Multi-period supplier selection with price uncertainty

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Keyword(s):

Supplier selection; Quantity discounts; Price uncertainty; multi-stage stochastic programming

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1. Introduction

Procurement is a core strategic function that affects the profitability of a firm as the cost of goods and services acquired through procurement typically constitutes a majority of operating expenses. A fundamental problem in procurement is supplier selection or sourcing, i.e., how to allocate the firm's business across suppliers, considering factors such as cost, quality, responsiveness and risk.

The industry has seen two shifts in supplier selection in the past two decades. First, with the advent of the Internet, procurement organizations moved away from manual bidding processes and negotiations to electronic sourcing. For example, between 2001 and 2006,

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Hewlett-Packard increased its total spend through e-sourcing events to $30 billion, a 100-fold increase (Carbone, 2004, Moody, 2006). Electronic sourcing not only allowed firms to expand their supply pool to more competition, but also made the supplier selection process faster and more structured, enabling a simultaneous evaluation and negotiation of supplier offers. A second shift is an organizational change with which corporations started to procure centrally to leverage economies of scale of their global business. For example, Hewlett-Packard combined even its indirect and services procurement globally and started to use category based sourcing to leverage its total spend of $16.5 billion (Avery, 2008). While these two shifts increased the potential value of the procurement function, they also made the decisions faced by procurement managers more complex and challenging. A procurement manager now needs to distribute a large volume of many items required at multiple locations across many global suppliers that approach the firm with various terms and offers.

A complicating feature of this problem is regarding how the suppliers present their price offers in a procurement environment. Many suppliers often exhibit economies of scale and scope in their production activities. Some others have growth and market share targets for a specific group of items. Suppliers express these internal efficiencies and pressures by offering discounts to the buyer. These offers are usually complex and are contingent on meeting various conditions on total available market, volume or spend for a single item or a set of items and the discounts may be applicable to the same or a different set of items.

It is not possible to incorporate these complex discount offers into a simple reverse auction. Ignoring these (potential) offers and selecting the lowest bidding supplier for each item or lot would lead to inefficiencies. We have recently seen efforts to develop tools that would enable suppliers to express these offers and buyers to evaluate them. The success of the software tool CombineNet is described in Sandholm (2007). Between 2001 and 2006, 447 bidding events are administered totaling a spend of $35 billion. It is believed that CombineNet delivered savings of $4.4 billion in these events. Bichler et al. (2011) proposed a bidding language and an optimization model to express and evaluate more complex offers.

Both of these efforts and many other research in this area assume that the important parameters of the problem are perfectly known in advance. However, procurement environments are replete with uncertainties. A primary uncertainty is in the volume that needs to be procured. Since the demand for end-products is often volatile, it is also very hard to predict the amounts of goods and services that need to be procured to make them. For example, global shipments of personal computers declined in the first quarter of 2011, by 3.2% according to an estimate by International Data Corp., and by 1.1% according to an another estimate by Gartner Inc., while both tracking firms previously predicted an increase (Sherr, 2011).

Another important uncertainty is in the prices of components that need to be procured. For example, prices of many components that are used in personal computers fluctuate heavily and these shifts in prices are also very hard to predict. The price of the DRAM memory that Hewlett-Packard uses dropped by over 90 percent in 2001, and then more than tripled in early 2002 (Nagali et al., 2008). Supplier offers usually state a commitment to base prices and discounts that will be given if certain conditions are met. However, price uncertainty in the market may still have an impact on how a procurement manager would evaluate such offers through a number of ways. First, if the procurement is through a contracted supplier, the future prices of this supplier to other customers may have an impact on an existing contract based on certain clauses. These clauses usually refer to what is known as a most-favored-customer (MFC) status. A customer who obtains a MFC status from a company is guaranteed to receive the best price the company gives to anyone. If the supplier lowers price to someone else, then the customer's price will be lowered to match. In some cases, a customer may need to purchase a minimum volume of a set of items over a specified time period to obtain this status. In other cases, the customer (such as a government agency) may demand MFC status for any contract without any condition. Lowest prices can be verified and contract compliance can be ensured through third-party audits.

MFC clauses are commonly used in procurement contracts in many industries. For example a contract (Sample Business Contracts, 2012) between Cisco Systems Inc. and one of its suppliers, Frontier Software Development Inc., stipulates "Frontier represents and warrants to Cisco that the product prices/license fees offered to Cisco under this agreement are no less favorable than the product prices/license fees offered to any other party purchasing or licensing similar quantities. In the event Frontier offers more favorable product prices/license fees to any other party, Frontier will promptly notify Cisco of such event and offer such more favorable product prices/license fees to Cisco commencing upon the date such more favorable product prices/license fees were offered to the other party."

As an example of a contract that guarantees a government agency to purchase at the lowest price, we note the following price reductions clause stated in a contract between Hewlett-Packard and US Department of Defense (DoD-ESI, 2012): "The prices under this BPA (blanket purchase agreement) shall be at least as low as the prices that the contractor has under any other contract instrument under like terms and conditions. If at any time the prices under any other contract instrument become lower than the prices in this BPA, this BPA will be modified to include the lower prices". Governments usually enforce the compliance to these clauses strictly. In a recent settlement, Oracle accepted to pay the US government $199.5 million after a file suit claiming that Oracle was not providing the government discounts that were as deep as some other customers were receiving (Montalbano, 2011).

