

THE ACTIVE CONTACT BINARY TY UMA REVISITED: IS IT A QUADRUPLE STAR?

K. LI^{1,2}, S.-M. HU¹, D.-F. GUO¹, Y.-G. JIANG¹, Y.-G. GAO¹, X. CHEN¹, AND ANDREW P. ODELL³¹ Shandong Provincial Key Laboratory of Optical Astronomy and Solar-Terrestrial Environment, Institute of Space Sciences, Shandong University, Weihai 264209, China; kaili@sdu.edu.cn, likai@ynao.ac.cn, husm@sdu.edu.cn, difu@sdu.edu.cn² Key Laboratory for the Structure and Evolution of Celestial Objects, Chinese Academy of Sciences, Kunming 650011, China³ Dept of Physics and Astronomy, Northern Arizona University, Flagstaff, AZ 86011, USA

Received 2014 September 5; accepted 2015 January 22; published 2015 March 5

ABSTRACT

TY UMa is an F-type eclipsing binary star. Four-color light curves and radial velocities of this system were presented and simultaneously analyzed using the W–D code. It is found that TY UMa is a W-subtype shallow contact binary system ($f = 13.4\%$) with a mass ratio of $q = 2.523$. In order to explain the asymmetric light curve of this binary, a dark spot on the less massive component was employed. Our newly determined 31 times of minimum light, including those collected from the literature, have been used to analyze orbital period changes of TY UMa. The complicated period variation could be sorted into a secular period increase at a rate of $dp/dt = +5.18(\pm 0.21) \times 10^{-7}$ days yr^{-1} , a 51.7 yr periodic modulation ($A_3 = 0.0182$ days), and a very small amplitude cyclic oscillation with a period of 10.0 yr ($A_4 = 0.0015$ days). The long-term increase of the period can be explained by mass transfer from the less massive component to the more massive one. The Applegate mechanism may impossibly explain the two cyclic components in the period. The two cyclic variations are very likely caused by the light travel time effect of third and fourth components, suggesting that TY UMa is a quadruple system.

Key words: binaries: close – binaries: eclipsing – stars: individual (TY UMa)

Supporting material: machine-readable and VO table

1. INTRODUCTION

The light variability of TY UMa was first discovered by Belijawsky (1933) and was classified as a W UMa-type eclipsing binary. The first photoelectric observations obtained in 1967 and 1981 were published by Broglia & Conconi (1983). Meanwhile, Broglia & Conconi (1983) analyzed the two sets of light curves and found that TY UMa is a totally eclipsing system and exhibits seasonal light curve variation. They also suggested that TY UMa is a contact binary with a mass ratio of $q = 0.4$. After that, the light curves of TY UMa were investigated by Lister et al. (2000), Stoddard et al. (2001), and Kang et al. (2002).

The change in orbital period of W UMa contact binaries is one of many interesting problems in the study of such systems. Many W UMa-type binary stars show cyclic orbital period variations, which can usually be explained due to either the magnetic activity of one or both components (e.g., Applegate 1992) or the light travel time effect through the presence of a third body. A recent statistical analysis by Liao & Qian (2010) indicated that the presence of a third body is very frequent in W UMa binary systems. Many investigators have analyzed the orbital period change of TY UMa. Broglia & Conconi (1983) first analyzed the orbital period change of TY UMa and found that the period is changing slowly. Lister et al. (2000) suggested a decreasing period in TY UMa. Then, Stoddard et al. (2001) pointed out that the period shows a continuous increase. After that, Kang et al. (2002) reported that the period of TY UMa is composed of a secular increase and a cyclic oscillation. In summary, different investigators derived different results. The orbital period of TY UMa is worth further study.

In this paper, we presented new observed four-color light curves and radial velocities of TY UMa. Using the Wilson–Devinney (W–D) code, we analyzed the new observed light curves and radial velocities simultaneously. Moreover, the

orbital period variations of TY UMa were analyzed based on all available times of minimum light, and the properties of the suspected additional companions have been discussed.

2. OBSERVATIONS

CCD observations of TY UMa were taken with the Weihai Observatory 1.0 m telescope of Shandong University (WHOT). Observations were carried out on three nights (UT 2014 February 20 and 25 and May 21). The PIXIS 2048B CCD camera was used (Hu et al. 2014). The CCD field is $12' \times 12'$. Standard BVR_cI_c filters in the Johnson–Cousin–Bessel $UBVR_cI_c$ filter set were used during the observation. The typical integration time was 50 s for the B filter, 30 s for the V filter, 15 s for the R_c filter, and 10 s for the I_c filter. Two nearby stars GSC 03836-00293 ($\alpha_{2000.0} = 12^{\text{h}}08^{\text{m}}16^{\text{s}}.6$, $\delta_{2000.0} = 56^{\circ}02'55''.1$) and GSC 03836-00294 ($\alpha_{2000.0} = 12^{\text{h}}08^{\text{m}}30^{\text{s}}.7$, $\delta_{2000.0} = 56^{\circ}02'58''.8$) were selected as the comparison and the check. Using the following equation,

$$\text{Min. I} = HJD\ 2456714.1341 + 0.35454813E, \quad (1)$$

the orbital phases were calculated. The BVR_cI_c light curves of TY UMa are plotted in Figure 1. The curves have a clear O'Connell effect and the maximum at phase 0.25 is brighter than phase 0.75 by ~ 0.012 mags in V band. Four times of minimum light were determined from our observation.

