Selective spin coupling through a single exciton

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We present a novel scheme for performing a conditional phase gate between two spin qubits in adjacent semiconductor quantum dots through delocalized single exciton states, formed through the inter-dot Förster interaction. We consider two resonant quantum dots, each containing a single excess conduction band electron whose spin embodies the qubit. We demonstrate that both the two-qubit gate, and arbitrary single-qubit rotations, may be realized to a high fidelity with current semiconductor and laser technology.

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Quantum information processors would provide us with revolutionary algorithms for a wide range of appications [1]. Until recently, schemes for the implementation of such devices within semiconductor quantum dots (QDs) usually fell into distinct categories based upon their proposed qubit. For example, quantum gates based on exciton qubits [2–4] and spin qubits [5] have been put forward. However, a new field of 'hybrid' schemes is now emerging with the aim of marrying the advantageous aspects of these individual candidate systems [6–8].

In this Letter, we analyse the possibility of all-optical selective coupling of electron spins in adjacent QDs via intermediate excitonic states. Motivated by the work of Refs. [7, 8], where the static dipole-dipole interactions between two excitonic states were exploited, our scheme also benefits from the fast (picosecond) time scales of excitonic interactions, along with the relative stability of spin qubits to decoherence [9, 10]. In contrast to the previous work, we consider the inter-dot resonant energy transfer (Förster) interaction [4] and find that a simple two qubit gate requires the excitation of *single exciton states only*. This requires only one laser pulse, and so we believe that it may be more readily implemented with current semiconductor and laser technology, and that it represents a significant step towards a working spin-based optical quantum logic gate.

We consider two resonant QDs, each of which are n-doped so that they contain a single excess conduction band electron; samples of this type have already been made [11]. The qubit basis $|0\rangle$ and $|1\rangle$ is defined by the electron spin states $m_z = -1/2$ and $1/2$ respectively. Ideally, we would like to implement a controlled phase (CPHASE) gate given by: $\{|00\rangle \rightarrow |00\rangle, |01\rangle \rightarrow |01\rangle, |10\rangle \rightarrow |10\rangle$ and $|11\rangle \rightarrow -|11\rangle\}$. To perform this operation, we consider a single laser radiating both QDs with σ^+ polarized light, resonant with the s-shell heavy-hole exciton creation energies (the formulation for light-holes is equivalent). These excitons are necessarily in the z-angular momentum states given by $|3/2^{hh}, -1/2^e\rangle$. As noted in Refs. [7, 8], such a polarized laser will only create an exciton on a QD if its excess electron is in the spin state $|m_z = 1/2\rangle$, due to the Pauli blocking effect for the two conduction band electrons. We denote such a combined exciton-spin (trion) state by $|\mathrm{X}\rangle$.

The Hamiltonian for our two resonantly coupled QDs interacting with the single classical laser field may be written as:

$$\begin{aligned} H(t) &= \omega_a(|\mathrm{X}\rangle\langle\mathrm{X}| \otimes \hat{I} + \hat{I} \otimes |\mathrm{X}\rangle\langle\mathrm{X}|) + V_{\mathrm{XX}}|\mathrm{XX}\rangle\langle\mathrm{XX}| \\ &\quad + V_{\mathrm{F}}(|1\mathrm{X}\rangle\langle\mathrm{X}1| + \mathrm{H.c.}) \\ &\quad + \Omega\cos\omega_l t(|1\rangle\langle\mathrm{X}| \otimes \hat{I} + \hat{I} \otimes |1\rangle\langle\mathrm{X}| + \mathrm{H.c.}) \quad (1) \end{aligned}$$

where the ordering of each operator is $|\mathrm{dot}\,1\rangle \otimes |\mathrm{dot}\,2\rangle$. ω_a is the exciton creation energy for each dot, V_{F} is the inter-dot Förster coupling strength, V_{XX} is the biexcitonic energy shift due to exciton dipole-dipole interactions, Ω is the time-dependent coupling between laser and dot (taken to be the same for both QDs), and ω_l is the laser frequency. We have assumed that the energy difference between states $|0\rangle$ and $|1\rangle$ is negligible on the exciton energy scales. Single-particle tunneling between the dots is also neglected. The frequency dependence of the Förster interaction [12] can also be ignored for this two dot case, but will become important for a chain of dots in a scaled up device.