Another contract clause that price uncertainty can have an effect is what is called meet-the-competition-clause (MCC). An MCC clause (sometimes also referred to as meet-or-release clause) in a procurement contract gives the seller an option to retain the customer's business by matching any lower price offer that may be coming in the future. A third contract type that may lead to uncertainty in price is price indexing. In this case, the contract price of a product is indexed to the price of a commodity or an official price index (such as consumer price index). For example, in the UK, 85% of natural gas is sold under long-term contracts in which prices are indexed to the spot market (Neumann and von Hirschhausen, 2004). Uncertainty in the commodity prices or price index obviously creates uncertainty on the prices that the seller would charge to the buyer. Finally, if some of the products under consideration are commodity-like products and can also be procured from the spot market, an uncertainty in spot prices has a clear and direct effect on how much the manager should procure from the spot market or using a fixed-price contract now and in the future.

In this study, we develop a model that incorporates price uncertainty to the supplier selection problem when the suppliers offer complex discount offers. The formal problem we consider may be stated as follows. A buyer needs to purchase a large volume of multiple items over a planning horizon (a quarter or a year) that consists of multiple periods (months). The demand for each item can be different in each period, but is known. There are multiple suppliers that are qualified to offer all or a portion of the items in consideration. Each supplier provides a base price for each item that it offers. In addition, suppliers propose various discount offers to the buyer that are contingent on meeting various conditions on a single item or a group of items, over a single period or multiple periods. An offer may provide per unit discounts that can be applied to a single item or a group of items, over a single period or multiple periods and on all units purchased (all-units discount) or units purchased above a threshold (incremental discounts). Alternatively, an offer may provide the buyer a lump sum. Some of these offers may be tied to realization of random events in the future which can be mutually verified. The buyer's problem is to select suppliers and determine the amount of each item to be procured from each supplier in each period of the planning horizon to minimize its expected procurement and inventory holding costs while satisfying item demands in each period. There could be also capacity constraints which limit how much the buyer can procure from a supplier. In addition, the buyer may also enforce certain side constraints (e.g., enforce a minimum and maximum number of suppliers for each item) to properly manage other procurement risks. While the procurement decisions for the first period are executed immediately, the decisions in the latter periods will be contingent on the realization of random events in those periods (i.e., recourses).

We formulate the buyer's problem as a multi-stage stochastic mixed-integer program using a scenario tree. To our knowledge, this is the first model for the supplier selection problem that simultaneously considers uncertainty and discount offers of combinatorial nature. This is also one of the first multi-period models and allows the buyer to consider discount rules defined over multiple periods and carry inventory from one period to another to be eligible for a favorable discount. The formulation is also very general in two aspects. First, we can represent a variety of random events that have direct or indirect effects on the discounts that the buyer gets from the suppliers. Second, we can represent many different forms of supplier offers with very complex conditions and discounts. For example, the model supports the separation of items (periods) for which the conditions are imposed and items (periods) on which the discounts apply, pricing with multiple price breaks and incremental or all-units discounts.

We also suggest two certainty-equivalent heuristics that can be used for this problem. In both of these heuristics, a deterministic version of the problem is solved by setting the prices in later periods to their expected values. The static heuristic solves the problem only once at the beginning of the horizon and does not respond to the actual realizations of events in the later periods. The dynamic heuristic, on the other hand, resolves the problem at each period. We show how one compute the performance of these heuristics by using our stochastic formulation.

The multi-stage stochastic programming model is used to evaluate MFC status benefits and regular discount offers for three bidding events that took place in 2010 at a major manufacturing company. The results of this case study show that considering MFC terms in addition to the regular discount offers may lead to substantial savings. In some of the events, the incremental savings by taking MFC terms into account may be larger than the savings that can be obtained by only evaluating regular discount offers. The results also show that the price certainty-equivalent heuristics, the static version in particular, fail to capture the benefits of MFC terms in contracts.

The rest of the paper is organized as follows. In Section 2, we analyze a single-item, two-period problem with two suppliers to gain insight into the trade-offs. In Section 3, we review the literature on supplier selection problem. In Section 4, we present our model. In Section 5, we propose the two certainty-equivalent heuristics. In Section 6, we provide the results of our case study and analyze the effects of various model elements and parameters on the benefits of considering MFC terms and effectiveness of the two heuristics. We conclude in Section 7.

2. A Motivating Example

In order to explain the basic trade-offs in evaluating discount offers under uncertainty and to show the need to use a formal stochastic model to support decision making in this context, we provide the following stylized example.

A company needs to procure an item over a two-period horizon. The demand in the first and second periods are δ_1 and δ_2. There are two suppliers that offer this item. Supplier a charges μ_a per unit and offers a most favored customer clause in the contract. Under this contract, if the firm procures $100\,m$ percent of its demand from supplier a in period 1, it will benefit from any possible reduction in price (to other customers) in period 2. The price per unit is to reduce by π_a with a probability γ and to remain constant with probability $1-\gamma$. Supplier b charges μ_b per unit and offers a volume discount contract. Under this contract, if the firm procures a total of ρ in two periods from supplier b, it will receive a discount of π_b per unit. We assume that

$$\delta_1 + \delta_2 \geq \rho > (1-m)\delta_1 + \delta_2.$$

That is, *i-)* the firm can always qualify for a volume discount from supplier b by buying enough and *ii-)* the firm cannot qualify for the volume discount offer from supplier b and buy enough from supplier a to benefit from a possible price drop at the same time. We also assume that

$$\mu_b < \mu_a < \mu_b - \pi_b + \pi_a,$$

i.e., firm will choose supplier a over b, if MFC clause in supplier a is used, and supplier b over supplier a, otherwise. If the firm buys enough from supplier a to benefit from a potential price drop, its expected cost will be

$$\Phi_0^a = \mu_a m\delta_1 + \mu_b(1-m)\delta_1 + \gamma(\mu_a - \pi_a)\delta_2 + (1-\gamma)\mu_b\delta_2.$$