We obtained nine spectra of TY UMa on UT 2014 November 3 with the B&C spectrograph on the Steward Observatory 2.3 m Bok telescope. Spectra were also obtained for spectral and radial velocity standard stars. The spectrograph configuration was such as to produce 1.35 \AA resolution (2 pixel), using the 832 mm^{-1} grating in second order and a slit width of 1.5 arcsec . Wavelength coverage was about 3850 to 4700 \AA .

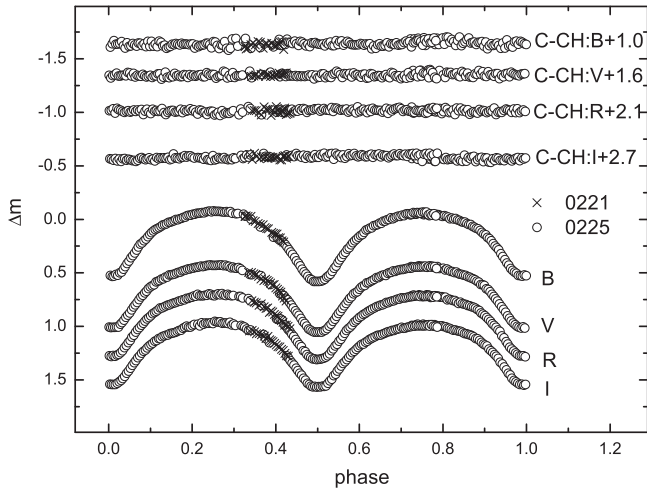


Figure 1. CCD photometric light curves of TY UMa in the $BVR_C I_C$ bands obtained using the 1.0 m telescope at Weihai Observatory of Shandong University. Different symbols represent different days.

The spectra were reduced with IRAF and radial velocities were obtained with task fxcor. The spectral type of TY UMa was concluded to be F8 V. The template spectrum was of HD 222368, a radial velocity standard of spectral type F7, similar to the primary of TY UMa. Two peaks were visible in all of the spectra and the deblending feature of fxcor was used to derive the velocities of each star. The radial velocities of TY UMa are listed in Table 1. Due to the lack of observing time, we did not obtain radial velocities after phase 0.75.

3. LIGHT CURVE SOLUTIONS

The light curve of TY UMa is asymmetric and changes seasonally. Investigators have provided various explanations for the asymmetries. Broglia & Conconi (1983) proposed that TY UMa has a thin common unstable envelope. Lister et al. (2000) believed there was extensive spot activity on TY UMa. Kang et al. (2002) suggested that the variation of spot location and size was the main reason for the changing shape of light curves.

In this paper, we analyzed the newly determined $BVR_C I_C$ light curves together with the radial velocities of TY UMa using the fourth version of the W-D program (Wilson & Devinney 1971; Wilson 1990, 1994). The spectral type of TY UMa was determined to be F8 V based on spectral observations, and the Terrell et al. (2012) $BVR_C I_C$ study of contact binaries agrees with this result. Therefore, the temperature of the primary component of TY UMa was chosen to be 6250 K. The bolometric and bandpass limb-darkening coefficients were taken from van Hamme (1993). The gravity-darkening and bolometric albedo coefficients were set to be $g_{1,2} = 0.32$ and $A_{1,2} = 0.5$ following Lucy (1967) and Ruciński (1969), which are appropriate for stars having convective envelopes. Extensive testing has certified that TY UMa is a contact binary, and so the solutions are made in mode 3. During our solutions, the adjustable parameters were as follows: the orbital inclination i , the mass ratio q , the effective temperature of the secondary component T_2 , the monochromatic luminosity of Star 1 L_1 , and the dimensionless potential of the two components $\Omega_1 = \Omega_2$.

Figure 1 shows constant light in the primary eclipse, indicating that TY UMa should be a W-subtype W UMa

Table 1
Heliocentric Radial Velocities of TY UMa

JD (Hel.) 2450000+	Phase ^a	RV ₁ (km s ⁻¹)	Errors (km s ⁻¹)	RV ₂ (km s ⁻¹)	Errors (km s ⁻¹)
6965.0179	0.6156	-75.4	±9.1	87.7	±27.4
6965.0259	0.6382	-79.5	±10.4	169.1	±17.2
6965.0289	0.6467	-86.0	±9.7	185.1	±17.7
6965.0363	0.6675	-93.9	±8.2	200.0	±22.2
6965.0393	0.6760	-99.3	±8.5	223.2	±8.9
6965.0424	0.6847	-96.0	±6.1	247.0	±33.9
6965.9482	0.2395	108.3	±7.0	-265.8	±13.5
6965.9629	0.2810	108.1	±6.6	-275.3	±7.5
6965.9748	0.3146	105.6	±6.6	-256.2	±15.3

^a Phases are computed with Equation (1).

