Original Article

JOURNAL OF TEXTILES AND FIBROUS MATERIALS

Journal of Textiles and Fibrous Materials
Volume 1: 1–7

DOI: 10.1177/2515221118785917
journals.sagepub.com/home/tex
SAGE

A mathematical formula to find the wing band length of a bra to fit the comfort zone of the wearer

Sapadhi Abeysuriya and Thanuja Paragoda

Abstract

This work concerns on finding a formula of the length of a wing band within the comfort zone. Bra band elongation (BBE) test is one of the main methods that is being used to validate the comfort level of a bra bottom band when it is on the body. This test validates the suitable bra bottom band length, which is within the comfort zone for the wearer. However, BBE test by trial and error method is time-consuming and cost ineffective. Therefore, this research carries on an analytical method of obtaining a formula for the elongation of a wing band of a bra, which is assumed to be a rectangular thin plate. We obtain a new formula for the weighted elongation of the wing band using the Navier equation and compare the analytical weighted elongations and experimental elongation with different force values for the different bra sizes and for the different fabrics. With the weighted parameters, the model on elongation of the wing band fits with actual data and we provide specific different weighted parameters for different bra sizes to find the exact original length of the wing band of a bra.

Keywords

Bra band elongation, Navier equations, weighted elongation

Introduction

The study of theory of elasticity in fabrics is a fascinating subject and plays a prominent role in textile science and engineering.[1–4] The article[5] studied the mechanical properties of a woven fabric, the shear deformation analysis and Poisson's ratios for woven fabrics in 2005. It provides a guideline for the design of a woven fabric. Mirjalili et al.[6] studied the garment pressure on the human body using finite elements. It presents a detailed analysis of the contact pressure between an elastic garment and some parts of the human body whose geometries are cylindrical and conical, which is based on contact mechanics and the results are compared using a finite element method as well as an experimental approach. Liu et al.[7] studied the fabric mechanical surface properties of compression hosiery and their effects on skin pressure magnitudes when worn, and Qiaoling et al. studied the effects of mechanical properties of fabrics on clothing pressure in which they observed how the elastic modulus, elongation and relaxation time of fabric influence clothing pressure, the pressure on cylinder model.[8]

The ABC company, a well-known bra manufacturing company in Sri Lanka, uses bra band elongation (BBE) test to find the length of the bra bottom band of a bra by a trial and error method and it is time-wasting and cost ineffective. Therefore, the company needs an explicit formula to find the length of the bra bottom band when it is on the body. Since the stretched length in the comfort zone can be easily measured, the original length of the wing band can be easily computed from the elongation. Therefore, we provide our own mathematical formula for the weighted elongation of the wing band of a bra when it is stretched and is in the range of the comfort zone.

Department of Mathematics, Faculty of Applied Sciences, University of Sri Jayewardenepura, Western, Sri Lanka

Corresponding author:
Thanuja Paragoda, Department of Mathematics, Faculty of Applied Sciences, University of Sri Jayewardenepura, Nugegoda, Western 10100, Sri Lanka.
Email: thanujap@sjp.ac.lk

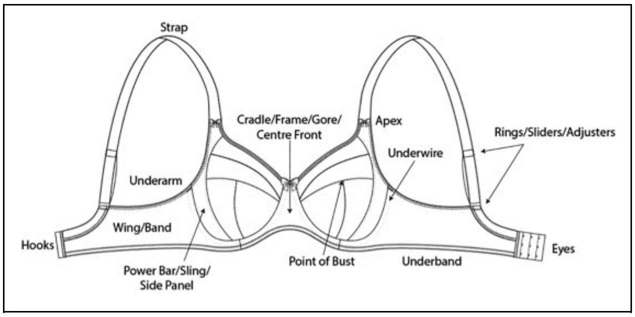

Figure 1. Components of the bra. .

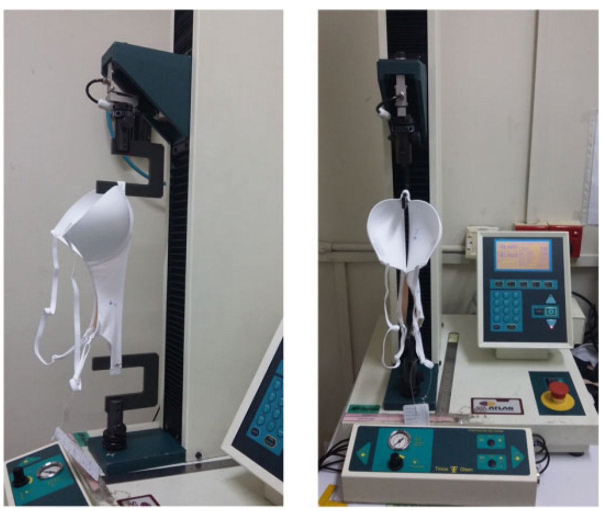

Figure 2. Bra band elongation.

