

# Optimal Relay Station Placement in Broadband Wireless Access Networks

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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## Abstract

With the development of IEEE 802.16j multihop relay protocol, the requirement to enhance the network capacity in a wireless network has been met effectively. In this thesis, we study the capacity enhancement problem for a broadband wireless access network which is achieved by optimal placement of Relay Stations (RSs) along with the presence of a Base Station (BS) and multiple Candidate Positions (CPs). We present a mixed integer programming formulation for the crucial task of RS placement. Weighted objective is also explored to include preferential RS placement. The proposed formulations are solved in a matter of seconds. It is observed that with preferential RS placement, the same demand can be met with 73% fewer RSs with a slight, 6%, decrease in the overall network capacity. Moving forward, the objective is broadened to combine and include joint BS and RS placements for a given network. This model formulation provides better overall capacity than combined capacities of RS placement formulations. Maximin objective is introduced to distribute the excess bandwidth to all subscriber stations (SS) rather than assigning it to only one SS. With this approach, bandwidth allocated to each SS is increases by an average of 35.18%.

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## **Dedication**

This is dedicated to my family and my friends.

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# Chapter 1

## Introduction

With more than seven billion people on this earth [1], the telecommunication industry is an ever-growing industry. Telecommunication networks are under constant developments to cope up with the increasing customer demand. The demand for higher capacity of the network is motivated by the presence of service hot-spots (high demand areas) and because of demand for content-rich web-based applications. Also, there is a need in wireless networks to provide services at lower cost and to satisfy customer demands while maintaining quality-of-service (QoS). Higher capacity of the network can be achieved either by laying high capacity cables and installing new equipment or by using new technologies that can work on existing infrastructure without much modification.

The communication networks can be broadly divided into three categories; Local Area Network (LAN), Metropolitan Area Network (MAN) and Wide Area Network (WAN). LAN is a computer network that connects computers in a small area such as a home

or a school. On the other hand, MAN links multiple LANs and extends the reach of the network which spans over a large campus or even a city. WAN connects multiple LANs and MANs and covers a broad area. Internet is the best example of a wide area network. Any network that does not connect to computers via a cable is known as a wireless network. A wireless telephone network is a telecommunications network used for telephone calls where the users are mobile and can roam anywhere within a fixed area or cell site. A Mobile user in a cell site is connected to its home base station (BS) and is switched to a neighbouring BS when he/she moves from one cell site to another. Currently, problems such as low signal-to-noise ratio (SNR) at the cell boundary, coverage holes due to shadowing (signal fading due to obstacles) existence of hotspots (densely populated areas) and varied QoS requirements are making signal transmission weak [6], [13]. These problems apply in all wireless communication networks; wireless LAN, MAN, WAN as well as wireless telephone networks. In this thesis, focus is on wireless telephone networks.

Until 2009, various techniques such as splitting cells and increasing the number of channels [10] are used to enhance the capacity of a network within the physical layer of the communication network. But, with the presence of the IEEE 802.16j protocol, multihop relaying technique is being used to achieve enhanced capacity gains [17]. Subscriber Stations (SSs) located at the edge of a wireless telecommunication network communicate with a Base Station (BS) at a low data rate which results in poor quality of service. Using relaying technology, the signal is sent from the BS to the SSs via a Relay Station (RS) and this re-transmission is known as two-hop transmission due to presence of two hops (BS  $\rightarrow$  RS  $\rightarrow$  SS). A Relay Station forwards the data and improves the signal quality for subscriber stations by replacing one long-distance low-rate link with two short-distance