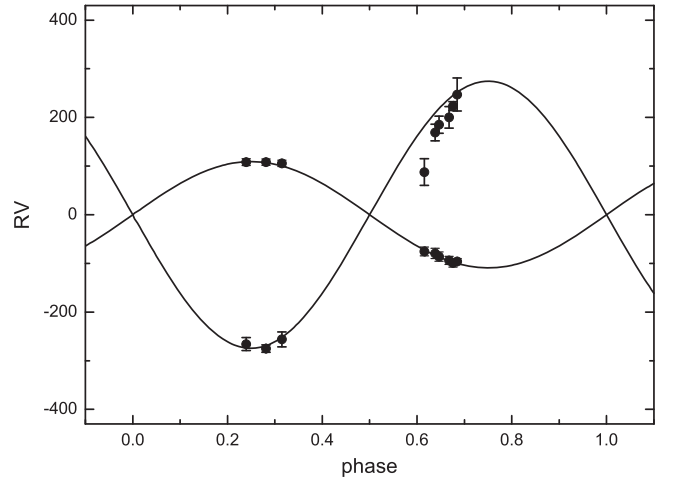


Figure 2. Radial velocity curve of TY UMa fitted by a sine curve.

system. From the solutions, the mass ratio of TY UMa was determined to be 2.523 corresponding to the photometric light curves. As in the case of other late-type contact binary stars (e.g., EQ Tau; Li et al. 2014), a deep convective envelope along with fast rotation can produce a strong magnetic dynamo and solar-like activity, including photospheric dark spots. Therefore, the asymmetry light curves of TY UMa can be plausibly explained by the presence of dark spots. Then, we used the spot model of the W-D program to fit the asymmetry light curves. Extensive testing shows that a dark spot on the primary component leads to the best fit. The derived photometric elements are listed in Table 2. The residual of the solution with spots is much smaller than that without spots. Therefore, we adopted the dark spot model as the final solution. The radial velocity curve of TY UMa fitted by a sine curve is displayed in Figure 2. The theoretical light curves computed with the dark spot model are shown in Figure 3. The geometrical structure of TY UMa is plotted in Figure 4.

4. ORBITAL PERIOD INVESTIGATION

The orbital period of TY UMa has been analyzed by many investigators. Broglia & Conconi (1983) found that the period of TY UMa was continuously increasing. Lister et al. (2000) reported a decreasing period in TY UMa. Stoddard et al. (2001) suggested a continuous increase of the period. Afterwards, Kang et al. (2002) reported a cyclic change

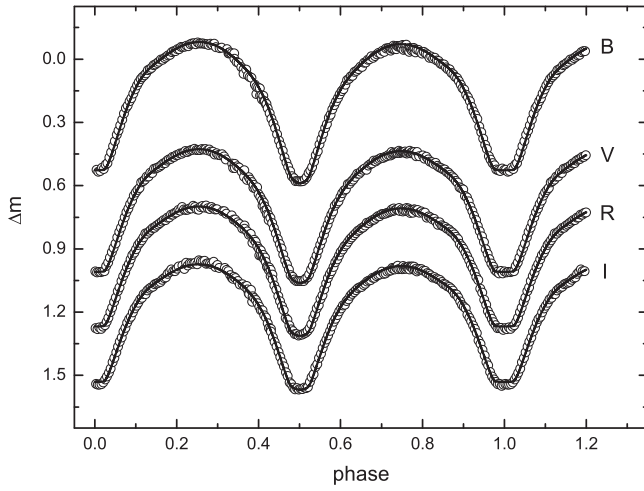


Figure 3. Comparison between observations (open symbols) and their theoretical values (solid lines) for TY UMa.

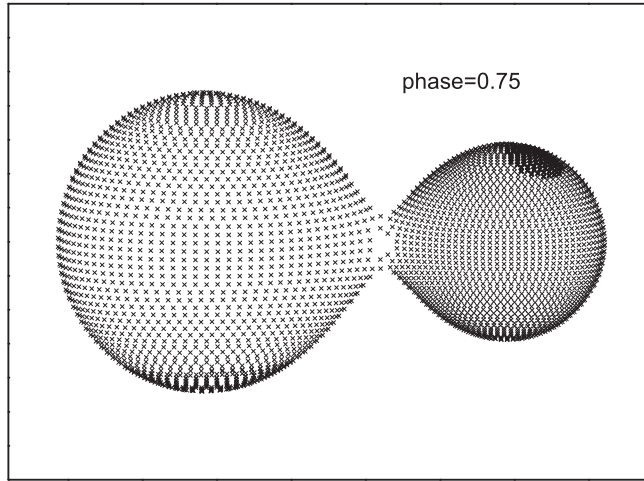


Figure 4. Geometrical structure of TY UMa at phase 0.75.

superimposed on a long-term increase in the period of TY UMa. In brief, different investigators derived discordant results.

