

# **Differential Geometric Approach to the Stress Aspect of a Fault: Airy Stress Function Surface and Curvatures**

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## **A b s t r a c t**

We considered the two-dimensional stress aspect of a fault from the viewpoint of differential geometry. For this analysis, we concentrated on the curvatures of the Airy stress function surface. We found the following: (i) Because the principal stresses are the principal curvatures of the stress function surface, the first and the second invariant quantities in the elasticity correspond to invariant quantities in differential geometry; specifically, the mean and Gaussian curvatures, respectively; (ii) Coulomb's failure criterion shows that the coefficient of friction is the physical expression of the geometric energy of the stress function surface; (iii) The differential geometric expression of the Goursat formula shows that the fault (dislocation) type (strike-slip or dip-slip) corresponds to the stress function surface type (elliptic or hyperbolic). Finally, we discuss the need to use non-biharmonic stress tensor theory to describe the stress aspect of multi-faults or an earthquake source zone.

**Key words:** stress function, curvature, Mohr's circle, faults, Goursat formula.