high-rate links. Decode-and-forward (D-F) cooperative relaying is a relaying technology in which the received signal from BS is decoded at the RS and forwarded to the SS using a different coding scheme. At SS, the original signal as well as the forwarded signal are received and the best one is selected. It should be noted that relay stations are useful for enhancing the strength of the signal for users that are not close enough to the base station and act as an integral solution to the capacity/throughput enhancement [6]. RSs can be immobile or mobile. Immobile RSs not only have constant access to power supply, but also keeps the distance between a BS and a RS fixed, which results in a relatively static link. It is worth mentioning that RSs do not have a direct access to the network and rely entirely on a BS for that purpose. Also, the relay stations can be developed at considerably lower cost due to lower complexity (lack of call routing and handling) and lower installation and maintenance cost as compared to a base station. RSs possess omni-directional antenna that make the coverage of the entire network a comparatively easy task as compared to traditional RSs with one directional antenna. Focus of this thesis is on two-hop D-F cooperative relaying and immobile RSs. The user demand is proportional to the application needs and if the suggested RS placement is able to fulfill the user demands with allocated bandwidth; then it is assumed that the QoS requirements are met.

One of the determining factors in relay transmission is to observe and examine the impact of different RS locations on link reliability and system capacity. The challenge in the location planning for relay stations is to find a middle ground between BS-RS connection reliability and the overall system capacity while fulfilling the user demands. This makes RS placement an essential component of the wireless network planning and deployment. It is also important to keep connectivity and bandwidth limitations into

consideration when deciding on the RS placement as un-optimized placements of RSs may not produce expected benefits. It is desirable to achieve the required degree of coverage to maximize system capacity and minimize the overall cost.

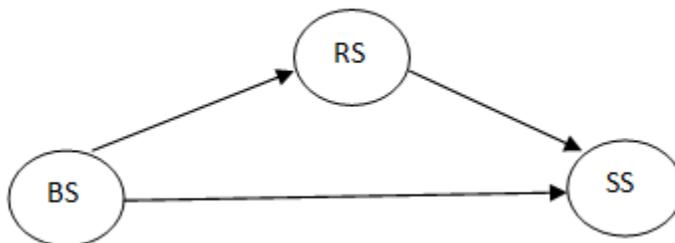


Figure 1.1: A 3 node network

Figure 1.1 shows a 3 node architecture which can be used as a guideline for the operational procedures on how various components of the network behave in a multi RS scenario. A base station acts as a central component that not only connects the cell-site to the network backbone, but also deals with the call flow and routing in the entire network. A SS is an accumulated sum of user connections that are localized to the subscriber station. The number of users connected with a SS can vary from a few hundred in a housing neighbourhood to a few thousand in a hotspot like a concert or a game.

It is observed that with proper deployment of RSs in a network, significant performance gains can be achieved in terms of user throughput as well as the cellular coverage compared to conventional single-hop cellular networks[8]. In this thesis, we aim to determine the optimal placement of relay stations to meet the demand as well as to maximize system capacity. Several formulations are developed to capture various planning policies that

govern RS placement in the network. We propose a new model that solves the relay placement problem using D-F cooperative relaying in less time as compared to the original model. Then, we introduce a location model with a weighted objective to implement preferential RS placement. We extend the models to locate both BS and RS and we develop a maxi-min objective to balance bandwidth assignments between SSs. The results obtained prove the worth of our formulations in terms of better system capacity and reduced computational time.

The rest of this thesis is organised as follows; Chapter 2 discusses the literature review. In Chapter 3, the problem formulation is developed for RS location planning of a single cell site where the SS-BS connection is operational via D-F cooperative relaying. An extension of the formulation is provided with preferential RS placement. Chapter 4 models the case of RS and BS location planning with D-F cooperative relaying. This formulation applies to a multiple cell site scenario. Chapter 5 models the RS placement problem with a maxi-min objective. The final conclusion and discussion is covered in Chapter 6. Table 1.1 gives a summary of the proposed model formulations.

