The light produced by firefly acts as a signal that attracts other fireflies or preys. Firefly colony can be view as a natural manifestation of swarm intelligence, in which the cooperation (or competition) of simpler and less intelligent members generates a higher degree of intelligence unattainable by any individual member. Recently, the social behavior of fireflies and their swarm intelligence have fascinated and inspired scientists working in other fields.

Inspired by the aforementioned social behavior, in 2009 Yang developed the firefly algorithm with the following assumptions [17]:

1- Fireflies are unisexual, therefore all fireflies will be attracted to each other. The attraction intensity (*I*) has been defined as follows:

$$I(r) = I_0\, e^{-\gamma r^2} \tag{13}$$

Where I_0 is the initial light intensity, γ is the constant light absorption coefficient, and r is the distance between the two fireflies.

Since attractiveness of a firefly is proportional to its light intensity as viewed by others, the attractiveness function $\beta(r)$ has been defined as follows:

$$\beta(r) = \beta_0\, e^{-\gamma r^2} \tag{14}$$

In this function, β_0 is the attractiveness at $r = 0$.

2- Attractiveness of a firefly is proportional to its brightness. Thus for any two fireflies, the less bright one will move toward the brighter one. But the perceived brightness and attractiveness is inversely proportional to distance. Thus, as distance increases, perceived brightness decreases, and so does the attractiveness.

The distance between the fireflies i and j located at x_i and x_j is given by the following equation:

$$r_{ij} = |x_i - x_j| = \sqrt{\sum_{k=1}^{d}\left(x_{i,k} - x_{j,k}\right)^2} \tag{15}$$

Where $x_{i,k}$ is the k-th spatial coordinate of firefly i, and d is the number of problem variables.

3- The brightness of each firefly is determined by the objective function defined for the problem.

Finally, each firefly (i) moves toward the most attractive firefly in its view (j) in accordance with the following equation:

$$x_i^{t+1} = x_i^t + \beta_0\, e^{-\gamma r_{i,j}^2} \left(x_j^t - x_i^t\right) + \alpha \varepsilon_i^t \tag{16}$$

In this equation, the second term represents the relocation resulting from attraction and the third term introduces some randomness into the movement. The coefficient α is the motion randomization parameter, and ε_i is a random vector with normal distribution in the interval [0, 1]. In most optimization problems, $\beta_0 = 1$.

The value of the parameter γ plays a critical role in algorithm's rate of convergence and its behavior. Although in theory $\gamma \in [0,\infty)$, in practice, γ must be between 0.1 and 10.

The primary merit of firefly algorithm (FA) is its very good accuracy in solving optimization problems, but it is rather slow when implemented with large populations. In contrast, the particle swarm optimization algorithm (PSO), which is inspired by the collective behavior of bird swarms, exhibits higher speed in solving complex optimization problems.

In the proposed hybrid algorithm, first, firefly algorithm will be run for a specified number of iterations (FA_{iter}), and the best solution among all iterations will be registered. Then, this best solution will be imported into PSO to calculate the velocity of each particle and update its position accordingly. The output of PSO function for the new position will be compared with the current best solution and if a better solution is found, the best solution will be updated accordingly (standard PSO procedure). This PSO will be run for a specified number of iterations (PSO_{iter}). Finally, all of the above steps will be repeated for a specified number of iterations (HYB_{iter}). This algorithm is designed such that the ratio of combination of two algorithms and their population are adjustable. In this algorithm, the best solution is the minimum distance between the failure surface of the limit state function and the origin of the standard normal coordinate system. In summary, the reliability index and the failure probability are obtained by the use of Hasofer-Lind definition of the reliability index and the search for the optimal solution by the algorithm.

The pseudocode of the proposed FA-PSO hybrid algorithm is presented in Figure (1).

4. Problem

This section is divided into two parts, one dedicated to mathematical problems and the other dedicated to reference engineering problems. In each part, we use the proposed hybrid algorithm to solve a number of typical problems already solved through different methods and with different algorithms.

```
Objective function f(x), x = (x1... xd) T.
Generate an initial population of n fireflies' xi, (i = 1, 2... n).
Light intensity Ii at xi is determined by f (xi).
Define light absorption coefficient γ.
Generate an initial population of N particle Xi, (i = 1, 2... n).
Calculate velocity every particle.

While (t < Max Generation),
    For i = 1: n (number of fireflies)
        For j = 1: n (all n fireflies)
            If (Ii < Ij)
                Move firefly i towards j.
            End if
            Vary attractiveness with distance r via exp [−γr2].
            Evaluate new solutions and update light intensity.
        End for j
    End for i

    For i = 1: N (number of particle)
        Update velocity
        New position = new velocity + old position
        Evaluate new solutions
    End for i
    Rank the fireflies and particle.
    Find the current global best.
End while

Post process results and visualization.
```

Fig. 1. Proposed pseudocode FA-PSO.