From the WHOT observation, four new times of minimum light were determined. In addition, we newly derived 27 eclipses from the AAVSO (American Association of Variable Stars Observers) International Database.⁴ To analyze the period variation of TY UMa, we collected a total of 305 times of light time minimum (155 visual (vis), 18 photographic (phg), 43 photoelectric (pe), and 89 ccd) and listed them in the first column of Table 3. The times of minimum light span more than 84 yr. Using the same linear ephemeris as Kang et al. (2002),

$$\text{Min. I} = 2439532.4951 + 0.35453954E, \quad (2)$$

the $(O - C)_1$ values were determined and are listed in the fifth column of Table 3. The corresponding $(O - C)_1$ diagram is plotted in the top panel of Figure 5. As shown in Figure 5, the $(O - C)_1$ diagram reveals a cyclic oscillation apart from an upward parabolic variation. Therefore, a combination of a second-order polynomial and a periodic term are required to fit the $(O - C)_1$ curve satisfactorily. We used the following

Table 2
Photometric Solutions for TY UMa

Parameters	Photometric Elements		Photometric Elements	
	No Spot	Errors	With Spot	Errors
$g_1 = g_2$	0.32	Assumed	0.32	Assumed
$A_1 = A_2$	0.5	Assumed	0.5	Assumed
$x_{1bol} = x_{2bol}$	0.644		Assumed	
$y_{1bol} = y_{2bol}$	0.221		Assumed	
$x_{1B} = x_{2B}$	0.817		Assumed	
$y_{1B} = y_{2B}$	0.215		Assumed	
$x_{1V} = x_{2V}$	0.728		Assumed	
$y_{1V} = y_{2V}$	0.269		Assumed	
$x_{1R_c} = x_{2R_c}$	0.655		Assumed	
$y_{1R_c} = y_{2R_c}$	0.278		Assumed	
$x_{1I_c} = x_{2I_c}$	0.572		Assumed	
$y_{1I_c} = y_{2I_c}$	0.267		Assumed	
$T_1 (K)$	6250		Assumed	
$q (M_2/M_1)$	2.523	± 0.047	2.523	± 0.031
$T_2 (K)$	6277	± 42	6229	± 35
$i (\text{deg})$	83.9	± 1.5	84.9	± 1.1
$K_1 (\text{km s}^{-1})$	278.3		± 30.1	
$K_2 (\text{km s}^{-1})$	110.3		± 3.3	
Ω_{in}	5.9771		Assumed	
Ω_{out}	5.3667		Assumed	
$\Omega_1 = \Omega_2$	5.8220	± 0.0389	5.8956	± 0.0245
L_{1B}/L_B	0.3005	± 0.0053	0.3066	± 0.0042
L_{1V}/L_V	0.3016	± 0.0041	0.3054	± 0.0029
L_{1R_c}/L_{R_c}	0.3021	± 0.0036	0.3048	± 0.0018
L_{1I_c}/L_{I_c}	0.3026	± 0.0032	0.3043	± 0.0015
$\eta (\text{pole})$	0.2940	± 0.0048	0.2881	± 0.0039
$\eta (\text{side})$	0.3083	± 0.0060	0.3012	± 0.0048
$\eta (\text{back})$	0.3506	± 0.0111	0.3385	± 0.0084
$\eta (\text{mean})$	0.3167	± 0.0125	0.3085	± 0.0101
$r_2 (\text{pole})$	0.4450	± 0.0039	0.4395	± 0.0031
$r_2 (\text{side})$	0.4780	± 0.0053	0.4706	± 0.0042
$r_2 (\text{back})$	0.5093	± 0.0071	0.4996	± 0.0055
$r_2 (\text{mean})$	0.4767	± 0.0097	0.4693	± 0.0078
f	25.4%	$\pm 6.4\%$	13.4%	$\pm 4.0\%$
colatitude(deg)	28.6	± 3.5
longitude(deg)	135.1	± 6.1
diameter(deg)	19.7	± 2.2
$T - \text{factor}$	0.742	± 0.077
$\Sigma W(O - C)^2$	0.0110		0.0078	

equation (e.g., Irwin 1952; Paparo et al. 1988),

$$\begin{aligned}
 (O - C)_1 &= T_0 + \Delta T_0 + (P_0 + \Delta P_0)E + \frac{\beta}{2}E^2 \\
 &\quad + A \left[(1 - e^2) \frac{\sin(\nu + \omega)}{(1 + e \cos \nu)} + e \sin \omega \right] \\
 &= T_0 + \Delta T_0 + (P_0 + \Delta P_0)E + \frac{\beta}{2}E^2 \\
 &\quad + A \left[\sqrt{1 - e^2} \sin E^* \cos \omega \right. \\
 &\quad \left. + \cos E^* \sin \omega \right], \quad (3)
 \end{aligned}$$