	<b>D-F co-op relay only</b>	<b>Maxi-min Objective</b>
<b>location planning of RS only</b>	Model P2	Model P11
<b>location planning of RS + BS</b>	Model P5	Model P12

Table 1.1: Formulation Summary



# Chapter 2

## Literature Review

Relaying is a technique that enhances the signal strength of a wireless network while increasing the system capacity by strategically placing relay stations throughout the network. With the development of well integrated and advanced wireless communication technologies, relaying is an emerging technology to achieve better signal strength throughout a network. Relaying has been studied not only for internet wireless networks such as WLAN and WiFi, but also for cellular telecommunication networks. Table 2.1 gives an overview of different networks where user relaying technology can be used.

Different papers have proposed different models with varied objective functions to provide the best RS placement for a given network. The most common approach used is that of capacity enhancement, which can be achieved by maximizing the relay rate at each SS [6, 7, 11] or by maximizing the overall throughput [8, 13]. Lin et al. [7] study the capacity enhancement problem to accomplish an efficient design in broadband wireless

access networks. This is carried out by formulating the problem of joint RS placement and bandwidth allocation as a mixed-integer linear program (MILP). Performance benefits of relaying technique are achieved by developing a framework to maximize the capacity of the cell and to meet the traffic demands by each SS. In order to reduce the interference of the BS-RS and RS-SS signals, decode-and-forward (D-F) cooperative relaying is used at relay stations during the initial setup which demodulates and decodes the data packets received from the source and forwards them to the destination using a different coding scheme. Lin et al. [6] incorporate cooperative relaying technology to formulate the problem as an optimization problem. The optimal RS location and relay time allocation are both considered in a single stage. The authors focus on maximum data rate at the destination that can be achieved using cooperative relay scheme. Instead of only focusing on D-F cooperative relay strategy, authors also work with compress-and-forward (C-F) cooperative relaying strategy which sends a compressed version of the signal it receives to the destination.

Paper	Network	Modelling Approach
Yu et al. [17]	IEEE 802.16j multi-hop relay network	Minimizing cost of the network
Lin et al. [7]	wireless metropolitan area network (WMAN)	Capacity enhancement
Lin et al. [6]	IEEE 802.16j mobile multi-hop relay (MMR) networks	Capacity enhancement
So A. and Liang B. [13]	wireless local area network (WLAN)	Minimizing packet transaction time
Wang et al. [14]	multi-hop cellular systems	Relay selection rules
Chandra et al. [3]	multi-hop wireless network	Minimizing number of RSs required
Lu et al. [8]	IEEE 802.16j WiMAX Networks	Max throughput

Table 2.1: Networks

Aaron and Liang [13] present a formulation to minimize the expected packet transaction time from a BS to a SS which in turn enhances the efficiency of underlying technology being utilized. A new relaying architecture that exploits the multi-rate ability of the WLAN physical layer is also proposed. The authors solve the p-median location problem

by analyzing the network expected packet transaction time with respect to various RS placements. Wang et al. [14] try to maximize the system capacity based on the decision whether to use a two hop transmission or not. To help in this decision process, two selection rules are used: Signal Strength-Oriented (SSC) and Throughput Oriented (TO). TO selection rule forwards the data via a two-hop transmission if a higher bit rate can be achieved via a RS. On the other hand, SSC selection rule forwards the data via a two-hop transmission if the received signal strength from a RS is stronger than that from the BS. All the SSs communicate with the BS in a specific time-slot, one at a time. Two different time-slot allocation schemes are also studied by the authors. First is the Equal Time-Duration Allocation where each user is allocated an equal time for the data transmission irrespective of whether data is transmitted directly from the BS or through a two-hop transmission process. Second scheme is called Equal User-Throughput Allocation and it allocates time-slots such that all users have same throughput. In general, this scheme is less preferred as a user with low transmission rate will be allocated more resources to achieve the same throughput and this in return decreases the system capacity. This thesis also utilizes the capacity enhancement approach, but expands the objective from just one cell-site to multiple cell-sites. This approach gives us a better overview of the entire network and how multiple cell-sites influence the RS placement.