We studied the theory of elasticity on fabrics and using fundamental equations of elasticity and Navier equations for plane stress and then we derive a formula for the weighted elongation of the wing band of a bra within the comfort zone. In numerical section, we provide the error between the analytical elongation and the experimental elongation according to the model and graphical figures are presented.

Components of a bra

Fabrics and elastics are the main components used in the production of a bra. In addition to that, pad cups, hooks and eyes, rings, sliders and wires are also used based on the type of the bra as shown in Figures 1 and 3.

Technical aspects of material

There are three main features that we need to consider when selecting fabrics and elastics for a bra. They are the modulus, the stretch/elongation and the density of the fabric/elastic. Based on the design and the requirements of the bra, we need to select fabrics and elastics with correct values for above features. The modulus of the fabric measures the tensile elasticity or the tendency of an object to deform along an axis when opposing forces are applied along that axis; it is defined as the ratio of tensile stress to tensile strain. It is often referred to simply as the elastic modulus. The stretch/elongation provides the average increase in length of the sample at its break (rupture) point.

BBE test

BBE test is one of the main tests that is being used to decide the actual length of a bra band within the comfort zone at the ABC company. The BBE test measures the force (lbs) exerted on the bottom band of a bra at the smallest and largest wing band adjustments as shown in Figure 2. Once the garment is subjected to the BBE test and two measurements lie in the ranges given in Table 1, then it is said that the actual length of the bra band is fit for use.

If the BBE result is greater than the above value range, then the garment is too tight for use and vice versa. Currently, the sample production team of the development centre arrives at an acceptable bra band length through a trial and error method. They produce garments with several bra band lengths subject to BBE test, until they put a result, which is in acceptable range. Because of trial and error method, the company has to spend a lot of time to figure out the correct length for the wing band of a bra which leads to waste a lot of money and time for the production. Hence, we derive a formula to find an elongation of the wing band when it is stretched and using that formula, original length of the wing band can be determined.

Mathematical model

We first provide a brief introduction to the theory of elasticity. We describe coaxial stresses and strains of the deformed rectangular-shaped thin plate. Then, we present matrix representation of stresses and strains and the generalized Hook's law. Finally, we present the fundamental equations for a thin plate in plane stress and Navier equations for the plane stress.

Theory of elasticity

The theory of elasticity is a branch of continuum mechanics. The classical theory of elasticity is valid for isotropic, linearly elastic materials subjected to small deformations.[9–11]

We now assume small deformations and small displacements such that the strains may be given by the strain tensor for small deformations $\boldsymbol{E}$, which, in a Cartesian coordinate system, Ox has components

$$E_{ik} = (u_{i,k} + u_{k,j}) \tag{1}$$

where u_i are the components of the displacement vector $\boldsymbol{u}$, and $u_{i,j}$ are the displacement gradients. E_{ii} (not summed) are longitudinal strains in the directions of the coordinate