to fit the $(O - C)_1$ curve. During the fitting process, the weight

⁴ <http://www.aavso.org/>

Table 3
Observed Times of Minimum Light for TY UMa

JD (Hel.)	Type	Min.	Epoch	$(O - C)_1$	$(O - C)_2$	$(O - C)_3$	Residuals	References
2400000+								
26093.2840	phg	s	-37906.5	0.1420	-0.0162	-0.0017	...	(1)
26423.3600	phg	s	-36975.5	0.1417	-0.0079	0.0071	...	(1)
26450.3130	phg	s	-36899.5	0.1497	0.0008	0.0158	...	(1)
26455.2740	phg	s	-36885.5	0.1471	-0.0017	0.0134	...	(1)
27283.4430	phg	s	-34549.4	0.0763	-0.0519	-0.0360	...	(2)
27360.3750	phg	s	-34332.5	0.1087	-0.0177	-0.0017	...	(3)
38112.3920	vis	s	-4005.5	0.0050	0.0188	0.0094	...	(4)
39532.4971	pe	p	0	0.0020	0.0170	0.0012	0.0020	(5)
39532.6727	pe	s	0.5	0.0003	0.0153	-0.0005	0.0003	(5)
39533.5720	phg	p	3	0.0133	0.0283	0.0125	...	(6)
39561.5685	pe	p	82	0.0012	0.0162	0.0002	0.0012	(5)
39562.4545	pe	s	84.5	0.0008	0.0158	-0.0002	0.0008	(5)
39563.5179	pe	s	87.5	0.0006	0.0156	-0.0004	0.0006	(5)

References. (1) PZ 4, 196, (2) PZ 4, 235, (3) PZ 5, 108, (4) Pohl & Kizilirmak (1964), (5) Broglia & Conconi (1981), (6) Götz (1969), (7) Stoddard et al. (2001), (8) BBSAG 21, (9) BBSAG 23, (10) Samolyk (1992), (11) GEOS 3, (12) BBSAG 47, (13) Broglia & Conconi (1983), (14) Hoffmann (1981), (15) BBSAG 59, (16) BBSAG 60, (17) BBSAG 61, (18) BBSAG 65, (19) BBSAG 66, (20) BBSAG 71, (21) BBSAG 76, (22) BBSAG 77, (23) BBSAG 80, (24) BBSAG 81, (25) BBSAG 84, (26) BBSAG 83, (27) BBSAG 86, (28) BBSAG 94, (29) BBSAG 95, (30) BBSAG 97, (31) BBSAG 98, (32) BBSAG 101, (33) BBSAG 103, (34) BBSAG 104, (35) BBSAG 106, (36) BBSAG 108, (37) Baldwin & Samolyk (1999), (38) BBSAG 107, (39) BBSAG 113, (40) BBSAG 109, (41) BBSAG 111, (42) Agerer & Huebscher (1998), (43) BBSAG 112, (44) BBSAG 114, (45) BBSAG 115, (46) BBSAG 117, (47) This paper (AAVSO), (48) Lister et al. (2000), (49) O-C Gateway, (50) Agerer & Huebscher (2002), (51) BBSAG 124, (52) Kang et al. (2002), (53) Agerer & Huebscher (2003), (54) Nelson (2004), (55) VSOLJ 42, (56) Hubscher (2005), (57) JAAVSO 42, (58) Krajci (2005), (59) Hubscher (2005), (60) Nelson (2006), (61) Nelson 20, (62) AAVSO 12, (63) Hubscher et al. (2006), (64) Hubscher (2007), (65) Hubscher et al. (2009), (66) AAVSO 79, (67) AAVSO 86, (68) VSOLJ 48, (69) Diethelm (2009), (70) AAVSO 109, (71) Hubscher et al. (2010), (72) Dvorak (2011), (73) AAVSO 130, (74) Diethelm (2010), (75) Hubscher & Monninger (2011), (76) BRNO 37, (77) Hubscher et al. (2012), (78) Diethelm (2011), (79) AAVSO 151, (80) Hubscher & Lehmann (2013), (81) BRNO 38, (82) Nelson (2012), (83) Diethelm (2012), (84) VSOLJ 56, (85) This paper (WHOT).

(This table is available in its entirety in machine-readable and Virtual Observatory (VO) forms.)