On the other hand, if the firm chooses to use supplier b and benefit from the volume discount offer, its cost will be

$$\Phi^b = (\mu_b - \pi_b)(\delta_1 + \delta_2).$$

The firm will opt for supplier a's MFC clause ($\Phi_0^a < \Phi^b$) if and only if

$$\gamma > \gamma_0 = \frac{\pi_b(\delta_1+\delta_2) + (\mu_a - \mu_b)m\delta_1}{(\pi_a + \mu_b - \mu_a)\delta_2}.$$

An alternative to using the stochastic formulation is to use a certainty-equivalent argument and assume that supplier a's second period price will be the expected price $\mu_a - \gamma\pi_a$. In this case, we can write firm's cost if it chooses to use the MFC clause as

$$\Phi_1^a = \mu_a m\delta_1 + \mu_b(1-m)\delta_1 + (\mu_a - \gamma\pi_a)\delta_2.$$

In this case, the firm will opt for supplier a's MFC clause ($\Phi_1^a < \Phi^b$) if and only if

$$\gamma > \gamma_1 = \frac{\pi_b(\delta_1+\delta_2) + (\mu_a - \mu_b)m\delta_1 + (\mu_a - \mu_b)\delta_2}{\pi_a\delta_2}.$$

Denoting $\alpha = \pi_b(\delta_1 + \delta_2) + (\mu_a - \mu_b)m\delta_1$, $\beta = (\pi_a + \mu_a - \mu_b)\delta_2$ and $\gamma = (\mu_a - \mu_b)\delta_2$, we have $\gamma_0 = \alpha/\beta$ and $\gamma_1 = (\alpha+\gamma)/(\beta+\gamma)$. It is then easy to see that γ_0 is strictly smaller than γ_1 if and only if $\gamma_0 < 1$, i.e., unless it is never optimal for the firm to consider the supplier a which offers the MFC clause. In this case for $\gamma_0 < \gamma \leq \gamma_1$, the firm's optimal action is to procure $m\delta_1$ from supplier a in period 1. However, this action will not be taken if a certainty-equivalent approach is used. In general, the decisions taken using a certainty-equivalent approach would be different from optimal decisions (using stochastic formulations) and therefore lead to expected costs that are higher than optimal.

3. Literature Review

The impact of quantity discounts on replenishment and procurement decisions of a company is well-studied in the operations management literature (Munson and Rosenblatt, 1998). Most of the basic textbooks in this area include a section on extensions of the economic order quantity (EOQ) model that consider quantity discounts (e.g., Silver et al., 1998, §5.5). Another line of research focuses on sourcing, i.e., how a company should select and allocate its spend to different suppliers based on different factors such as cost, quality, lead time and reliability (Chopra and Meindl, 2013, §13).

While the extensions of the EOQ model for the case of quantity discounts and single-item, single-supplier problems are usually tractable, the problem becomes difficult, even for the case of a single item when there are multiple suppliers whose price offers are functions of the quantity purchased. For example, consider a buyer who needs to purchase a predetermined amount of a single item from a set of suppliers. Each supplier offers a certain price, but the price is valid only if the quantity purchased is in a specific interval, reflecting the cost and capacity structure of the supplier. Chauhan et al. (2005) show that the problem is NP-hard.

For reviews on the supplier selection problem, we refer the reader to Benton and Park (1996), Munson and Rosenblatt (1998) and Aissaoui et al. (2006). Here we briefly summarize some examples on different variants of the problem.

Most studies on supplier selection for multiple items consider single period problems. Goossens et al. (2007) study the problem of deciding on purchase quantities for multiple items from multiple suppliers that offer total quantity discounts based on total purchase quantities. The authors prove that this problem is NP-hard even for some specific discount structures. They present an MIP formulation and model the LP relaxation as a min cost network flow problem. They extend their results to variants of the problem with market share constraints, limited number of winning suppliers and multiple periods. Manerba and Mansini (2012) study the same problem under capacity constraints and present valid inequalities and a branch and cut algorithm. Qin et al. (2012) study a distribution planning problem where shipping companies offer total quantity discounts. Crama et al. (2004) consider the supplier selection problem with alternative product recipes and Mansini et al. (2012) incorporate transportation costs. A different setting where suppliers offer their products in bundles is studied by Murthy et al. (2004). In this problem, decisions regarding the purchase quantities for different items are related not only through bundles but also through fixed costs of buying from suppliers. A Lagrangian relaxation based heuristic is proposed to solve this problem. Sadrian and Yoon (1994) and Katz et al. (1994) present MIP formulations for the problem in the presence of business volume discounts. Bichler et al. (2011) introduce a comprehensive bidding language that allows for elaborate discount structures. Total quantity and incremental quantity discounts as well as lump sum discounts and markups with conditions on spend or purchase quantities can be expressed with this bidding language. The authors present a MIP model to solve the supplier selection problem and report the results of their experiments in solving the model under different scenarios.

There are few studies on the supplier selection problem with multiple periods and dynamic demand. Tempelmeier (2002) presents formulations and a heuristic solution approach for the single-item problem with both total quantity discounts and incremental discounts. van de Klundert et al. (2005) study the problem of selecting telecommunications carriers under total quantity discounts. The discounts are given based on the total call-minutes over the planning horizon, but lower and upper bounds are imposed on call-minutes per period routed via each carrier.

The multi-item problem with dynamic demand is studied by Stadtler (2007). Different from our study, discount rules involve a single item and decisions concerning different items are related through a fixed cost of buying from suppliers. Xu et al. (2000) study the single supplier problem in the presence of business volume discounts and setup costs.

The present study extends the literature on multi-item problems with multiple suppliers and dynamic demand by considering very general discount rules and price uncertainty.