Another approach utilized to find the optimal placement of relay stations in a network is to calculate the minimum number of RSs required in the network. Yu et al. [17] use this approach for 802.16j multi-hop relay networks by using clustering. Clustering is a technique which is used to divide the state space of the problem and is based on the geographical proximity of the locations. It breaks the large problem into small sub-

problems which are comparatively easy to solve. Along with the multiple RS and SS locations, the formulation also considers several locations for BS. The costs associated with establishing a BS and a RS are also incorporated into the model along with a weight parameter  $\lambda$  which is used to determine the weight that is assigned to the installation cost versus the transmit power requirements. Chandra et al. [3] optimize the placement of relay stations in a wireless neighbourhood network. The placement of RSs is crucial in such an environment and depends upon the layout of the network, demand from users and wireless link characteristics. Subscriber stations have been given the functionality to route their traffic through other SSs to reach a RS, making it a case of multi-hop problem. At the same time, each SS has an upper bound on the amount of traffic that can pass through it at a given time. For this thesis, we utilize the p-median approach which gives us a fixed number of RSs required in the network.

A few heuristic approaches have also been provided for RS optimal placement. [7] proposes a heuristic solution which focuses entirely on the achievable maximum data rate. The search space for the algorithm has been also been reduced by using a constraint to cap the upper bound on the cell capacity. Yu et al. [17] discuss a heuristic algorithm that iteratively picks a RS which maximizes the total demand satisfied when opened with the RSs chosen in the previous iteration. The authors were able to show that the number of RSs increases as the number of SSs increase, which is as expected. But at the same time, if the communication radius is increased for each RS, this requirement decreases substantially.

Now, we present the different formulations proposed.

# Chapter 3

## The Relay Station Placement Problem

The relay station placement problem with multiple candidate positions (CPs) and subscriber stations (SSs) is studied in this chapter. Given the locations and traffic demands, denoted as  $\rho_i$ , of  $I$  SSs, finite locations of  $J$  candidate positions (CPs) for positioning relay stations (RSs), total bandwidth allotted to the cell site (BW); the objective is to maximize the cell capacity ( $C$ ), by positioning a fixed number of  $K$  ( $K \leq J$ ) RSs and allocating at least a minimum required bandwidth to each SS.

In order to formulate the problem, we use the following notation:

### Indices

$i \in N_{SS} = \{1, \dots, I\}$  ;  $N_{SS}$  = Set of SSs

$j \in N_{CP} = \{1, \dots, J\}$  ;  $N_{CP}$  = Set of CPs

### Parameters

$r_{ij}$  = The achievable D-F cooperative relaying rate for SS  $i$  via RS  $j$  and is calculated as in [7].

$\rho_i$  = User demand at SS  $i$ .

BW = The upper bound of radio bandwidth allocated to the cell.

K = The number of RSs to be deployed within the cell.

### Decision Variables

We use a binary decision variable  $x_{ij}$  to indicate whether a SS  $i$  is assigned to a RS at CP

$j$

$$x_{ij} = \begin{cases} 1 & \text{if SS } i \text{ is relayed via an RS located at CP } j, i \in N_{SS}, j \in N_{CP} \\ 0 & \text{otherwise} \end{cases}$$

a binary location variable  $y_j$  to indicate if an RS is located at CP  $j$

$$y_j = \begin{cases} 1 & \text{if an RS is placed at CP } j, j \in N_{CP} \\ 0 & \text{otherwise} \end{cases}$$

a continuous variable  $w_i$  which is the bandwidth allocated to SS  $i$ , and a continuous

variable  $b_{ij}$  which takes the value  $w_i$  if SS  $i$  is relayed via CP  $j$  and 0 otherwise. In other

words

$$b_{ij} = w_i \text{ if } x_{ij} = 1$$

$$\text{and } b_{ij} = 0 \text{ if } x_{ij} = 0$$

The formulation proposed in [7] is:

$$(P1) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J r_{ij} b_{ij} \quad (3.1)$$

subject to:

