

Dispersion of sound in dilute suspensions with nonlinear particle relaxation

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Abstract: The theory accounting for nonlinear particle relaxation (viscous and thermal) has been applied to the prediction of dispersion of sound in dilute suspensions. The results suggest that significant deviations exist for sound dispersion between the linear and nonlinear theories at large values of $\omega\tau_d$, where ω is the circular frequency, and τ_d is the Stokesian particle relaxation time. It is revealed that the nonlinear effect on the dispersion coefficient due to viscous contribution is larger relative to that of thermal conduction.

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1. Introduction

Sound attenuation in fluids, representing the dissipation of acoustic energy from a sound wave, occurs through a number of physical processes involving molecular viscosity, thermal conductivity, and other dissipative or relaxation processes.¹⁻⁷ When a fluid contains inhomogeneities such as suspended particles (solid particles, drops and bubbles) additional viscous and heat conduction losses occur in the immediate neighbourhood of the suspended particles.^{1-3,8} Particle relaxation in a dilute suspension is the process by which particles adjust to fluctuations (velocity and temperature) of the surrounding fluid, leading to attenuation and dispersion of a sound wave. A comprehensive review of the physics and scientific history of

acoustic interactions with particulate mixtures is provided by Challis et al.⁹ The acoustic intensity I of a plane wave propagating through an absorbing medium is expressed by

$$I = I_0 e^{-\alpha_i x} \quad (1)$$

where x is the distance traversed, I_0 the intensity at $x = 0$, and α_i the intensity attenuation coefficient for the medium. The quantity α_i depends on viscosity, thermal conductivity, and other factors such as molecular relaxation.

Sound propagation in aerosols and fog has been studied experimentally and theoretically by several investigators., since the pioneering work of Sewell¹⁰, with the aid of a scattering formulation on the assumption of immovable particles. Epstein¹¹ extended this theory for particles in motion, and Epstein and Carhart¹² additionally considered heat conduction effects. Allegra and Hawley¹³ provided further extensions by including liquid-liquid as well as liquid-solid systems.

Temkin and Dobbins,¹⁴ in their classical work involving a coupled-phase formulation, theoretically considered particle attenuation and dispersion of sound in a manner which illustrates explicitly the relaxation character of the problem. Basset history and added mass terms were included in an elegant coupled phase formulation by Harker and Temple.¹⁵ Coupled phase effects were also treated by Evans and Attenborough^{16,17} in an extension to the work of Harker and Temple¹⁵ to incorporate thermal conduction.

The absorption of sound in suspensions of irregular (nonspherical) particles was considered by Urick¹⁸. Experimental and theoretical studies (extension of Urick's model) on sound absorption involving irregular particles were also considered by Richards et al.^{19,20}

The particulate relaxation models for the sound attenuation are all based on Stokes drag (linear drag law) and pure conduction limit (linear heat transfer). Recently the author²¹ investigated sound attenuation in dilute suspensions, and extended the theory of Temkin and

Dobbins¹⁴ by considering nonlinear drag and heat transfer laws applicable to relatively large-sized droplets. The absorption coefficient per unit frequency predicted by the nonlinear theory is compared with that indicated by the theory of Temkin and Dobbins¹⁴ in Fig. 1. In Fig. 1,

$$\bar{\alpha} = c_0 \alpha / (C_m \omega) = f(\omega \tau_d, \text{Pr}, c_{pp} / c_{pg}, \gamma) \quad (2a)$$

where
$$\tau_d = d_p^2 \rho_p / (18 \mu_g) \quad (2b)$$

and
$$C_m = n_0 m_p / \rho_g \quad (2c)$$

In the above, $\bar{\alpha}$ is the attenuation per unit frequency per unit mass fraction, α the amplitude attenuation coefficient, c_0 the speed of sound in the gas phase, ω the circular frequency, C_m the mass concentration, τ_d is the dynamic relaxation time of the particle (relating to particle-fluid velocity lag), n_0 the mean number of particles per unit volume of mixture, and m_p the mass of one particle, ρ_p the mean particle density, ρ_g the mean density of gas, μ_g the mean dynamic viscosity of gas, and d_p the particle diameter. Also the quantity Pr refers to Prandtl number of the gas, c_{pg} the specific heat of gas, c_{pp} the specific heat of the particle, and γ the isentropic exponent (specific heat ratio). Note that $\alpha = \alpha_i / 2$. The results shown in Fig. 1 correspond to $c_{pp} / c_{pg} = 4.17$, $\text{Pr} = 0.71$, $\gamma = 1.4$ (representative of water droplets in air¹⁴). With the aid of this nonlinear model, the existence of the spectral peak in the linear absorption coefficient α has been demonstrated²¹.