4. The Multi-stage Stochastic Programming Model

A firm (buyer) needs to procure a set of items $\mathcal{I}$ over a set of periods $\mathcal{T}$. For each item $i \in \mathcal{I}$ and period $t \in \mathcal{T}$, the firm has to satisfy demand denoted by δ_{it} (a deterministic quantity) without a backlog. The firm works with a set of suppliers $\mathcal{N}$. For each item i, there is a subset of suppliers $\mathcal{N}_i \subseteq \mathcal{N}$ that are qualified. Supplier j in $\mathcal{N}_i$ charges a unit price μ_{ijt} and has a capacity κ_{ijt} for item i in period t. The firm can also carry inventory from one period to the next by incurring an inventory holding cost of η_{it} for each unit of item i's ending inventory in period t.

A set of discount rules $\mathcal{D}$ and a set of lump sum rebate rules $\mathcal{L}$ are available. These rules involve a set of conditions C on order quantities. The quantity ρ_c is the minimum order quantity that the firm needs to purchase from items in set $\mathcal{I}_c \subseteq \mathcal{I}$ over periods $\mathcal{T}_c \subseteq \mathcal{T}$ for condition $c \in C$ to be satisfied. Let $\mathcal{E}$ be the set of conflicting pairs of rules.

We define $\mathcal{R} = \mathcal{D} \cup \mathcal{L}$. The buyer can benefit from the rule r offered by supplier $j(r) \in \mathcal{N}$ if it satisfies the conditions $C_r \subseteq C$ and if it does not benefit from any other rule in $\mathcal{R}_r \subset \mathcal{R}$. If $r \in \mathcal{L}$, then a lump sum rebate of ω_r is offered. If $r \in \mathcal{D}$, the supplier provides a set of discounts $\mathcal{K}_r$. The discount $k \in \mathcal{K}_r$ reduces the price by π_k per unit for items in the set $\mathcal{I}_k \subseteq \mathcal{I}$ purchased over periods $\mathcal{T}_k \subseteq \mathcal{T}$ exceeding the quantity θ_k ($\mathcal{I}_{k1} \cap \mathcal{I}_{k2} = \emptyset$ for all $k_1, k_2 \in \mathcal{K}_r$, $k_1 \neq k_2$).

Let $\mathcal{V}$ be the set of nodes of the scenario tree with node 0 corresponding to the root and $\mathcal{V}_t$ be the set of nodes in layer $t \in \mathcal{T}$. For a given node $s \in \mathcal{V}$, let $\tau(s)$ be its layer, $a(s)$ be its predecessor in the scenario tree, $\mathcal{P}^s$ be the set of nodes on the path from the root to node s, and γ_s be its probability. We define the rules at the terminal nodes. Let $\mathcal{R}^s$ be the set of discount rules available at node s. Define $\mathcal{D}^s = \mathcal{D} \cap \mathcal{R}^s$ and $\mathcal{L}^s = \mathcal{L} \cap \mathcal{R}^s$. Let $\mathcal{E}^s$ be the set of pairs of conflicting rules at node s, i.e., $\mathcal{E}^s = \mathcal{E} \cap (\mathcal{R}^s \times \mathcal{R}^s)$.

We demonstrate the construction of the scenario tree for the example problem discussed in Section 1 in Figure 4.

At the beginning of horizon, the orders are placed for the first period. Recourse actions are taken at the beginning of each other period based on actual realizations. We define the following decision variables. The quantity x_{ijs} is the order quantity for item $i \in \mathcal{I}$ and supplier $j \in \mathcal{N}_i$ at node $s \in \mathcal{V}$. I_{is} stands for the ending inventory for item $i \in \mathcal{I}$ at node $s \in \mathcal{V}$. The binary variable z_r^s takes value 1 if rule $r \in \mathcal{R}^s$ applies at node $s \in \mathcal{V}$ and takes value 0 otherwise. Finally, y_k^s is the total amount of units of items in the set $\mathcal{I}^k$ that are discounted with discount

Period 1: Supplier a's lowest price μ_a

0

Constrai

due to cons

rules. Conc

compute the amount of discounted units (we assume that $\sum_{i \in \mathcal{I}_k} \sum_{\hat{s} \in \mathcal{P}^s : \tau(\hat{s}) \in \mathcal{T}_k} x_{ij(r)\hat{s}} \geq \theta_k$ is implied by the conditions for discount rule r and discount $k \in \mathcal{K}_r$). If, in scenario $s \in \mathcal{S}$, discount rule $r \in \mathcal{D}^s$ applies, then constraints (6) is redundant and the maximum of zero and the right hand side of constraint (5) is equal to the amount of discounted units. If this rule does not apply, then constraint (5) is redundant and constraints (6) and (11) force y^s_{kr} to zero. Finally constraints (7) ensure that conflicting rules do not apply at the same time. The objective function (1) is equal to the expected total cost.

The formulation can be strengthened by replacing conflict constraints (7) with inequalities corresponding to cliques in the conflict graph $\mathcal{G}^s = (\mathcal{R}^s, \mathcal{E}^s)$. In some cases, the same information can be used to strengthen constraints (4). We sketch this with a very simple example. Suppose that we consider a single period problem where supplier 1 offers total quantity discounts for item 1. The unit price reduces by a factor for every 1000 items purchased and the capacity is 4000. To handle this discount, we define four discount rules $r \in \{1, \ldots, 4\}$, each with a single condition. We replace constraint (4) with $x_{110} \geq \sum_{r=1}^{4} 1000 r z^0_r$ and use the clique inequality $\sum_{r=1}^{4} z^0_r \leq 1$.