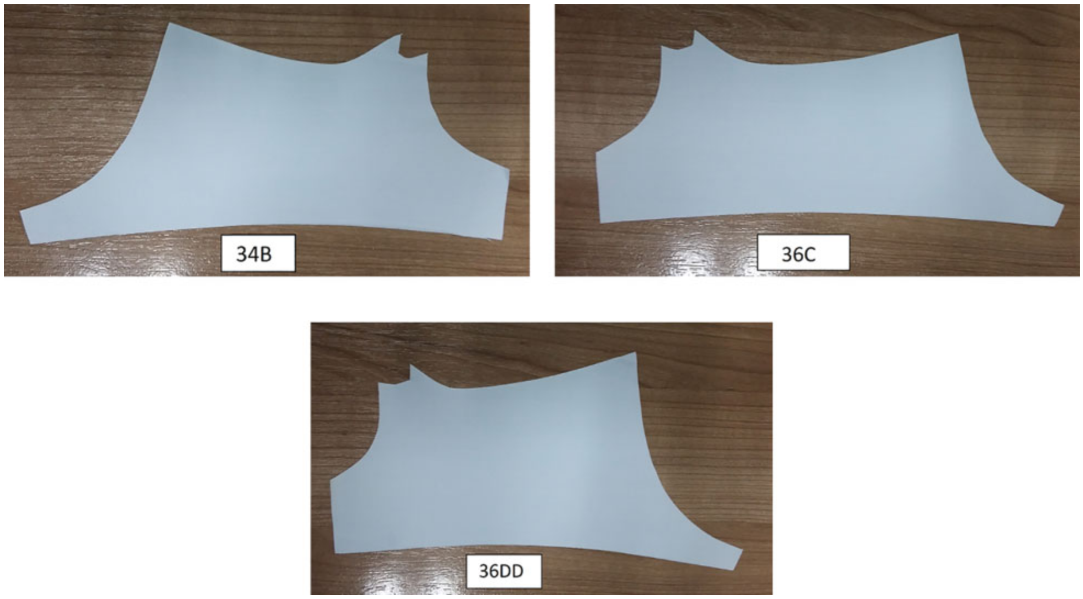

Figure 3. Wing bands of bra sizes 34B, 36C and 36DD.

Table 1. Force adjustments in BBE test.

Adjustment type	Value range (lbs)
Smallest adjustment	2.4–3.4
Largest adjustment	1.5–2.5

BBE: bra band elongation.

axes, and E_{ij} $(i \neq j)$ are half of the shear strains for the directions $\boldsymbol{e}_i$ and $\boldsymbol{e}_j$.

A material is said to be Cauchy-elastic, if the stresses $\boldsymbol{T}$ in a particle $\boldsymbol{r} = x_i\boldsymbol{e}_i$ function only of the strains in the particle

$$T_{ik} = T_{ik}(E, x) \Leftrightarrow \boldsymbol{T} = \boldsymbol{T}[\boldsymbol{E}, \boldsymbol{r}] \tag{2}$$

which is called the basic constitutive equations for Cauchy-elastic materials.

If the elastic properties are same in every particle in a material, then the material is called elastically homogeneous. If the elastic properties are same in all directions through one and the same particle, then the material is said to be elastically isotropic. Isotropic elasticity implies that the principal directions of stress and strain coincide. The stress tensor and the strain tensor are coaxial.

In tension or compression tests of isotropic materials, a test specimen is subjected to uniaxial stress σ and experiences the strains ε in the direction of the stress and ε_t in any transverse direction, that is, normal to the stress. For a linearly elastic material, the following relations may be stated:

$$\varepsilon = \frac{\sigma}{\eta}, \quad \varepsilon_t = -v\varepsilon = v\frac{\sigma}{\eta} \tag{3}$$

where η is the modulus of elasticity and v is Poisson's ratio.[12]

A linear elastic material in a state of uniaxial stress: $\sigma_1 \neq 0, \sigma_2 = \sigma_3 = 0$, obtains the strains

$$\varepsilon_1 = \frac{\sigma}{\eta}, \quad \varepsilon_2 = \varepsilon_3 = \varepsilon_t = -v\frac{\sigma_1}{\eta} \tag{4}$$

In a general state of triaxial stress, with principal stresses σ_1, σ_2 and σ_3, the principal strains are

$$\varepsilon_1 = \frac{\sigma_1}{\eta} - \frac{\upsilon}{\eta}(\sigma_2 + \sigma_3) = \frac{1+\upsilon}{\eta}\sigma_1 - \frac{\upsilon}{\eta}(\sigma_1 + \sigma_2 + \sigma_3) \tag{5}$$

Using the fact that the relations between stresses and strains are linear such that the principal of superposition applies.

Therefore, in any Cartesian coordinate system, Ox has the representation

$$E_{ik} = \frac{1+v}{\eta}T_{ik} - \frac{v}{\eta}T_{jj}\delta_{ik} \tag{6}$$

which is called the generalized Hook's law of the Hookean material or Hookean solid.

The material parameter μ is called the shear modulus which is given by

$$\mu = \frac{\eta}{2(1+\upsilon)} \tag{7}$$

Two-dimensional theory of elasticity

In the theory of elasticity, the general equations can be solved by elementary analytical methods. However, we may consider simplifications with respect to the state of stress or the state of displacements, such that a useful solution may be found by relatively simple means in many practical problems.