On the basis of this extension, good agreement was achieved with the recent data of Norum²² for sound attenuation in perfectly expanded supersonic jets containing suspended water droplets, which reveal that the linear absorption coefficient displays a spectral peak (Fig. 2). The data correspond to hot supersonic jet of air from a convergent-divergent (CD) nozzle operation at a jet total temperature $T_t = 867$ K, and a jet exit Mach number $M_j = 1.45$. The jet Mach number is

defined as $M_j = u_j / c_j$, where the subscript j refers to the nozzle exit conditions. The mass flow rate (maximum considered) of water to that of the jet is about 0.85. The angle θ is measured from the jet inlet axis. In the data, water is injected at 45 deg.

Similar spectral peaks in the attenuation coefficient have been observed in the measurements by Krothapalli et al.²³ for microjet injection of water into high speed exhaust jets. Spectral peaks in the linear absorption coefficient in dilute suspensions were also noticed in the experimental data of Richards et al.^{19,20} covering a frequency range of 50-150 kHz.

In the preceding work by the author²¹ results were presented only for sound attenuation with nonlinear particle relaxation, and the effect of nonlinearity on the dispersion effects was not considered. In practical applications such as exhaust jets with injected water droplets, in addition to sound attenuation the dispersion effects (phase velocity changes) are also important from the standpoint of sound radiation. The present work applies the nonlinear theory for the dispersion of sound in dilute suspensions, and compares the linear and nonlinear theories for sound dispersion.

2. Sound dispersion

The present analysis for sound dispersion is similar to that proposed by the author²¹ for sound attenuation, and will be briefly presented as below. Without any loss of generality the dispersion of sound for large particle Reynolds numbers with nonlinear particle relaxation may be expressed with the aid of Temkin and Dobbins¹⁴ results as follows:

$$\bar{\beta} = \left[\left(\frac{c_0}{c} \right)^2 - 1 \right] / C_m = \frac{1}{1 + \omega^2 \tau_{d1}^2} + (\gamma - 1) \left(\frac{c_{pp}}{c_{pg}} \right) \frac{1}{1 + \omega^2 \tau_{l1}^2} \quad (3)$$

In the above the quantity $\bar{\beta}$ is the dimensionless dispersion coefficient, c_p the specific heat, c the actual speed of sound in two-phase medium (phase velocity), and γ the isentropic exponent (specific-heat ratio). The subscripts g and p respectively denote the gas and the particle.

The relaxation times τ_{d1} and τ_{t1} correspond to those under nonlinear drag conditions (generally representative of large-sized particles). Physically the dynamic relaxation time τ_{d1} is a measure of the time scale in which the particles follow (respond to) the fluctuations in the fluid motion.³ Likewise, the thermal relaxation time τ_{t1} is a measure of the thermal response time of the particles to follow the fluctuations in the temperature of the fluid. They are related to the relaxation times τ_d and τ_t by the relations²¹

$$\tau_{d1} = \tau_d \psi_1(\text{Re}_p) \quad \tau_{t1} = \tau_t \psi_2(\text{Re}_p) \quad (4a)$$

where $\psi_1(\text{Re}_p) = C_{D1} / C_D$, $\psi_2(\text{Re}_p, \text{Pr}) = Nu_1 / Nu$ (4b)

with C_{D1} standing for the nonlinear drag coefficient, and Nu_1 for the nonlinear heat transfer (Nusselt number). In the above, the quantity τ_d is given by Eq. (2b), and

$$\tau_t = \frac{m_p c_{pp}}{2\pi d_p k_g} = \frac{\text{Pr} c_{pp} d_p^2 \rho_p}{12 \mu_g c_{pg}} = \left(\frac{3}{2} \right) \left(\frac{c_{pp}}{c_{pg}} \right) \text{Pr} \tau_d \quad (4c)$$

Also the particle Reynolds number Re_p is defined by

$$\text{Re}_p = \rho_g |u_g - u_p| d_p / \mu_g \quad (4d)$$

where u_g and u_p denote the velocity of the particle and the gas respectively. Also the quantity $\text{Pr} = c_{pg} \mu_g / k_g$ stands for the Prandtl number of gas, where k_g stands for the thermal conductivity of the gas.