$$\sum_{j \in N_{CP}} r_{ij} b_{ij} \geq \rho_i, \quad \forall i \in N_{SS} \quad (3.2)$$

$$\sum_{j \in N_{CP}} x_{ij} = 1, \quad \forall i \in N_{SS} \quad (3.3)$$

$$x_{ij} \leq y_j, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.4)$$

$$b_{ij} \leq BW x_{ij}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.5)$$

$$b_{ij} \leq BW(1 - x_{ij}) + w_i, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.6)$$

$$\sum_{j \in N_{CP}} y_j = K \quad (3.7)$$

$$\sum_{i \in N_{SS}} w_i \leq BW \quad (3.8)$$

$$b_{ij} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.9)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.10)$$

$$y_j \in \{0, 1\}, \quad \forall j \in N_{CP} \quad (3.11)$$

The objective function (3.1) maximizes the cell capacity. Constraint (3.2) ensures that the throughput of each SS is not less than its minimum traffic load. Constraint (3.3) makes sure that a SS is serviced by exactly one RS. Constraint (3.4) enforces an RS to be placed at CP  $j$  if SS  $i$  is associated with it. Constraints (3.5) and (3.6) define the decision variable  $b_{ij}$ . They work as follows: if  $x_{ij} = 0$ , then  $b_{ij} \leq 0$ , and with (3.9),  $b_{ij} = 0$ . On the other hand, if  $x_{ij} = 1$ , then together with the objective function  $b_{ij} = w_i$ . Constraint (3.7) is used if we want to have a fixed number of RSs in place within a given network. Constraint (3.8) is the bandwidth constraint and ensures that the bandwidth allocated to each SS is

not more than the overall bandwidth allocated to BS or cell site. Constraints (3.9) are the nonnegativity constraints on  $b_{ij}$ . Constraints (3.10) and (3.11) are binary constraints on  $x_{ij}$  and  $y_j$ .

This model selects  $K$  locations out of  $J$  candidate locations which makes it a facility location model of  $p$ -median type. It is different from classical facility location models in that in location literature terms, the objective is to maximize the throughput of the system given an available capacity  $BW$ , a minimum required demand  $\rho_i$  and a conversion rate  $r_{ij}$ .

In the next section, we propose a tighter formulation of the same problem. While in Section 3.2, we suggest a fixed-charge type location model.

### 3.1 Proposed Formulation

Let us analyze how constraints (3.5), (3.6) and (3.8) work together:

$$\begin{array}{l|l}
 \text{For } x_{ij}=1 & \text{For } x_{ij}=0 \\
 b_{ij} \leq BW & b_{ij} \leq 0 \\
 b_{ij} \leq w_i & b_{ij} \leq BW \\
 \sum_i w_i \leq BW &
 \end{array}$$

Since only one variable  $x_{ij} = 1$  for each  $SS_i$  (by 3.3), then one can enforce the same condition while eliminating the variable  $w_i$  and replacing (3.5), (3.6) and (3.8) by (3.16) and (3.17) as in model [P2] below.

$$(P2) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J r_{ij} b_{ij} \quad (3.12)$$

subject to:

$$\sum_{j \in N_{CP}} r_{ij} b_{ij} \geq \rho_i, \quad \forall i \in N_{SS} \quad (3.13)$$

$$\sum_{j \in N_{CP}} = 1, \quad \forall i \in N_{SS} \quad (3.14)$$

$$x_{ij} \leq y_j, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.15)$$

$$b_{ij} \leq BW x_{ij}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.16)$$

$$\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} b_{ij} \leq BW, \quad (3.17)$$

$$\sum_{j \in N_{CP}} y_j = K \quad (3.18)$$

$$b_{ij} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.19)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.20)$$