The fundamental equations for a thin plate in plane stress are given by

$$\operatorname{div}\mathbf{T} + \rho\mathbf{b} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2} \Leftrightarrow T_{\alpha\beta,\beta} + \rho b_\alpha = \rho\frac{\partial^2 u_\alpha}{\partial t^2} \tag{8}$$

which is called the Cauchy equations of motion and $\mathbf{b}$ is the body force vector and $\frac{\partial^2\mathbf{u}}{\partial t^2}$ represents the second material derivative of $\mathbf{u}$ with respect to time t. Furthermore, the strain–displacement relations are given by

$$E_{\alpha\beta} = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha}) \tag{9}$$

In an analytical solution to a problem in the theory of elasticity, it is customary to choose either displacements or stresses as the primary unknown functions. When the displacements are selected as the primary unknowns, the fundamental equations are transformed as follows.[12]

Substituting equation (9) into equation (6), we have

$$T_{\alpha\beta} = \mu\left[u_{\alpha,\beta} + u_{\beta,\alpha} + \frac{2v}{1-v}u_{\rho,\rho}\delta_{\alpha\beta}\right] \tag{10}$$

Substituting equation (10) into equation (8), we get

$$u_{\alpha,\beta\beta} + \frac{1-v}{1+v}u_{\beta,\beta\alpha} + \frac{1}{\mu}\rho\left(b_\alpha - \frac{\partial^2 u_\alpha}{\partial t^2}\right) = 0 \tag{11}$$

which is said to be the Navier equations for plane stress.

If we consider the state of displacements is axisymmetrical, then the state of deformation is irrotational, or in other words, the result is a state of pure strain

$$u_{\alpha,\beta} = u_{\beta,\alpha} \Rightarrow u_{\alpha,\beta\beta} = u_{\beta,\alpha\beta} = u_{\beta,\beta\alpha}$$

Then, equation (11) can be transformed into

$$\varepsilon_{A,\alpha} + \frac{1-v}{2\mu}\rho\left(b_\alpha - \frac{\partial^2 u_\alpha}{\partial t^2}\right) = 0 \tag{12}$$

where ε_A is the area strain which is invariant and represents the change in area per unit area in the plane of the plate and $\varepsilon_{A,\alpha}$ is the derivative of the area strain with respect to α.[12]

As shown in Figure 4, we assume that the wing band of a bra is a rectangular thin plate and the garment is approximated as an isotropic and homogeneous material. The stress in the fabric thickness is assumed to be zero. The length of the hooks and eyes is included in the length of the wing band of the bra. Since the straps are disconnected with the bottom band of the bra and even through the strap stretch, it doesn't have any effect on the bottom band of the bra, and other components such as wires and centre fronts have negligible elasticity compared to the elasticity of the bottom band; we assume that they don't stretch hence no effect from them. Furthermore, we assume that the state of displacements of the wing band is axisymmetrical which implies the state of deformation is irrotational. This model is based on Hook's law, the theory of elasticity and the plane displacements.

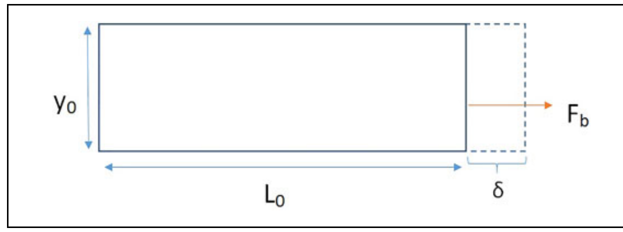

Figure 4. Rectangular wing band.

The strain in x-direction is given by

$$\varepsilon_x = \frac{\delta}{L_0} \tag{13}$$

where L_0 is the original length of the wing band of the bra and δ is the elongation of the wing band

$$\varepsilon_A = \frac{\delta}{L_0}y_0 \tag{14}$$

Where ε_A is the area strain and y_0 is the height of the wing band.