The drag coefficient and the Nusselt number in Eq. (4b) are defined by

$$C_D = 2F_p / \left(\rho_g \frac{\pi}{4} d_p^2 u_g^2 \right), \quad Nu = h_g d_p / k_g \quad (5)$$

where F_p is the particle drag force, and h_g the gas-droplet convective heat transfer coefficient.

For rigid particles, the linear droplet drag and heat transfer are respectively obtained from

$$F_p = 6\pi\mu_g(u_p - u_g) \quad (6a)$$

$$Q_p = 2\pi d_p k_g (T_p - T_g) \quad (6b)$$

which correspond to the zero droplet Reynolds number limit ($Re_p \rightarrow 0$). According to Temkin and Dobbins,¹⁴ Stokes linear drag law can be justified for $0 \leq \omega\tau_d \approx 1$, provided that $\rho_g / \rho_p \ll 1$ and $(\omega d_p^2 / 8\nu_g)^{1/2} \ll 1$.

The expressions for ψ_1 and ψ_2 have been taken as²¹

$$\psi_1(Re_p) = 1 + \frac{Re_p}{24} \left(\frac{6}{1 + \sqrt{Re_p}} + 0.4 \right) \quad (7)$$

$$\psi_2(Re_p, Pr) = Nu_1 / Nu = 1 + 0.3 Re_p^{0.5} Pr^{0.33} \quad (8)$$

Eq. (8) is obtained from Ref. 24, and Eq. (23) is based on the Ref. 25.

The determination of particle Reynolds number required in the evaluation of the functions ψ_1 and ψ_2 in Eqs. (7) and (8) respectively is exceedingly complex. There exists relatively little information on the dependence of particle Reynolds number on the particle characteristics in two-phase flows. In this connection the author²¹ postulated that the particle Reynolds number depends only on the particle relaxation time, and is independent of the particle to fluid density ratio, and the following power law relation is proposed on the basis of work of Ref. 26:

$$Re_p = f(\omega\tau_d) = c(\omega\tau_d)^3 \quad (13)$$

The adjustable constant c is determined from a correlation of the theory with the test data. A value of $c = 10$ was found to be satisfactory based on the data of Norum²¹ for water droplets in a supersonic air jet (Fig. 2).

3. Results and comparison

Figure 3 illustrates a comparison of the predictions for the dispersion coefficient between the linear and nonlinear theories for particle relaxation. The results are shown for $c_{pp}/c_{pg} = 4.17$, $Pr = 0.71$, $\gamma = 1.4$ (representative of water droplets in air¹⁴). The contributions of viscosity and the thermal conductivity along with their combined effect on the dispersion coefficient are shown for both linear and nonlinear relaxation. It is seen that the nonlinear effects become important for $\omega\tau_d > 0.2$ (where heat conduction effects become important). With regard to the viscous contribution, nonlinearities become manifest for somewhat higher values of $\omega\tau_d$ in excess of about 0.4. The results also suggest that the nonlinear effect is more significant in the viscous contribution relative to that of thermal conduction. With nonlinear particle relaxation, the total dispersion coefficient approaches zero for smaller values of $\omega\tau_d$ than those in the case of linear relaxation.

The predictions for the dependence of the dispersion coefficient (yielding the phase velocity c) on the frequency for various values of the dynamic relaxation time τ_d is presented in Fig. 4. For example, the dynamic relaxation time τ_d for silica particles in air is about 10^{-3} sec for a $5 \mu m$ particle, and about 10^{-1} sec for a $50 \mu m$ particle.⁸ On the other hand, the thermal relaxation time for particles in air is about 5×10^{-4} sec for a $5 \mu m$ particle, and about 5×10^{-2} sec for a $50 \mu m$ particle. For comparison purposes, the molecular and thermal relaxation time scales are of the order of 10^{-10} sec for gases. The results suggest that as the particle relaxation time increases, the dispersion curve is shifted to lower frequencies, as is to be expected.