$$y_j \in \{0, 1\}, \quad \forall j \in N_{CP} \quad (3.21)$$

The formulation [P2] models the same setting as [P1] and maximizes the cell capacity. Constraint (3.16) defines decision variable  $b_{ij}$ ; if  $x_{ij} = 0$  then  $b_{ij} = 0$  and if  $x_{ij} = 1$  then  $b_{ij} \leq BW$  which is a redundant upper bound. In fact, when  $x_{ij} = 1$ , the value of  $b_{ij}$  is determined by constraints (3.13) and the objective function: for  $x_{ij} = 1$ ,  $b_{ij} \geq \frac{\rho_i}{r_{ij}}$  to satisfy the minimum required bandwidth. Its final value will be set so that  $\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} r_{ij} b_{ij}$  is maximized. We have removed the decision variable  $w_i$  and replaced the need with constraint (3.17). The number of variables is decreased by  $I$  and number of constraints are reduced by  $I \times J$ .

## 3.2 Weighted Objective Formulation

Model P2 is able to provide us with the best RS placement locations in the network. It's optimal solution, however, lacks an important realistic component. In a real-life scenario, there might be few candidate positions (CPs) that are preferred over other locations due to several parameters such as proximity to the city centre, steady user demand, zoning by-laws, etc. Also, cost of construction of a relay station can vary due to price for land acquisition among other factors. These factors motivated us to extend the model to include the cost of locating an RS at a CP in the objective function.

Given the fixed cost ( $f_j$ ) for location  $j$  and a preference weightage ( $\gamma$ ), the proposed location model with weighted objective is:

$$(P3) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J r_{ij} b_{ij} - \gamma \sum_{j=1}^J f_j y_j \quad (3.22)$$

subject to:

$$\sum_{j \in N_{CP}} r_{ij} b_{ij} \geq \rho_i, \quad \forall i \in N_{SS} \quad (3.23)$$

$$\sum_{j \in N_{CP}} x_{ij} = 1, \quad \forall i \in N_{SS} \quad (3.24)$$

$$x_{ij} \leq y_j, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.25)$$

$$b_{ij} \leq BW x_{ij}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.26)$$

$$\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} b_{ij} \leq BW, \quad (3.27)$$

$$b_{ij} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.28)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (3.29)$$

$$y_j \in \{0, 1\}, \quad \forall j \in N_{CP} \quad (3.30)$$

The model is similar to [P2]. The difference is that the constraint (3.18) which sets the number of RSs located to  $K$  is dropped and the objective function is replaced by a weighted one. The first term in the objective function maximizes cell capacity while the second term minimizes location cost. Since, both terms are not measured in the same units and may have different ranges of magnitude, a weight  $\gamma$  multiplies the second term to balance capacity and cost. By varying  $\gamma$ , one can construct a trade off curve between achieved capacity and RS location cost which is useful for decision makers to decide on the number of RSs to locate.

### 3.3 Numerical Results

#### 3.3.1 Data Sets

A base station caters to a cell site which has a hexagonal structure. Cell site structure can be circle, square, etc., but hexagonal cells are conventionally used by the industry and we use the same to formulate our scenarios. The data is generated randomly using the procedure given in [7]. Each scenario is assigned a bandwidth of  $BW = 20$  MHz. For all the three scenarios, we generate locations of SSs and CPs inside a cell site and random load for each SS between a fixed range of 1-4. For all the scenarios, BS is located at  $(0,0)$ ; the CPs are distributed uniformly between  $x = (0.1-0.4)$  and  $y = (0.1-0.4)$  and the SSs are distributed uniformly between  $x = (0.4-0.8)$  and  $y = (0.4-0.8)$ . To simulate the proposed models, three instances are generated with  $(I, J) = (22,40),(40,60),(65,100)$ .

$I$	$J$	$K$	Model P1				Model P2			
			No. of variables	No. of constraints	CPU time	Best Objective	No. of variables	No. of constraints	CPU time	Optimal Objective
22	40	12	1,822	3,566	3600	71.57	1800	2686	2.34	72.69
40	60	16	4,900	9,682	3600	–	4860	7282	4.35	79.4
65	100	32	13,165	26,132	3600	–	13100	19632	8.56	75.98

Table 3.1: Comparison of Model P1 and Model P2

### 3.3.2 Results

We now report on computational experiments and results for models P1, P2 and P3. The formulations are solved by Gurobi 4.5.1, using a laptop with Intel Core i3 2.27 GHz and 4 GB RAM.