4.1. Numerical Examples

The accuracy and performance of the proposed algorithm are expressed by the following mathematical functions. These function are used for comparison with other methods according to the number of involved statistical parameters and variables.

$$G(X_1, X_2) = 5 - 0.5(X_1 - 0.1)^2 - X_2 \quad (17)$$

$$G(X_1, X_2) = \exp(0.2X_1 + 1.4) - X_2 \quad (18)$$

$$G(X1, X2) = \exp\left(0.4\left(X_1 + 2\right) + 6.2\right) - \exp\left(0.3X_2 + 5\right) - 200 \quad (19)$$

The results concerning the reliability index and the failure probability function for the above mathematical functions are presented in Table (1). For mathematical problems, the reliability index is obtained through Hasofer-Lind method. Comparison of the results demonstrate the good accuracy and performance as well as the high speed of the proposed algorithm.

Table 1. Comparison of results in numerical examples.

Limit Function	Variable	Algorithm	U^*	μ	σ	β	P_f
Eqe. 17	X_1 X_2	PSO [20]	(-2.7416، 0.9625)	0	1	2.905	0.00183
		CSS [21]	(-2.7408، 0.9607)			2.906	0.00183
		FA-PSO	(2.7551، 0.9241)			2.905	0.00183
18	X_1 X_2	PSO [20]	(-1.6882, 2.8931)	0	1	3.497	0.000405
		CSS [21]	(-1.6797, 2.8981)			3.35	0.000404
		FA-PSO	(-1.6826, 2.8964)			3.349	0.000404
19	X_1 X_2	PSO [20]	(-2.5477, 0.9235)	0	1	2.709	0.00337
		CSS [21]	(-2.5396, 0.9453)			2.71	0.00336
		FA-PSO	(-2.5783, 0.8407)			2.709	0.00330

4.2. Engineering Problems

4.2.1. Simply-Supported Beam

In this problem instance, a simple beam is subjected to a point load (Figure 2). For this problem the limit state function is defined as:

$$G(E,I,P)=48EI-100PL^2 \tag{20}$$

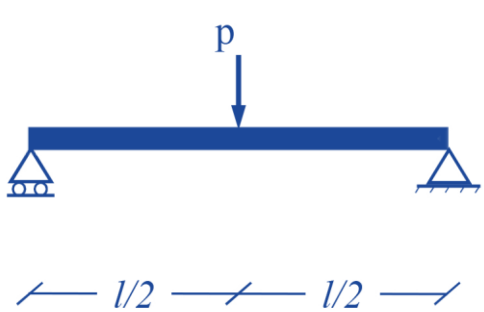

Fig. 2. Simply-Supported Beam.

The beam has a length of 6 meters. The mean values and standard deviations of variables *P*, *E*, and *I* are given in Table (2).

Table 2. Random variable for Simply-Supported Beam.

Variable	Mean	Standard Deviation	Distribution
P (kN)	2	0.6	normal
E (kN/m^2)	2×10^7	3×10^6	normal
I (m^4)	2×10^5	2×10^{-6}	normal

In Table (3), the optimum value of the reliability index β and the failure probability P_f as calculated by the proposed algorithm are compared with the results of previous methods.

Table 3. Comparison between Risk Analysis and FA-PSO algorithms.

Method	Optimum Point	β	P_f
Risk Analysis [20]	----	3.15	0.00081
FA-PSO	P = 3.12 E = 1.316×10^7 I = 1.77×10^5	3.148	0.00082

As shown in Table (3), the results obtained from the Modified algorithm are highly accurate and give a conservative estimation of beam failure probability. The convergence diagram of the proposed algorithm is plotted in Figure (3).