It is believed the nonlinear particle relaxation effects on sound attenuation and dispersion will be of interest in the prediction of jet noise reduction by water injection.²⁷⁻²⁹

4. Conclusion

The theory of nonlinear particle relaxation proposed previously for sound attenuation in dilute suspensions has been extended to investigate the effect of nonlinearity on sound dispersion. The results reveal that significant nonlinear effects are noticed at relatively large particle relaxation times. It is also observed that the nonlinear effect on dispersion due to viscous contribution is larger relative to that of thermal conduction.

Acknowledgments

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Figure Captions

Fig. 1. (color online) Predicted absorption coefficient with nonlinear particle relaxation processes.

Fig. 2. (color online) Comparison of the predictions for the linear absorption coefficient with test data of Norum²².

Fig. 3. (color on line) Comparison of the predictions for the dispersion coefficient between the linear and nonlinear theories for particle relaxation.

Fig. 4. (color online) Variation of dispersion coefficient with frequency for various values of particle relaxation time.

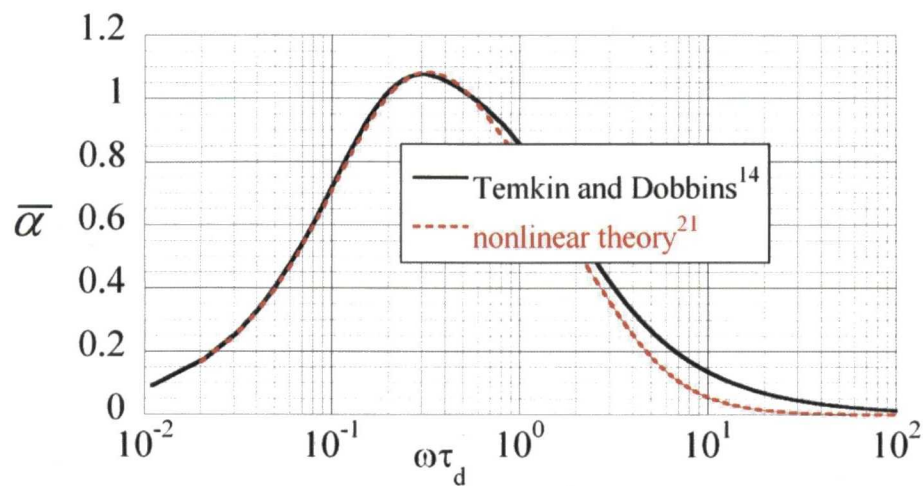


Fig. 1. (Color online) Predicted absorption coefficient with nonlinear particle relaxation processes.