Under the above assumptions, we consider equation (12) for the plane stress of the wing band and obtain a formula for the weighted elongation in terms of the stretched length, shear modulus, Poisson's ratio and the density of the fabric. Since we assume that the material derivative is constant, then the second material derivative is equal to zero. Moreover, since the strain in the transverse direction is equal to zero, Poisson's ratio is also zero from equation (3). Then, equation (12) becomes

$$\frac{\mathrm{d}}{\mathrm{d}L}\left(\frac{\delta y_0}{L-\delta}\right) + \frac{1}{2\mu}\rho F_b = 0 \tag{15}$$

where ρ is the grams per square meter, F_b (>0) is the force applied on the fabric and L is the stretched length of the wing band of the bra. After a few calculations, we obtain the following quadratic equation for δ, which is given by

$$\delta^2 + \left(-\frac{2\mu y_0}{\rho F_b} - 2L\right)\delta + L^2 = 0 \tag{16}$$

Then, the roots of equation (6) are given by

$$\delta = A \pm \sqrt{A^2 - L^2} \tag{17}$$

where

$$A = \frac{\mu y_0}{\rho F_b} + L$$

We only consider the negative elongation. Then, we consider the weighted elongation δ, which is given by $\delta_w = C_1\delta - C_2$, where C_1 and C_2 are the weighted parameters which depend on the size of the bra and the fabric material.

Derivation of Hook's law from the model

We deduce Hook's law from our model equation (6). We assume that the elongation is infinitesimal small, and then

Table 2. Error between the analytical weighted elongation and the experimental elongation of 34B bra size.

Force (F_b; lbs)	Analytical weighted elongation (δ_w; in)	Experimental elongation (δ_e; in)	Error $E = \lvert\delta_w - \delta_e\rvert$ (in)
1.04	0.8865	0.88	0.0065
1.24	1.1283	1.08	0.0483
1.46	1.3737	1.28	0.0937
1.56	1.4790	1.38	0.0990
1.88	1.7928	1.68	0.1128
2.09	1.9819	1.88	0.1019
2.63	2.4175	2.38	0.0375
3.21	2.8205	2.88	0.0595
3.43	2.9591	3.08	0.1209

Table 3. Error between the analytical weighted elongation and the experimental elongation of 36B bra size.

Force (F_b; lbs)	Analytical weighted elongation (δ_w; in)	Experimental elongation (δ_e; in)	Error $E = \lvert\delta_w - \delta_e\rvert$ (in)
1.46	0.8520	0.88	0.0280
1.7	1.0395	1.08	0.0405
2.02	1.2674	1.38	0.1126
2.39	1.5044	1.68	0.1756
2.62	1.6396	1.88	0.2404
3.23	1.9614	2.38	0.4186
3.89	2.2609	2.88	0.6191

we get $\delta^2 \approx 0$ in which equation (6) implies

$$L^2 \approx 2\delta(X + L) \tag{18}$$

where $X = \frac{\mu y_0}{\rho F_b}$.

From equation (18), we have

$$L_0^2 - \delta^2 \approx 2\delta(\frac{K}{F_b}) \tag{19}$$

Where $K = \frac{\mu y_0}{p}$. Hence, we obtain

$$F_b \approx \delta(\frac{2K}{L_0^2}) \tag{20}$$

which implies Hook's law by considering $\frac{K}{L_0^2}$ as the modulus of the elasticity.

Numerical results

We compared the analytical weighted elongations for different bra sizes with the experimental elongations for different force values, while we kept the height of the wing band as a constant. We have also represented the absolute error between the analytical weighted elongations computed from the model and the experimental elongation for different bra sizes when the force varies as shown in Tables 2 to 4.

Table 4. Error between the analytical weighted elongation and the experimental elongation of 36DD bra size.

Force (F_b)	Analytical weighted elongation (δ_w; in)	Experimental elongation (δ_e; in)	Error $E = \lvert\delta_w - \delta_e\rvert$ (in)
1.1800	0.8720	0.8800	0.0080
1.4000	1.0979	1.0800	0.0179
1.7200	1.3961	1.3800	0.0161
2.1000	1.7115	1.6800	0.0315
2.3200	1.8781	1.8800	0.0019
2.9000	2.2712	2.3800	0.1088
3.6200	2.6859	2.8800	0.1941