The mixed-integer program given in (1-11) can be used to model various forms of regular discount offers and other discounts that are contingent on realization of random events. We discuss some of these here. First, traditional quantity discounts schemes can be easily modeled. Consider for example an incremental discount rule r that requires condition c and applies a discount k. The condition and discount are applied on the same item set and same period set, i.e., $\mathcal{I}_c = \mathcal{I}_k$, and $\mathcal{T}_c = \mathcal{T}_k$. Thresholds are then also set to be the same, i.e., $\rho_c = \theta_k$. The rule is defined in all terminal nodes. Multiple price breaks can be modeled using multiple rules that are disjoint. An all-units discount rule can be modeled similarly except that now we set $\theta_k = 0$. One can also separate the periods (items) for which the conditions are imposed and the periods (items) for which the discounts are applied on.

In addition to MFC terms which is explained by an example in Figure 4, various other uncertainties can be modeled. For the case of index pricing, various scenarios can be created for the value of the index in the later periods. For each scenario, we define a discount rule which provides a discount in the amount of price difference without a condition. One can also model spot price uncertainty by defining a dummy supplier for spot purchases and creating a scenario for each possible price change in the spot market. We can then define a discount rule for every terminal node whose path from the root node has a price change. These rules will also have no conditions and will provide a discount in the amount of price change for all units purchased in periods after the price change took place. Finally, any potential regular discount offer in the future (for example, if the buyer thinks that there is a chance that one of the suppliers will offer a new discount in the middle of the planning horizon) can be easily incorporated in the model provided that the conditions and discounts can be properly estimated.

5. Heuristics

A usual approach in practice to solve problems under uncertainty is to use certainty-equivalent heuristics. These heuristics solve a deterministic version of the problem in which the random elements are replaced by their average values. The approach can also be used for our problem. For example, for the case of a possible price drop in a given future period that the buyer will also benefit due to MFC clauses in the contract, one can set the discount amount to be equal to the difference between the current price and the expected price in that period. Note that to be eligible for this discount, the buyer still has to abide by the rules of obtaining MFC status. For the case of spot price uncertainty, one can use the expected spot price in each period as the price for that period in the deterministic formulation. For the case of index pricing, one can index the product prices to the expected price of the commodity or index.

We consider two versions of the certainty-equivalent heuristic. In the first version, certainty-equivalent deterministic problem is solved only once at the beginning of the planning horizon and the decisions are never changed. In the second version, the deterministic problem is re-solved at the beginning of each period after random events for that period are observed. We next explain how one can compute the solution and obtain the expected cost for each heuristic.

5.1. Static Certainty-Equivalent Heuristic

In this heuristic, the problem is solved only once at the beginning of the horizon (at node 0) and the solution is followed regardless of the actual realizations of the random events throughout the planning horizon. In order to compute the solution for the certainty-equivalent heuristic, one can solve the following mathematical program.

$$\min \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{N}_i}\sum_{t\in\mathcal{T}} \mu_{ijt}x_{ijt} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \eta_{it}I_{it} - \sum_{r\in\mathcal{D}}\sum_{k\in\mathcal{K}_r} \overline{\pi}_k^0 y_k - \sum_{r\in\mathcal{L}} \overline{\omega}_r^0 z_r \tag{12}$$

$$\text{s.t. } I_{it} = I_{i,t-1} + \sum_{j\in\mathcal{N}_i} x_{ijt} - \delta_{it} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, \tag{13}$$

$$x_{ijt} \le \kappa_{ijt} \qquad \forall i\in\mathcal{I}, j\in\mathcal{N}_i, t\in\mathcal{T}, \tag{14}$$

$$\sum_{i\in\mathcal{I}_c}\sum_{t\in\mathcal{T}_c} x_{ij(r)t} \ge \rho_c z_r \qquad \forall r\in\mathcal{R}, c\in\mathcal{C}_r, \tag{15}$$

$$y_k \le \sum_{i\in\mathcal{I}_k}\sum_{t\in\mathcal{T}_k} x_{ij(r)t} - \theta_k z_r \qquad \forall r\in\mathcal{D}, k\in\mathcal{K}_r, \tag{16}$$

$$y_k \le \left(\sum_{i\in\mathcal{I}_k}\sum_{t\in\mathcal{T}_k} \kappa_{ij(r)t} - \theta_k\right) z_r \qquad \forall r\in\mathcal{D}, k\in\mathcal{K}_r, \tag{17}$$

$$z_r + z_{r'} \le 1 \qquad \forall \{r, r'\}\in\mathcal{E}, \tag{18}$$

$$x_{ijt} \ge 0 \qquad \forall i\in\mathcal{I}, j\in\mathcal{N}_i, t\in\mathcal{T}, \tag{19}$$

$$I_{it} \ge 0 \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, \tag{20}$$

$$z_r \in \{0,1\} \qquad \forall r\in\mathcal{R}, \tag{21}$$

$$y_k \ge 0 \qquad \forall r\in\mathcal{D}, k\in\mathcal{K}_r. \tag{22}$$

Note that the formulation above is the same as the model in (1-11) except that the scenarios are removed. In this formulation, x_{ijt} is the order quantity for item i from supplier j in period t and I_{it} is the ending inventory for item i in period t. The binary variable z_r takes value 1 if rule $r \in \mathcal{R}$ applies and takes value 0 otherwise. The variable y_k is the total amount of items in the set $\mathcal{I}_k$ that are discounted with discount $k \in \mathcal{K}_r$ as a part of rule $r \in \mathcal{D}$. The only parameters that are different from the model in (1-11) are $\overline{\pi}_k^0$ which stands for the *expected* value of the discount given by the discount k and $\overline{\omega}_r^0$ which stands for the *expected* lump sum discount given by the rule r. Both of these expectations are taken at node 0 (unconditional expectation).