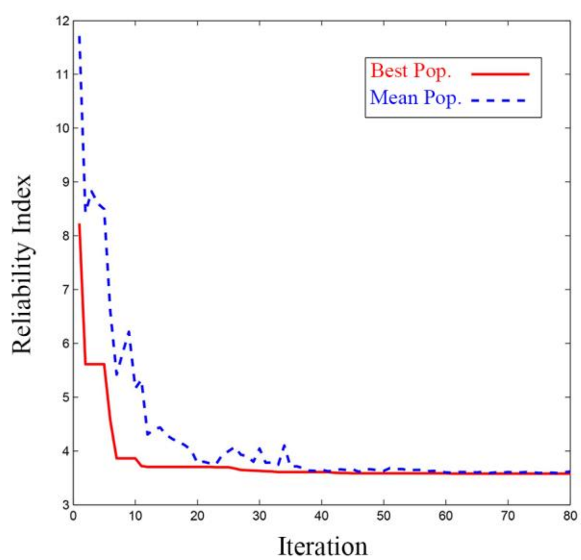

Fig. 3. The convergence diagram of the algorithm.

4.2.2. three-span continuous beam

In this problem instance, a three-span beam of 15 meters in length (5 meters per span) is subjected to a distributed load (Figure 4).

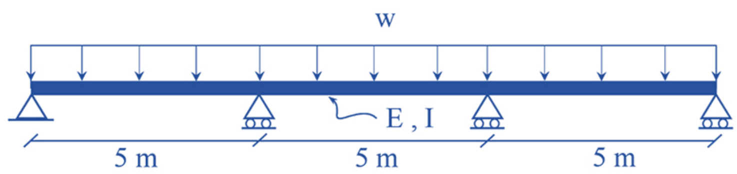

Fig.4. 3-span continuous beam subjected to a distributed load

In accordance with AISC regulations, the allowable displacement in this beam is L/360, and its limit state function is:

$$G(w,E,I)=\frac{L}{360}-0.0069\frac{wL^4}{EI} \tag{21}$$

This problem involves three variables *w*, *E*, and *I*, which all have a normal distribution function.

Table 4. Random variable for continuous beam.

Variable	Mean	Standard Deviation	Distribution
w (kN/m)	10	0.4	normal
E (kN/m^2)	2×10^7	0.5×10^{-7}	normal
I (m^4)	8×10^{-4}	1.5×10^{-4}	normal

The results obtained for this problem instance are presented in Table (5).

Table 5. Comparison between the HL-RF and FA-PSO algorithms.

Method	Optimum Point	β	P_f
HL-RF [22]	w = 10.044 E = 4368200 I = 7.139×10^{-4}	3.1805	0.000735
FA-PSO	w = 10.043 E = 4374200 I = 7.128×10^{-4}	3.1805	0.000735

The convergence diagram of the proposed algorithm is shown in Fig. 5.

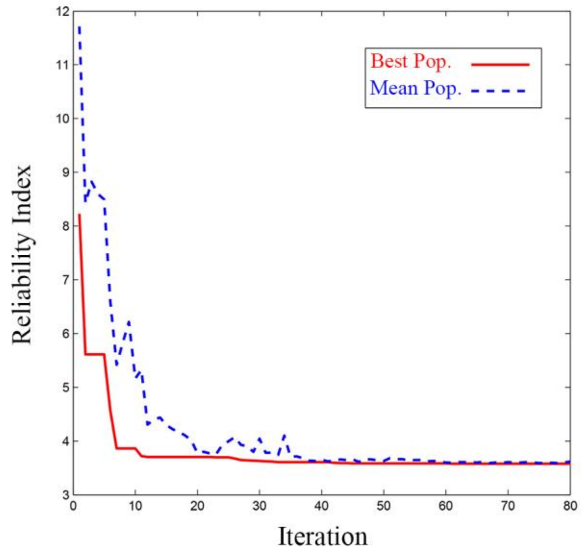

Fig.5. the convergence diagram of the algorithm

5. Conclusion

This paper introduces a hybrid algorithm consisting of firefly algorithm (FA) and particle swarm optimization algorithm (PSO) for determination of reliability index and failure probability. The good optimization performance of both FA and PSO has been acknowledged in many publications, thus, a combination of these two can be remarkably effective in reliability analysis. In the proposed algorithm, FA-PSO combination ratio is adjustable, and can therefore be fine-tuned to optimize the runtime and accuracy according to problem requirements and characteristics. The capability of the proposed algorithm in solving reliability theory problems without the limitations of classical approach is verified with the help of a set of mathematical and structural problems. The satisfactory performance and accuracy and acceptable speed of the proposed algorithm are demonstrated by comparing its results with those of alternative methods.

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