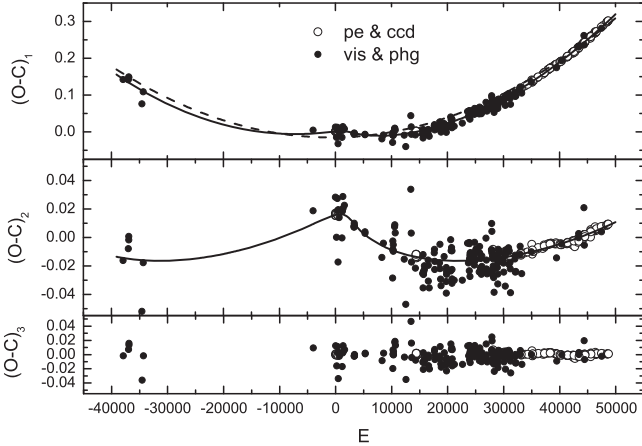


Figure 5. $O - C$ diagram of TY UMa. The upper panel shows the $(O - C)_1$ curve constructed with Equation (2). The middle panel gives the $(O - C)_2$ curve from the quadratic term of Equation (3) (solid line). The $(O - C)_3$ values from the whole effect of Equation (3) are displayed in the bottom.

of the vis and phg data is 1 and that of the pe and ccd data is 8. The parameters and the derived values are shown in Table 4. The first and second terms in the equation refer to the revisions on the initial epoch and the orbital period, respectively. The quadratic term indicates that the orbital period is increasing, a continuous period increase rate at $dp/dt = 5.18(\pm 0.21) \times 10^{-7}$ days yr^{-1} was calculated. The periodic term suggests a cyclic oscillation with an amplitude of 0.0182 days and a period of 51.7 yr. The $(O - C)_2$ values are plotted in the middle panel of Figure 5 when removing the quadratic term, where the cyclic oscillation can be seen more

Table 4 Parameters for the Fit of Times of Minimum Light				
Parameters	Values	Errors	Values	Errors
	A_3		A_4	
ΔT_0 (days)	-0.0150	± 0.0017
ΔP_0 (days)	1.96×10^{-7}	$\pm 0.64 \times 10^{-7}$
β (days yr^{-1})	5.18×10^{-7}	$\pm 0.21 \times 10^{-7}$
$A_{3,4}$ (d)	0.0182	± 0.0029	0.0015	± 0.0002
e	0.73	± 0.16	0	Fixed
$P_{3,4}$ (yr)	51.7	± 7.3	10.0	± 0.9
ω ($^\circ$)	126.8	± 7.1
T_p (HJD)	2440280.0	± 310.7

clearly. After both the long term increase and the cyclic change are subtracted, the $(O - C)_3$ values are displayed in the lowest panel of Figure 5. We found that the pe and ccd data of the $(O - C)_3$ values may show a cyclic oscillation. A nonlinear least-squares solution for the pe and ccd data of $(O - C)_3$ values leads to the following equation:

$$(O - C)_3 = 0.00051(\pm 0.00004) + 0.0015(\pm 0.0002) \times \sin(0.0350(\pm 0.0029)E + 31.8(\pm 1.7)). \quad (4)$$

A cyclic oscillation with a very small amplitude of 0.0015 days and a period of 10.0 yr is determined. The corresponding fitting curve is shown in the upper panel of Figure 6. When the cyclic term is removed, the residuals are displayed in the lower panel of Figure 6 where no changes can be traced.

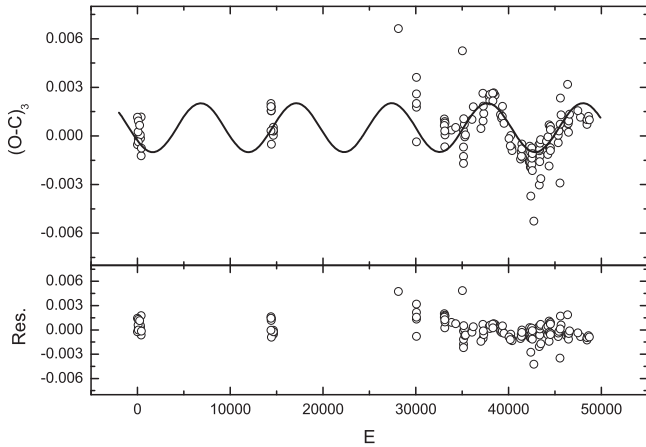


Figure 6. Pe and ccd data of the $(O - C)_3$ values phased with Equation (4) for TY UMa.

5. DISCUSSION AND CONCLUSIONS

According to our study of the four-color light curves and radial velocities of TY UMa, it is found that TY UMa is a W-subtype shallow contact binary with a fill-out factor of $13.4 \pm 4.0\%$. Additionally, the discovery of a high-latitude dark spot at phase 0.75 makes TY UMa an interesting system to study. Combining the W-D solutions and radial velocities of TY UMa, absolute parameters can be determined as follows:

$$\begin{aligned} a &= 2.74 \pm 0.23 R_{\odot}, & M_1 &= 0.62 \pm 0.07 M_{\odot}, \\ M_2 &= 1.57 \pm 0.14 M_{\odot}, & R_1 &= 0.85 \pm 0.07 R_{\odot}, \\ R_2 &= 1.29 \pm 0.10 R_{\odot}, & L_1 &= 0.99 \pm 0.10 L_{\odot}, \quad \text{and} \\ L_2 &= 0.63 \pm 0.05 L_{\odot}. \end{aligned}$$

By combining our newly acquired times of minimum light with those compiled from literatures, we analyzed the orbital period change of TY UMa. It is shown that the orbital period changes in a complicated mode, i.e., a secular period increase superimposed with two cyclic variations.