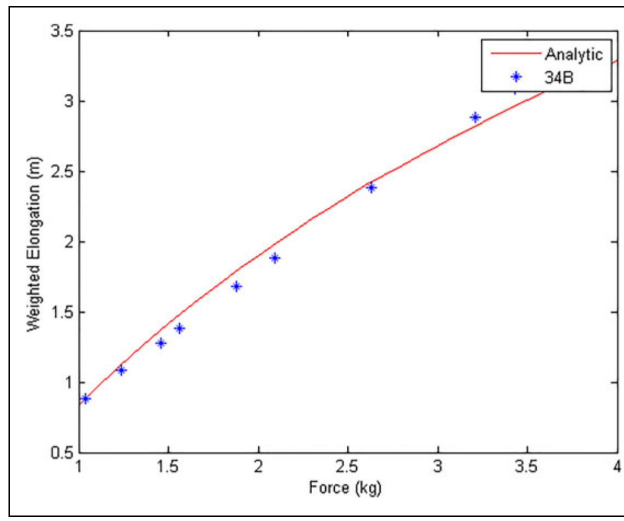

Figure 5. Variation of weighted elongation with force in small adjustments for 34B bra size.

Variation of the weighted elongation with the force for different bra sizes

We have plotted the variation of the weighted elongation with the force for different bra sizes using MATLAB. We basically considered three bra sizes such as 34B, 36C and 36DD. Figure 5 represents the variations of weighted elongation of the wing band of 34B bra size with force in small adjustments. The actual data points have been marked in * in blue, while the analytical weighted elongation is denoted by red. The other similar graphs are plotted for the bra sizes of 36C and 36DD. Here, the force varies from 1 to 3.5 with interval length 0.1, and we obtained the experimental elongation from BBE test for force values, which are shown in Table 2. Then, we have evaluated the absolute error between the analytical weighted elongation and the experimental elongation. In this case, we considered $\rho = 170\ \ \text{g/m}^2,\ \ \mu = 0.7\ \text{lbs/m}^2,\ \ L = 10.5$ in and $y_0 = 8$ in. Here, the weighted parameters are $C_1 = 50$ and $C_2 = 0.8$, respectively. and

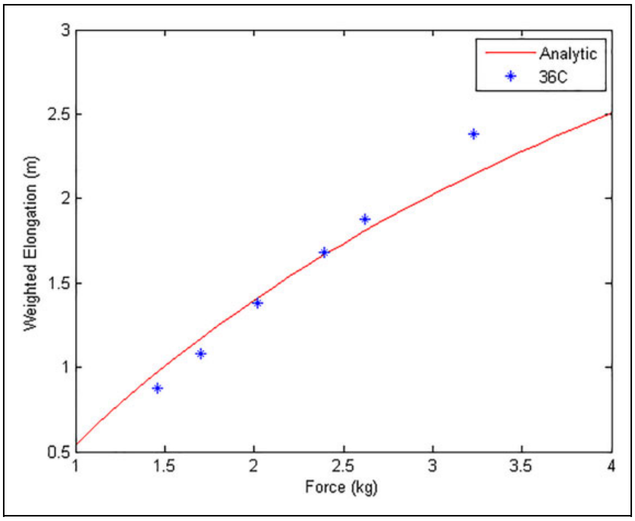

Figure 6. Variation of weighted elongation with force in small adjustments for 36C bra size.

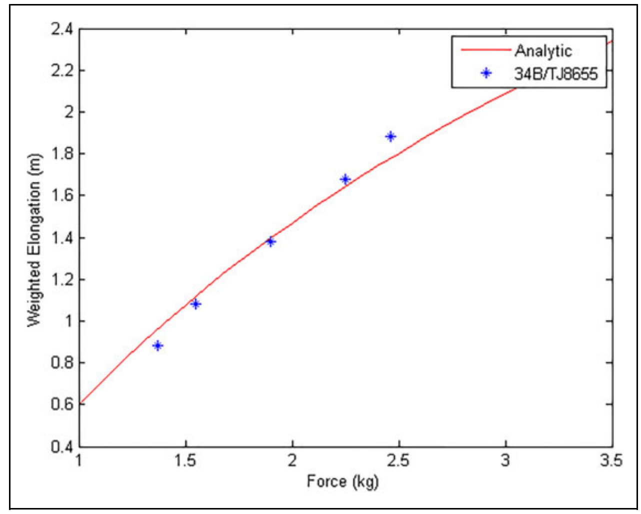

Figure 8. Variation of weighted elongation with force in small adjustments for the fabric 34B/TJ8655.