Our QDs are assumed to be smaller than the bulk exciton radius and hence within the strong confinement regime. Therefore, mixing of single-particle electron and hole states due their Coulomb interactions may be neglected and any energy shift can be incorporated into ω_a [13]. However, the Coulomb energies are still important as they ensure that the resonance condition for single-particle tunneling is not the same as that for resonant exciton transfer.

As a consequence of the Pauli exclusion principle, there are no matrix elements which can cause transitions between $|0\rangle$ and $|\mathrm{X}\rangle$ on either dot. Additionally, to first order, the Förster process, which is non-magnetic, couples only the states $|1\mathrm{X}\rangle$ and $|\mathrm{X}1\rangle$ and exchanges no spin information; hence single delocalized excitons may only exist on pairs of dots *in the same spin state*, and this is the essence of our scheme.

We can see that the Hamiltonian of Eq. 1 may be decoupled into four separate sub-

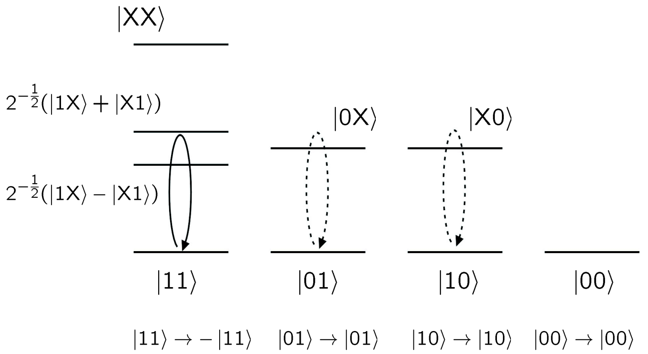

FIG. 1: Schematic of the energy level structure of two resonantly coupled QDs, showing its dependence on the spin state of the excess electron on each dot. When the spins are both in state $|1\rangle$ an energy shift of the single exciton states occurs due to their Förster coupling. Hence a phase shift may be accumulated using a laser resonant with the dipole allowed transition. This does not occur for any of the other states shown due to the Pauli blocking mechanism.

spaces with no interactions between them: $\{|00\rangle\}, \{|01\rangle, |0X\rangle\}, \{|10\rangle, |X0\rangle\}, \{|11\rangle, |1X\rangle, |X1\rangle, |XX\rangle\}$. We are primarily interested in the dynamics of the $\{|11\rangle, |1X\rangle, |X1\rangle, |XX\rangle\}$ subspace where the coupling between the states $|11\rangle, |1X\rangle$, and $|X1\rangle$ will allow us to generate a phase shift on the input state $|11\rangle$. In this subspace the Hamiltonian is:

$$H_{sub}(t) = \begin{pmatrix} 0 & \Omega\cos\omega_l t & \Omega\cos\omega_l t & 0 \\ \Omega\cos\omega_l t & \omega_a & V_{\rm F} & \Omega\cos\omega_l t \\ \Omega\cos\omega_l t & V_{\rm F} & \omega_a & \Omega\cos\omega_l t \\ 0 & \Omega\cos\omega_l t & \Omega\cos\omega_l t & 2\omega_a + V_{\rm XX} \end{pmatrix}. \quad (2)$$

As we are considering the case of two resonant QDs it is sensible to first rewrite the Hamiltonian of in a basis of its eigenstates when $\Omega = 0$. These are $|\psi_+\rangle = 2^{-1/2}(|1X\rangle + |X1\rangle)$ and $|\psi_-\rangle = 2^{-1/2}(|1X\rangle - |X1\rangle)$. We obtain:

$$H_{sub}(t) = \begin{pmatrix} 0 & \Omega'\cos\omega_l t & 0 & 0 \\ \Omega'\cos\omega_l t & \omega_a + V_{\rm F} & 0 & \Omega'\cos\omega_l t \\ 0 & 0 & \omega_a - V_{\rm F} & 0 \\ 0 & \Omega'\cos\omega_l t & 0 & 2\omega_a + V_{\rm XX} \end{pmatrix}, \quad (3)$$

where the only dipole allowed transitions are between the states $|11\rangle$ and $|\psi_+\rangle$, and between $|\psi_+\rangle$ and $|XX\rangle$, and $\Omega' = \sqrt{2}\Omega$.