Let x_{ijt}^0 be the optimal order quantity for item i from supplier j in period t obtained by solving (12-22) (superscript 0 is used to denote that this is the certainty-equivalent solution obtained at node 0). Note that the optimal value of the model in (12-22) is not the true expected value obtained by following the ordering decisions obtained by solving (12-22). To calculate the expected cost of the certainty-equivalent heuristic, we need to reinstate the scenarios and solve the following mathematical program.

$$\min \sum_{s \in \mathcal{V}} \gamma_s \left(\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{N}_i} \mu_{ij\tau(s)} x_{ijs} + \sum_{i \in \mathcal{I}} \eta_{i\tau(s)} I_{is} - \sum_{r \in \mathcal{D}^s} \sum_{k \in \mathcal{K}_r} \pi_k y_k^s - \sum_{r \in \mathcal{L}^s} \omega_r z_r^s \right)$$

$$\begin{aligned} \text{s.t. } & (2-11), \\ & x_{ijs} = x_{ij\tau(s)}^0 \qquad && \forall s \in \mathcal{V}, i \in \mathcal{I}, j \in \mathcal{N}_i. \end{aligned}$$

5.2. Dynamic Certainty-Equivalent Heuristic

In this heuristic, the problem is re-solved at the beginning of each period to account for realizations of all random events and prior decisions until that node. At node 0, we solve the same model given in (12-22). We then need to re-solve the problem at every node (going layer by layer) as well. At node s, we need to solve the following mathematical program.

$$\min \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{N}_i} \sum_{t \in \mathcal{T}} \mu_{ijt} x_{ijt} + \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \eta_{it} I_{it} - \sum_{r \in \mathcal{D}} \sum_{k \in \mathcal{K}_r} \overline{\pi}_k^s y_k - \sum_{r \in \mathcal{L}} \overline{\omega}_r^s z_r \tag{23}$$

$$\begin{aligned} \text{s.t. } & (13-22), \\ & x_{ij\tau(\hat{s})} = x_{ij\hat{s}}^{\hat{s}} \qquad && \forall \hat{s} \in \mathcal{P}^s, i \in \mathcal{I}, j \in \mathcal{N}_i. \end{aligned} \tag{24}$$

The last set of constraints enforces that the order quantity decisions taken prior to node s (nodes in the path from node 0 to node s, $\mathcal{P}^s$) are followed. Note also that we use $\overline{\pi}_k^s$ for the expected discount given by discount k and $\overline{\omega}_r^s$ for the expected lump sum amount given by rule r. These reflect the fact that expectations are taken at node s (conditioning on the fact that we are already at node s). If a discount or a rule depends only on random events that materialize in time periods before and at node s ($t \leq \tau(s)$), this means that we know these parameters with certainty at node s.

Let x^s_{ijt} be the optimal order quantity for item i from supplier j in period t obtained by solving model (23, 13-22, 24) at node s. In order to compute the expected cost of the dynamic certainty-equivalent heuristic, we need to solve the following program.

$$\min \sum_{s\in\mathcal{V}} \gamma_s \left(\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{N}_i} \mu_{ij\tau(s)} x_{ijs} + \sum_{i\in\mathcal{I}} \eta_{i\tau(s)} I_{is} - \sum_{r\in\mathcal{D}^s} \sum_{k\in\mathcal{K}_r} \pi_k y^s_k - \sum_{r\in\mathcal{L}^s} \omega_r z^s_r \right)$$

$$\text{s.t. } (2-11),$$

$$x_{ijs} = x^s_{ij\tau(s)} \qquad \forall s\in\mathcal{V}, i\in\mathcal{I}, j\in\mathcal{N}_i.$$

The last constraint enforces that order quantity decisions at each node are taken according to the solution of the certainty-equivalent model at that node.

6. Case Study

We test our model and the implications of its use using data from actual bidding events that took place in 2010 at a global manufacturing company. Each event was a major quarterly event to procure an important family of components required for various products in different divisions of the company. These events are held for 40-45 items (components) and involve 3-5 suppliers. For most of the items, there are two or more suppliers that are competing for the buyer's business. Each supplier offers a base price for each of the items it is offering. In addition, suppliers also offer discounts that reflect their economies of scale in costs and market share targets. Each discount offer requires a minimum volume or a minimum spend on an item or a group of items and provides a discount on a set of items (which can be different from the set that the conditions are imposed on). The discounts are either incremental or all-units discounts.

In addition to these usual discount offers, at least one supplier in each bidding event provides most favored customer benefits in its offers. If the supplier offering the MFC clause reduces the price in the middle of the quarter to other customers, it would extend the price reduction to the buyer in consideration as well if the buyer has already procured a minimum fraction of its demand (for this item or a group of items) from the supplier until that time. In order to model these possible discounts, we split the quarter into two periods and created scenarios to represent possible price reductions in items for which MFC clauses apply.

Table 1 The impact of discount offers and MFC clauses on total spend

Test	Events					
	1a	1b	2a	2b	3a	3b
Ignore MFC clauses (regular discount offers only)	0.93	1.26	2.29	2.31	3.22	3.23
Consider MFC clauses when the probability of a price drop is						
low	0.97	1.67	2.29	2.31	3.70	3.71
medium	1.48	2.29	2.29	2.31	4.42	4.42
high	1.98	2.91	2.30	2.32	5.14	5.13

Table 1 summarizes our tests for these three bidding events. The buyer gets price and discount offers from the suppliers in two rounds. The suppliers submit initial bids in round 1 and revise them after getting some feedback from the buyer's procurement organization. We represent each round separately in Table 1 (*i*a and *i*b stand for the first and second round offers for bidding event *i*, respectively). Our baseline for each event is disregarding all discounts offers (regular and MFC clauses). This corresponds to selecting the supplier that offers the minimum base price for each item.