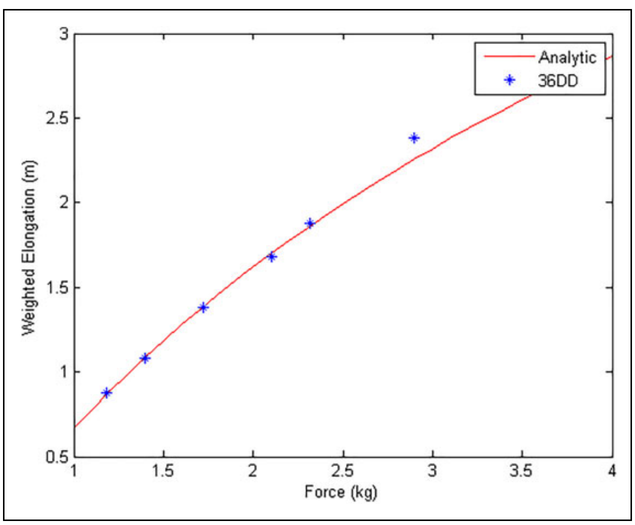

Figure 7. Variation of weighted elongation with force in small adjustments for 36DD bra size.

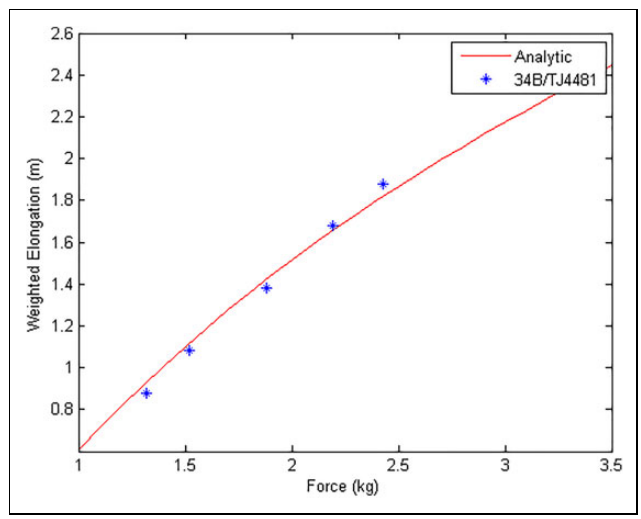

Figure 9. Variation of weighted elongation with force in small adjustments for the fabric 34B/TJ4481.

As shown in Figure 6 in the case of bra size 36C, we considered $\rho = 170$ g/m^2, $\nu = 0.5$, $\mu = 0.7$ lbs/m^2, $L = 10.94$ in and $y_0 = 8$ in. The weighted parameters in this case are $C_1 = 38$ and $C_2 = 0.8$, respectively.

We considered $\rho = 170$ g/m^2, $\mu = 0.7$lbs/m^2, $L = 10.47$ in and $y_0 = 8$ for bra size 36DD as shown in Figure 7. The weighted parameters in this case are $C_1 = 45$ and $C_2 = 0.8$, respectively.

Variation of the weighted elongation with the force for different fabrics of 34B size

We have plotted the variation of the weighted elongation with the force for different fabrics of 34B bra size. We considered the three different fabrics such as TJ8655, TJ4481 and KS812. Figure 8 shows the variation of the analytical weighted elongation with the force values according to the actual data for the fabric 34B/TJ8655. The force varies from 1 to 3.5 with interval length 0.1. We considered $\rho = 170$ g/m^2, $\nu = 0.5$, $\mu = 0.67$ lbs/m^2, $L = 10.5$ in and $y_0 = 8$ in. The weighted parameters in this case are $C_1 = 38$ and $C_2 = 0.8$, respectively.

As shown in Figure 9, we considered $\rho = 135$ g/m^2, $\nu = 0.5$, $\mu = 0.547$ lbs/m^2, $L = 10.5$ in and $y_0 = 8$ in. The weighted parameters in this case are $C_1 = 42$ and $C_2 = 0.8$, respectively.

Then as shown in Figure 10, we considered $\rho = 239$ g/m^2, $\nu = 0.5$, $\mu = 1.037$ lbs/m^2, $L = 10.5$ in and $y_0 = 8$ in for the fabric 34B/KS812. The weighted parameters in this case are $C_1 = 44$ and $C_2 = 0.8$, respectively.