We shall now transform this Hamiltonian into a frame rotating with the laser frequency ω_l with respect to both dipole allowed transitions. Within the rotating wave approximation, Eq. 3 becomes:

$$H'_{sub} = \begin{pmatrix} 0 & \Omega'/2 & 0 & 0 \\ \Omega'/2 & 0 & 0 & \Omega'/2 \\ 0 & 0 & -2V_{\rm F} & 0 \\ 0 & \Omega'/2 & 0 & V_{\rm XX} - 2V_{\rm F} \end{pmatrix}, \quad (4)$$

where we set our laser frequency to be resonant with the dipole allowed $|11\rangle$ to $|\psi_+\rangle$ transition, i.e. we set $\omega_l = (\omega_a + V_{\rm F})$, as this will allow us to generate the desired CPHASE gate (see Fig. 1). Under the condition

$$|\Omega'|/2 \ll |V_{\rm XX} - 2V_{\rm F}|, \quad (5)$$

the double excitation of $|11\rangle$ to $|XX\rangle$ is suppressed and we may use second order degenerate perturbation theory to decouple the two subspaces $\{|11\rangle, |\psi_+\rangle\}$ and $\{|\psi_-\rangle, |XX\rangle\}$. By doing this, we may write down an effective Hamiltonian in the degenerate subspace $\{|11\rangle, |\psi_+\rangle\}$ as

$$H_{eff} = \begin{pmatrix} 0 & \Omega'/2 \\ \Omega'/2 & -\Omega'^2/4(V_{\rm XX} - 2V_{\rm F}) \end{pmatrix}. \quad (6)$$

However, within the condition of Eq. 5 the difference in diagonal terms is very small compared to the magnitude of the off-diagonal terms. Therefore, the eigenstates of H_{eff} are given approximately by $2^{-1/2}\{|11\rangle + |\psi_+\rangle\}$ and $2^{-1/2}\{|11\rangle - |\psi_+\rangle\}$, with respective eigenvalues of $\Omega'/2$ and $-\Omega'/2$.

Transforming back to the lab frame leads to the following evolution of the initial state $|11\rangle$:

$$|11\rangle \to \cos\left[\frac{1}{2}\int_0^T \Omega'(t)\mathrm{d}t\right]|11\rangle + ie^{-i(\omega_a + V_{\rm F})}\sin\left[\frac{1}{2}\int_0^T \Omega'(t)\mathrm{d}t\right]|\psi_+\rangle. \quad (7)$$

Hence, for $\int_0^T \Omega'(t)\mathrm{d}t = 2\pi$ we generate the required phase change $|11\rangle \to -|11\rangle$.

If the CPHASE gate is to work, we must ensure that no basis state, other than $|11\rangle$, experiences a phase change within the gate time T. Returning to the Hamiltonian of Eq. 1 we see that the state $|00\rangle$ is completely uncoupled from any other state, and so it will undergo no phase change during the gate operation. The states $|10\rangle$ and $|01\rangle$ are not uncoupled and the transitions $|01\rangle \leftrightarrow |0X\rangle$ and $|10\rangle \leftrightarrow |X0\rangle$ are possible. Taking the $\{|01\rangle, |0X\rangle\}$ subspace as an example (the $\{|10\rangle, |X0\rangle\}$ subspace is entirely equivalent for two identical dots) and moving into the rotating frame as above, we may write:

$$H' = \begin{pmatrix} 0 & \Omega/2 \\ \Omega/2 & \omega_a - \omega_l \end{pmatrix} = \begin{pmatrix} 0 & \Omega/2 \\ \Omega/2 & -V_{\rm F} \end{pmatrix}, \quad (8)$$

when we insert the laser frequency $\omega_l = (\omega_a + V_{\rm F})$. Therefore, under the condition

$$|\Omega|/2 \ll |V_{\rm F}| \quad (9)$$

the initial state $|01\rangle$ will experience no phase shift due to the laser. [20]

In order to estimate the characteristic timescale of our CPHASE gate, we consider typical interaction strengths exploit Pauli blocking as above, a π-pulse of σ^+ light tuned to the exciton resonance gives us the transformation:

$$a|0\rangle + b|1\rangle \rightarrow a|0\rangle + b|\mathrm{X}\rangle. \quad (10)$$

If we now allow the system to evolve freely for a time T_s, and then de-excite via a further π-pulse, we obtain the state $a|0\rangle + b\exp(-i\omega_a T_s)|1\rangle$. Here, ω_a is the energy difference between $|\mathrm{X}\rangle$ and $|1\rangle$, and the final state corresponds to a rotation of the input state by $\phi = \omega_a T_s$ about the z-axis of the Bloch sphere.