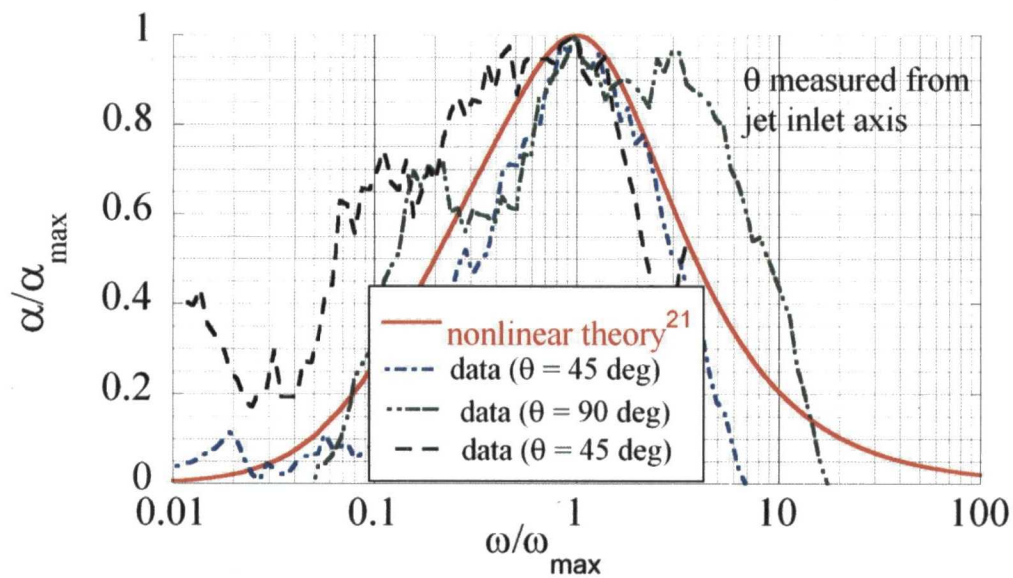


Fig. 2. (Color online) Comparison of the predictions for the linear absorption coefficient with test data of Norum²².

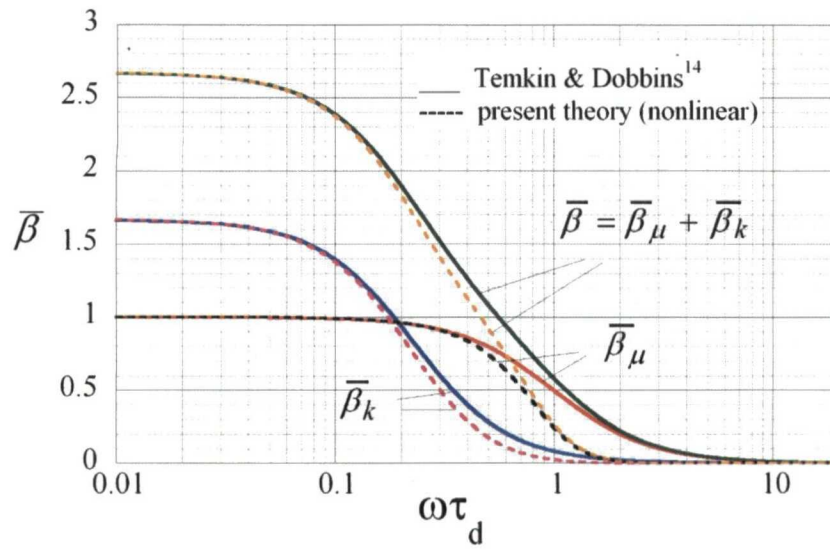


Fig. 3. (Color online) Comparison of the predictions for the dispersion coefficient between the linear and nonlinear theories for particle relaxation.

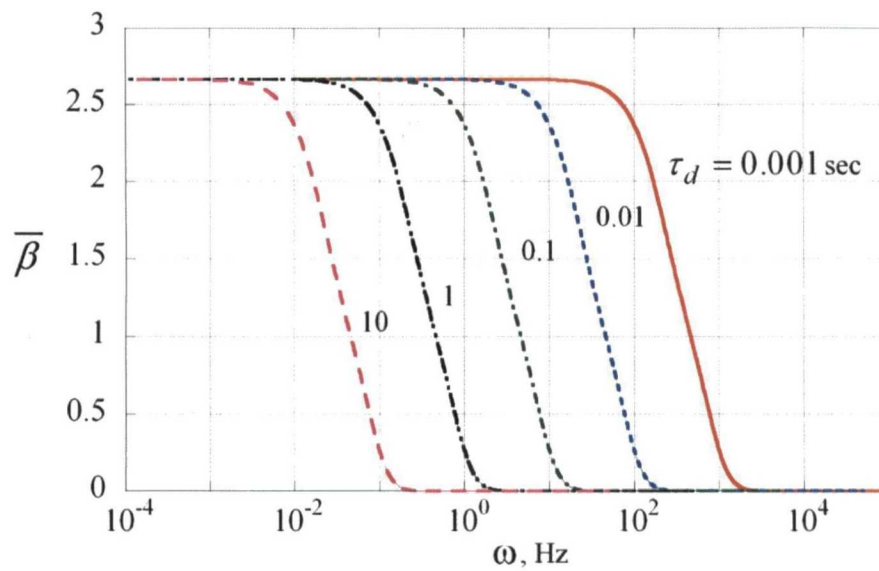


Fig. 4. Variation of dispersion coefficient with frequency for various values of particle relaxation time.