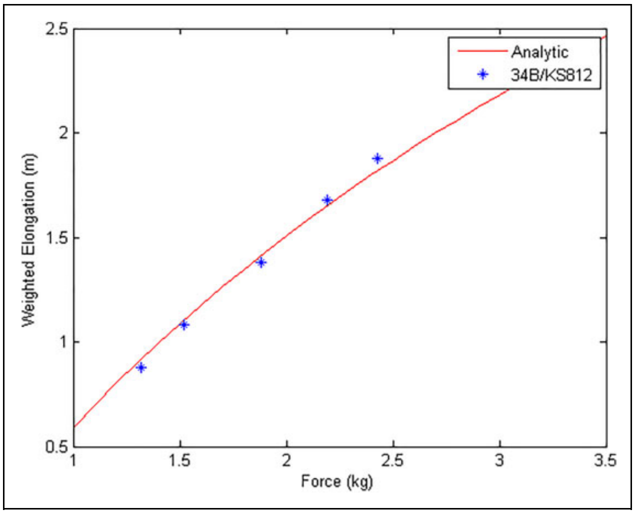

Figure 10. Variation of weighted elongation with force in small adjustments for the fabric 34B/KS812.

Conclusion

We have obtained a mathematical formula for the elongation of the wing band of the bra by assuming the wing band as a rectangular thin plate. We assumed that the fabric is elastically homogeneous and isotropic. The formula for the elongation of the wing band depends on the density of the fabric, stretched length, the height of the wing band, Poisson's ratio, the modulus of the fabric and the force applied on the wing band in the horizontal direction. Then, we have compared the analytical weighted elongations and experimental data with different force values for different bra sizes and different fabrics. We have used a weighted parameter for each bra size to match the actual data with the results of our model. So the weighted parameters are the values that bridge the gap between actual data and the results of our model. In order to define the comfort area, we need to analyse the smallest error between the analytical weighted elongation and the experimental elongation according to the force adjustments in BBE test. Then, we know how much force we applied to the elastic wing band which should be in the force adjustments in BBE test, then it is in comfort area. This model, which was formulated under many assumptions, computes values for elongation of the wing band for different bra sizes with errors in reasonable range. Therefore, we can expect the model to become more accurate if the wing of a bra is considered to be a rhombus thin plate instead of a rectangular thin plane and/or the human body to be cylindrical in shape. Through further research, it will be possible to give the exact range for the weighted parameter in the formula based on the physical characteristics of fabrics. This is a novel approach from a mathematical point of view compared to the existing literature in the study of elasticity of bras.

Acknowledgements

The authors are grateful to Dayal Dharmasena for his tremendous help and advices to improve this report and would like to acknowledge ABC Company for providing actual data.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

1. Yu A, Yick KL, Ng SP, et al. Numerical simulation of pressure therapy glove by using finite element method. *Burns* 2016; **42**(1): 141–151.
2. Kovar R. *Woven fabric engineering*. IntechOpen, 2010.
3. Mani S and Anbumani N. Dynamic elastic behavior of cotton and cotton/spandex knitted fabrics. *J Eng Fiber Fabr* 2014; **9**: 93–100.
4. Yongrong W, Peiua W and Yuan Y. Theoretical and experiments methods of dynamic clothing pressure performance. *Indian J Fiber Text Res* 2015; **40**(1): 94–99.
5. Sun H and Pan N. Mechanical properties of a woven fabric. In: *Third MIT Conference on Computational Fluid and Solid Mechanics*. Elsevier Science Ltd., 2005, pp. 507–511.
6. Mirjalili SA, Rafeeyan M and Soltanzadeh Z. The analytical study of garment pressure on the human body using finite elements. *Fibre Text East Eur* 2008; **16**(3): 69–73.
7. Li Y, Liu R, Kwok YL, et al. Fabric mechanical-surface properties of compression hosiery and their effects on skin pressure magnitudes when worn. *Fibre Text East Eur* 2010; **18**(279): 91–97.
8. Qiaoling Z, Dongsheng C, Hong L, et al. Effects of mechanical properties of fabrics on clothing pressure. *Przeg Elektrotech* 2013; **89**: 232–235.
9. Landau LD, Pitaevskii LP, Kosevich AM, et al. *Theory of elasticity*, Vol. **7**. Elsevier: Elsevier Science, 2010.
10. Atanackovic TM and Guran A. *Theory of elasticity for scientists and engineers*. Boston: Birkhäuser Boston, 2012.
11. Timoshenko S and Doodier JN. *Theory of elasticity*. 3rd ed. USA: McGraw-Hill Book Company, 1951.
12. Irgens F. *Continuum mechanics*. Berlin, Heidelberg: Springer-Verlag Berlin Heidelberg, 2008.