5.1. Mass Transfer and Possible Evolution

According to the orbital period analysis, we found that the orbital period of TY UMa secularly increases at a rate of $dp/dt = 5.18(\pm 0.21) \times 10^{-7} \text{ days yr}^{-1}$. Continuous period increase for binaries is generally caused by mass transfer from the less massive component to the more massive one. Considering a conservative mass transfer and combining the absolute parameters, the mass transfer rate of TY UMa can be estimated using the following equation:

$$\frac{\dot{P}}{P} = -3\dot{M}_1 \left(\frac{1}{M_1} - \frac{1}{M_2} \right), \quad (5)$$

the mass transfer at a rate of $dM_1/dt = -4.99(\pm 0.20) \times 10^{-7} M_{\odot} \text{ yr}^{-1}$ was determined. The negative sign implies that the less massive component, M_1 , is losing mass. With increasing period, the separation between both components may increase, which may cause the contact degree to decrease. TY UMa may evolve from the present shallow contact configuration to a non-contact phase.

5.2. TY UMa is a Quadruple Star?

When the continuous period increase is removed from the $(O - C)_1$ curve, two additional cyclic components may exist in

the $(O - C)_2$ values. These can be explained in two ways: the Applegate mechanism (Applegate 1992) and the light travel time effect.

According to Applegate (1992), the magnetic activity of one late-type component in the binary system can produce the orbital period modulation. As the active component goes through its magnetic cycle, the distribution of angular momentum changes, leading to a variation in the stellar oblateness. These changes are communicated to the orbit by gravity, resulting in the variation of the orbital period. Assuming that the two period oscillations are consequences of magnetic cycles in the primary component and using the relation $\frac{\Delta P}{P} = -9 \frac{\Delta Q}{Ma^2}$ (Lanza & Rodonò 2002), we can calculate the variations of the quadrupole moment for the primary component. We determined $\Delta Q_3 = 3.0 \times 10^{49} \text{ g cm}^2$ for the long-term cycle and $\Delta Q_4 = 1.4 \times 10^{49} \text{ g cm}^2$ for the short-term cycle. These two values are both two orders of magnitude smaller than typical values 10^{51} – 10^{52} g cm^2 for active close binaries. Therefore, the Applegate mechanism cannot explain the two cyclic period modulations for TY UMa.

Tokovinin et al. (2006) discovered that 96% of a sample of spectroscopic binaries which have periods less than three days are in triple or quadruple systems. Pribulla & Rucinski (2006) also claimed that most contact binary stars are in multiple systems. Many investigators (see, e.g., Steffen et al. 2011; Gies et al. 2012; Lee et al. 2013; Rappaport et al. 2013; Conroy et al. 2014) have successfully explained the orbital period modulation based on the light travel time effect of additional companions. Therefore, the two cyclic variations should be caused by the light travel time effect of third and fourth components, which means that TY UMa is a quadruple system. Using the following equation,

$$f(m) = \frac{(m_{3,4} \sin i)^3}{(m_1 + m_2 + m_{3,4})^2} = \frac{4\pi}{GP_{3,4}^2} \times (a_{12} \sin i)^3, \quad (6)$$

mass functions of $f_3(m) = 1.17(\pm 0.56) \times 10^{-2} M_{\odot}$ and $f_4(m) = 1.75(\pm 0.70) \times 10^{-4} M_{\odot}$ are determined for the assumed third and fourth bodies. If it is assumed that the two additional components are coplanar with that of the eclipsing pair of TY UMa ($i_{3,4} = 84^\circ 9'$), then the masses of the third and fourth objects should be $m_3 = 0.434(\pm 0.130) M_{\odot}$ and $m_4 = 0.097(\pm 0.022) M_{\odot}$. To verify the presence of the two additional companions, during the W-D solutions we added a third light (i.e., we set l_3 as an adjustable parameter), but l_3 is usually negative, indicating that the two additional companions are possibly too faint relative to the eclipsing pair to contribute to the whole system. According to the mass of the two additional companions, the third body may be a compact object such as a white dwarf, and the fourth body may be a brown dwarf.

Although the times of minimum light agree very well with the observed light travel time signal as caused by the presence of third and fourth companions, we cannot completely exclude the possibility of the Applegate mechanism. Future observations should be carried out to confirm these results.

This work is partly supported by the National Natural Science Foundation of China (Nos. 11203016, 11333002, 10778619, 10778701, U1431105), by the Natural Science Foundation of Shandong Province (No. ZR2012AQ008, ZR2014AQ019), and by the Open Research Program of Key

Laboratory for the Structure and Evolution of Celestial Objects (No. OP201303). We have used data from the AAVSO International Database. We appreciate Ai Jiaming for assisting with the observations of TY UMa. New CCD photometric observations of TY UMa were obtained with the 1 m telescope at Weihai Observatory of Shandong University. We thank the Steward Observatory for providing the spectroscopic observation time. We thank the referee very much for very useful comments.