Natural size and composition fluctuations in self-assembled dot samples allow for energy selective addressing of individual QDs and we must be able to move two coupled dots in and out of resonance in order to perform both single- and two-qubit manipulations. We propose the use of an inhomogeneous external electric field to enable us to move between the non-resonant and resonant cases on a timescale which is short compared to typical spin decoherence times [9, 10]. Field gradients of approximately 20 (MV/m)/μm have been obtained experimentally [19] with Stark shifts of about 2 meV seen in a field of 0.2 MV/m in the same experiment. Therefore, for adjacent dots spaced by 5 nm we can reasonable expect an energy selectivity of 1 meV. As any laser resonant with the $|1\rangle \rightarrow |\mathrm{X}\rangle$ transition on one dot will behave as a detuned laser on a second nearby dot, we may estimate the fidelity $\mathcal{F}$ of avoiding unwanted transitions in the second dot from $\mathcal{F} = 1 - 4(\Omega^2/\delta^2)$, where δ is the laser detuning. Hence, for $\delta = 1$ meV, to achieve a π-pulse with 99% fidelity requires an operation time of $T_s \sim 40$ ps (reducing to $T_s \sim 13$ ps for 90% fidelity). As the gate time scales as $1/\delta$ it would be significantly reduced in a well optimized experiment.

Optically induced single spin rotations about a second axis (for example about the X-axis) are a more difficult proposition. Raman transitions involving the light-hole levels $|m_z^{lh} = \pm 1/2\rangle$ and a single exciton within a QD have been proposed as a means to achieve direct optical transitions between the spin states $|0\rangle$ and $|1\rangle$ [7, 21]. Unfortunately, as the light-hole states are not hole ground states in GaAs based QDs they suffer from extremely short decoherence times. Pazy *et al.* [7] propose the use of II-IV semiconductors to shift the light-hole energy levels to become ground states. However, since GaAs based QDs are the current state-of-the-art we would like to be able to implement universal single qubit rotations in such systems. We therefore adapt the scheme of Ref. [7] by using detuned lasers to induce Raman transitions between $|0\rangle$ and $|1\rangle$ whilst exciting very little population to the fast decaying light-hole states. Fig. 3 shows a numerical simulation which demonstrates that the fidelity of the gate can be improved by detuning the lasers, but at the cost of a longer gate time. The simulations of Fig. 3 use an exciton decay time, τ_X of 1 ps, which is a very pessimistic estimate but is useful since it clearly

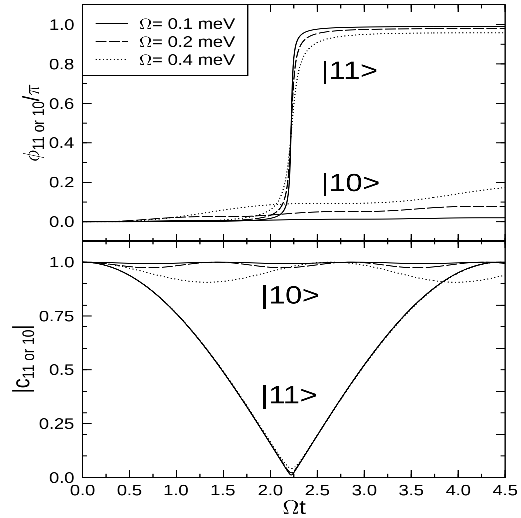

FIG. 2: Phase (top) and amplitude (bottom) of states $|10\rangle$ and $|11\rangle$ during the CPHASE gate operation, with V_F = 0.85 meV, $V_\mathrm{XX} = 5$ meV, and $\omega_a = 2$ eV.