In our first test for each bidding event, we run a version of our model where we ignore all MFC clauses. In the last three tests, we run our model considering the MFC clauses in the contracts. For each event, we consider three sub-scenarios: the prices for the items covered in MFC clauses may drop with a low probability, a medium probability and a high probability. The entries in Table 1 show the reduction in total procurement costs (as a percentage of total procurement costs when all discount offers are ignored). First notice that regular discount offers lead to savings in the range of 0.93-3.23% for the company. In absolute terms, these savings are substantial for the company.

Optimizing sourcing decisions by also considering MFC clauses lead to important additional savings for bidding events 1 and 3. Even under a scenario when the price drops are not very likely, MFC clauses lead to additional savings up to 0.48% of the total spend. When the price drop probability is medium, the additional savings for bidding events 1 and 3 are in the range of 0.55-1.20%. When the price drops are considered very likely, the incremental savings are in the range 1.05-1.92%. Under some scenarios, incremental savings through MFC clauses are more than savings that are possible with only regular discount offers. Once again, these additional percentage gains correspond to substantial monetary savings for the company. In event 2, the base prices and regular discount offers given by the MFC suppliers are either already very competitive (leading the buyer to select them even without MFC clauses) or very uncompetitive (leading the buyer to select other suppliers despite the possible MFC benefits).

6.1. Detailed Analysis

In order to test our model and the performance of heuristics proposed in Section 5 in various other settings, we study bidding event 1a and variations of it in more detail. This event was held for 44 items. There were 4 qualified suppliers which we name them as *A*, *B*, *C*, and *D*. Table 2 shows the number of items offered by different groups of suppliers. For example, 4

Table 2 Suppliers and their offerings in bidding event 1

Suppliers	A,B,C,D	A,B,C	A,B	A,C	A,D	A	C
Number of items	4	3	11	4	7	13	2

items are offered by all suppliers, while 3 items are offered by suppliers *A*, *B* and *C*, but not *D*. For 29 items, 2 or more suppliers compete for the buyer's spend. There may be considerable differences between the prices offered by competing suppliers. In this event, the maximum bid can be as much as 34.71% higher than the minimum bid. On the average, the maximum bids are 8.29% higher than the minimum bids. Note that there are 15 items that are offered by a single supplier (13 by supplier A and 2 by supplier B), but they still cannot be removed from the model and solved independently since some of the discount offers and MFC conditions involve these items together with other items offered by multiple firms.

Apart from base prices, suppliers propose various discount offers. In particular, supplier A submits 4, supplier B submits 3 and supplier C submits 1 discount offers. The offers involve multiple conditions on how much the buyer should buy from a set of items to qualify for the discount. The number of items in the condition sets is between 6 and 21. The discounts usually apply to the same set of items. However, some offers may apply a discount on items that are not in the condition set. Each offer provides a price reduction between of 3% and 7% of the base price.

In addition to these 8 discount offers, supplier A is offering a MFC term for 4 items in its contract. According to this term, if the buyer procures a certain percentage of total demand from supplier A, supplier A will ensure that the buyer will get the lowest price throughout the quarter. That is, if supplier A drops the price for some or all of the items during the quarter, the buyer will also benefit from these price drops. We consider various scenarios for the drop in prices for these 4 items.

Since the procurement costs are usually very large, we measure the effect of MFC clauses on procurement costs as a percentage of the savings obtained through regular discount offers. Let S be the total procurement spend in the absence of any discount offers. Let D be the procurement spend when only regular discount offers are utilized (those offers that are granted regardless of the price drops to other customers). In this particular event, $S - D$ is about 0.9321% of S. That is, regular discount offers lead to 0.9321% cost savings. Let D^* be the optimal expected procurement spend when MFC clauses are also used. Then we denote the effect of MFC terms as

$$J^* = 100 \times \frac{D - D^*}{S - D}. \tag{25}$$

That is, J^* measures the additional benefit of considering MFC clauses as a percentage of savings through regular discount offers.

Let D_{SCE} and D_{DCE} be the expected procurement spend if static and dynamic certainty-equivalent heuristics are used. Then, we measure the regrets of these heuristics (given that MFC terms provide savings, i.e., $D^* < D$) as follows

$$\Delta_{SCE} = 100 \times \frac{D_{SCE} - D^*}{D - D^*} \quad \text{and } \Delta_{DCE} = 100 \times \frac{D_{DCE} - D^*}{D - D^*}. \tag{26}$$

We first consider two-period instances with different scenario trees. In our simplest experiment, two scenarios are considered: the prices drop by π units in period 2 with probability γ or the prices remain the same with probability $1 - \gamma$. The corresponding scenario tree is depicted in Figure 2.

The results are given in Table 3 for different values of γ and π. First, notice that MFC terms lead to important savings in this bidding event and, as expected, the benefits increase as

Prices drop for all 4 items 2

1

Figure 2 Scenario Tree for Experiment 1

0

Figure 3 Scenario Tree for Experiment 2

0

The results for various values of γ_1, γ_2 and γ_3 are provided in Table 5. The results are not structurally different from the first two experiments, except that we now have more instances where the certainty-equivalent heuristic cannot capture any of the benefits of MFC terms.