REFERENCES

- Agerer, F., & Huebscher, J. 1998, IBVS, [4562](#), 1
Agerer, F., & Huebscher, J. 2002, IBVS, [5296](#), 1
Agerer, F., & Huebscher, J. 2003, IBVS, [5484](#), 1
Applegate, J. H. 1992, [ApJ](#), [385](#), 621
Baldwin, M. E., & Samolyk, G. 1999, in *Observed Minima Timings of Eclipsing Binaries No. 5*, (Cambridge, MA: AAVSO)
Beljawsky, S. 1933, NNVS, [4](#), 196
Broglia, P., & Conconi, P. 1981, IBVS, [1949](#), 1
Broglia, P., & Conconi, P. 1983, A&AS, [51](#), 97
Conroy, K. E., Prša, A., Stassun, K. G., et al. 2014, [AJ](#), [147](#), 45
Diethelm, R. 2009, IBVS, [5894](#), 1
Diethelm, R. 2010, IBVS, [5945](#), 1
Diethelm, R. 2011, IBVS, [5992](#), 1
Diethelm, R. 2012, IBVS, [6029](#), 1
Dvorak, S. W. 2011, IBVS, [5974](#), 1
Gies, D. R., Williams, S. J., Matson, R. A., et al. 2012, [AJ](#), [143](#), 137
Götz, W. 1969, MitVS, [5](#), 67
Hoffmann, M. 1981, IBVS, [1978](#), 1
Hubscher, J. 2005, IBVS, [5643](#), 1
Hubscher, J. 2007, IBVS, [5802](#), 1
Hubscher, J., & Lehmann, P. B. 2013, IBVS, [6070](#), 1
Hubscher, J., Lehmann, P. B., Monninger, G., Steinbach, H.-M., & Walter, F. 2010, IBVS, [5918](#), 1
Hubscher, J., Lehmann, P. B., & Walter, F. 2012, IBVS, [6010](#), 1
Hubscher, J., & Monninger, G. 2011, IBVS, [5959](#), 1
Hubscher, J., Paschke, A., & Walter, F. 2005, IBVS, [5657](#), 1
Hubscher, J., Paschke, A., & Walter, F. 2006, IBVS, [5731](#), 1
Hubscher, J., Steinbach, H.-M., & Walter, F. 2009, IBVS, [5874](#), 1
Hu, S. M., Han, S. H., Guo, D. F., & Du, J. J. 2014, [RAA](#), [14](#), 719
Irwin, J. B. 1952, [ApJ](#), [116](#), 211
Kang, Y. W., Oh, K.-D., Kim, C.-H., et al. 2002, [MNRAS](#), [331](#), 707
Krajci, T. 2005, IBVS, [5592](#), 1
Lanza, A. F., & Rodonò, M. 2002, AN, [323](#), 424
Lee, J. W., Hinse, T. C., & Park, J.-H. 2013, [AJ](#), [145](#), 100
Li, K., Qian, S.-B., Hu, S.-M., & He, J.-J. 2014, [AJ](#), [147](#), 98
Liao, W.-P., & Qian, S.-B. 2010, [MNRAS](#), [405](#), 1930
Lister, T. A., McDermid, R. M., & Hilditch, R. W. 2000, [MNRAS](#), [317](#), 111
Lucy, L. B. 1967, *Z. Astrophys.*, [65](#), 89
Nelson, R. H. 2004, IBVS, [5493](#), 1
Nelson, R. H. 2006, IBVS, [5672](#), 1
Nelson, R. H. 2007, IBVS, [5760](#), 1
Nelson, R. H. 2012, IBVS, [6018](#), 1
Paparo, M., Szeidl, B., & Mahdy, H. A. 1988, [Ap&SS](#), [149](#), 73
Pohl, E., & Kizilirmak, A. 1964, AN, [288](#), 69
Pribulla, T., & Rucinski, S. M. 2006, [AJ](#), [131](#), 2986
Rappaport, S., Deck, K., Levine, A., et al. 2013, [ApJ](#), [768](#), 33
Ruciński, S. M. 1969, *AcA*, [19](#), 245
Samolyk, G. 1992, JAAVSO, [21](#), 111
Steffen, J. H., Quinn, S. N., Borucki, W. J., et al. 2011, [MNRAS](#), [417](#), L31
Stoddard, M. L., Samec, R. G., Faulkner, D. R., & Walker, R. L. 2001, IBVS, [5169](#), 1
Terrell, D., Gross, J., Cooney, & Walter, R. 2012, [AJ](#), [143](#), 99
Tokovinin, A., Thomas, S., Sterzik, M., & Udry, S. 2006, [A&A](#), [450](#), 681
van Hamme, W. 1993, [AJ](#), [106](#), 2096
Wilson, R. E., & Devinney, E. J. 1971, [ApJ](#), [166](#), 605
Wilson, R. E. 1990, [ApJ](#), [356](#), 613
Wilson, R. E. 1994, [PASP](#), [106](#), 921