$V_\mathrm{F} = 0.85$ meV [14] and $V_\mathrm{XX} = 5$ meV [3, 4]. Conditions Eq. 5 and Eq. 9 imply that $\Omega \sim 0.1 - 0.2$ meV ($\Omega' \sim 0.14 - 0.28$ meV) at its maximum for a high fidelity operation (in the region of $95\% - 99\%$). This corresponds to a gate implementation time $T \sim 15 - 30$ ps for a square pulse, sufficient for many such operations to be performed within measured low-temperature exciton dephasing times [15–17]. In Fig. 2 we show a numerical simulation of Eq. 1 over a complete gate cycle, demonstrating the suppression of unwanted phase on the state $|10\rangle$ as the ratio Ω/V_F decreases.

The CPHASE operation outlined above, whilst being conceptually simple and the most straightforward gate to implement experimentally, places stringent conditions on the system dynamics. However, it is possible to relax the condition of Eq. 9 through the use of single qubit operations and by redefining the phase accumulated on state $|11\rangle$ as $|11\rangle \rightarrow e^{i\theta}|11\rangle$ [18]. Here, $\theta = \phi_{00} - \phi_{01} - \phi_{10} + \phi_{11}$ where ϕ_n is the phase change of $|n\rangle$ during the gate operation. Further, we can design the experiment so that no population leaks out of the computational basis after the completion of the CPHASE gate (for example, by making sure that the periods of evolution of the other qubit states are commensurate with the $|11\rangle$ evolution). In this case, we find that for $\theta = \pi$ we recover the simple CPHASE gate. Thus Eq. 9 is now no longer necessary, and we need only satisfy the condition of Eq. 5, which may be less restrictive [4].

To complete our proposal for a universal set of gates, we now move to single qubit operations. Optically, the most straightforward of these is a Z-rotation, where a relative phase is accumulated between $|0\rangle$ and $|1\rangle$. If we

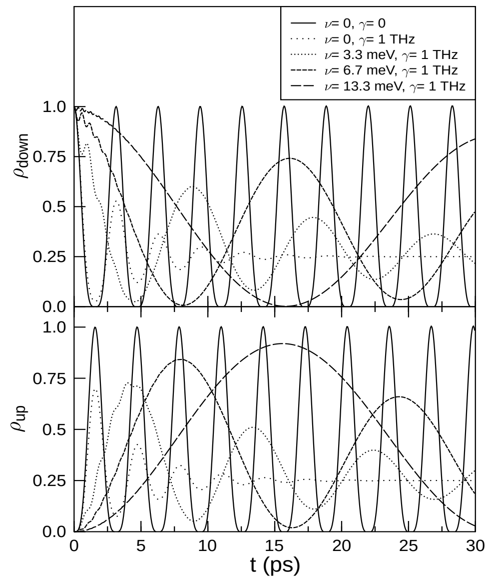

FIG. 3: Population of the state of a single qubit during the proposed X-rotation gate induced by a two-laser Raman transition. Various values of the exciton decay rate γ and laser detuning ν are shown with $\omega_a = 2$ eV, and $\Omega = 1.33$ meV for both lasers.

demonstrates the detuning effect. If we take $\tau_{\mathrm{X}} = 10$ ps, which is still much shorter than typical heavy hole exciton dephasing times [15], a π pulse fidelity of 99% can be achieved in 16 ps. It is sufficent for universal quantum computing to be able to perform this kind of gate globally, so long as the Z-gate, which we have described above, is qubit selective.

An important feature of our scheme is that the inherent coupling between optical transitions and spin states is just what is required for quantum measurements. This can proceed through spin-dependent resonance fluorescence, with or without shelving to a metastable level [22]. This technique is ideally a projective measurement and so could also be used for state preparation.

To summarize, we believe that the scheme outlined above allows for the realization, with available technology, of a CPHASE gate between two spins in adjacent quantum dots, and for arbitrary single-qubit manipulations. As our coupled-spin quantum logic gate requires the excitation of single exciton states only, it has a major advantage in the need for only one simple laser pulse to couple the qubits. Furthermore, we have shown that both types of gate may be performed to a good fidelity with the current semiconductor and laser technology; these gates therefore provide an immediate route to some of the less demanding applications of quantum processors [25]. Recent work on full-scale fault tolerant quantum computation (FTQC) [23] also indicates that our favourable ratio of exciton gate decay to spin qubit decoherence times could greatly decrease the threshold fidelity required for FTQC; indeed, recent estimates of this threshold have been around 99% [24]. Hence, we expect that it will be possible to use our scheme to build a full-scale quantum processor as quantum dot growth and characterization techniques progress.

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