Table 5 Results for Experiment 3

γ_1	γ_2	γ_3	J^*	Δ_{SCE}	Δ_{DCE}	γ_1	γ_2	γ_3	J^*	Δ_{SCE}	Δ_{DCE}
0.1	0.1	0.0	0.00	0.00	0.00	0.1	0.2	0.1	7.99	100.00	100.00
0.2	0.1	0.0	0.00	0.00	0.00	0.1	0.3	0.1	12.16	100.00	100.00
0.3	0.1	0.0	13.65	98.98	0.00	0.1	0.4	0.1	16.33	100.00	100.00
0.4	0.1	0.0	27.66	44.03	0.00	0.2	0.2	0.1	21.99	84.12	0.00
0.1	0.2	0.0	0.00	0.00	0.00	0.2	0.3	0.1	26.17	76.94	0.00
0.2	0.2	0.0	3.81	100.00	100.00	0.3	0.3	0.1	40.17	46.79	0.00
0.3	0.2	0.0	17.82	94.65	0.00	0.4	0.4	0.1	58.35	32.72	0.00
0.4	0.2	0.0	31.83	48.81	0.00	0.0	0.0	0.2	3.81	100.00	100.00
0.1	0.3	0.0	0.00	0.00	0.00	0.1	0.1	0.2	21.99	84.12	0.00
0.2	0.3	0.0	7.99	100.00	100.00	0.2	0.2	0.2	40.17	46.79	0.00
0.3	0.3	0.0	21.99	84.12	0.00	0.3	0.3	0.2	58.35	32.72	0.00
0.4	0.3	0.0	36.00	47.68	0.00	0.0	0.0	0.3	21.99	84.12	0.00
0.1	0.4	0.0	0.00	0.00	0.00	0.1	0.1	0.3	40.17	46.79	0.00
0.2	0.4	0.0	12.16	100.00	100.00	0.2	0.2	0.3	58.35	32.72	0.00
0.3	0.4	0.0	26.17	76.94	0.00	0.3	0.3	0.3	76.53	23.35	0.00
0.4	0.4	0.0	40.17	46.79	0.00	0.0	0.0	0.4	40.17	46.79	0.00
0.1	0.1	0.1	3.81	100.00	100.00	0.1	0.1	0.4	58.35	32.72	0.00
0.2	0.1	0.1	17.82	94.65	0.00	0.2	0.2	0.4	76.53	23.35	0.00
0.3	0.1	0.1	31.83	48.81	0.00	0.0	0.0	0.5	58.35	32.72	0.00
0.4	0.1	0.1	45.83	30.98	0.00	0.1	0.1	0.5	76.53	23.35	0.00

Our final experiment models three periods. Reflecting the regular pattern in practice, the demand in each of the periods 1 and 2 is assumed to be 30% of the total demand, while demand in period 3 is assumed to be 40%. Figure 5 shows the scenario tree for this experiment. In order to obtain the MFC status and benefit from a possible price drop in period 2, a minimum amount should be purchased in period 1. In order to obtain the MFC status and benefit from a possible price drop in period 3, the sum of purchases in period 2 and 3 should be above another threshold.

Figure 5 Scenario Tree for Experiment 4

Prices drop 5% for all 4 items

Prices remain the same

1

2

0

The results of experiment 4 are shown in Table 6 for different values of γ_1 and γ_2. Considering MFC clauses still leads to important savings in the three-period model and savings increase as the probability of a price drop increases. The percentage gap of the static certainty-equivalent heuristic can be significant, especially when the price drop probability is low. Structurally, the performance of the dynamic certainty-equivalent heuristic in the three-period problem is different than what we observe in the two-period problem. It is now possible that dynamic certainty-equivalent heuristic lead to a gap other than 0% and 100%.

Table 6 Results for Experiment 4

γ_1	γ_2	J^*	Δ_{SCE}	Δ_{DCE}
0.05	0.05	0.00	0.00	0.00
0.10	0.10	1.87	100.00	100.00
0.15	0.15	6.71	85.92	2.50
0.20	0.20	11.17	53.93	1.76
0.25	0.25	15.28	36.28	1.40
0.30	0.30	19.09	27.13	1.17
0.35	0.35	22.63	21.42	1.00
0.40	0.40	25.93	17.14	0.87
0.45	0.45	29.00	14.26	0.75
0.50	0.50	31.88	12.28	0.64
0.55	0.55	34.57	10.90	0.54
0.60	0.60	37.09	8.55	0.45

7. Conclusion

In this paper, we study the problem of a buyer who has to procure large volumes of multiple items over multiple periods and needs to evaluate discount offers from multiple suppliers for this purpose. Some of these discounts are tied to future realizations of random events. The objective of the buyer is to minimize his expected procurement and inventory holding costs subject to satisfying its demand and other various side constraints. We formulate the problem as a scenario based multi-stage stochastic optimization model. The formulation is very general in the sense that we can model various random events such as a supplier dropping price for other customers or a change in a price-index or spot price of a commodity. We can also model very complex offers that are frequently observed in industry such as those that involve conditions on multiple items and periods and apply incremental or all-units discounts to multiple items and periods that are potentially different from those for which the conditions are imposed on. The model also allows the buyer to carry inventory to benefit from a discount offer. Apart from the optimization model, we propose two certainty-equivalent heuristics that can be used for this problem and show how we can evaluate the performance of these heuristics. We use our model in a case study to see the effect of MFC status benefits on three bidding events that were administered by a global manufacturing company in 2010. The results show that taking the MFC terms into account using our model leads to significant savings for the company and using heuristics may fail to capture most of these savings.

There are many avenues for future research. First, one may consider modeling uncertainty on other parameters of the problem. Relevant uncertain parameters for procurement bidding events include item demands and supplier capacities. For some of these uncertain parameters, scenario based formulations may not be adequate and other approaches may be necessary to model uncertainty. Second, one may relax the assumption that the buyer is risk-neutral and consider alternative risk profiles. Finally, for very large problems, one may focus on developing efficient approaches for solving the mixed integer program that we propose.

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