Table 3.1 shows results for three instances indicating the number of variables, number of constraints, time in CPU seconds and the optimal /best objective function value achieved within the CPU time. For Model P1, each instance was run for one hour and the best objective was recorded. For instances 2 and 3, Gurobi failed to solve the model within one hour. For Model P2, all instances were solved within seconds. Figure 3.1 shows the network configuration of Instance-2 with 40 SSs and 60 CPs. Figure 3.2 shows the trade off curve between the objective function value and the number of RSs ( $K$ ) to be placed in the network. It is observed that after a fixed value of  $K$ , the threshold limit is reached and the objective cannot be improved further. For instance-1, that value is  $K=14$ , and for Instance-2, it is  $K=10$ . Table 3.2 shows the numerical results for the same.

For Model P3, Table 3.3 shows the computational results for all three scenarios.  $f_j$  is fixed and  $\gamma$  is varied from 0.01 to 1. We observe that as soon as we assign a weight to a CP, the number of RSs required decreases substantially, on average 73%, while achieved

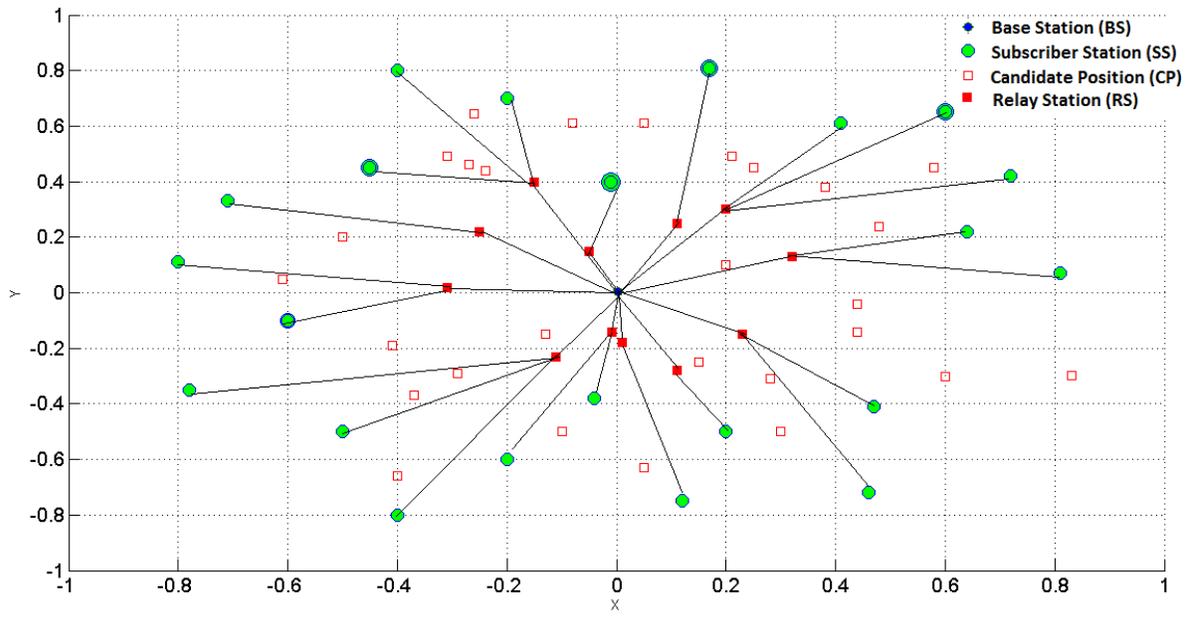


Figure 3.1: Network Configuration given by Model P2 for  $I=40, J=60$

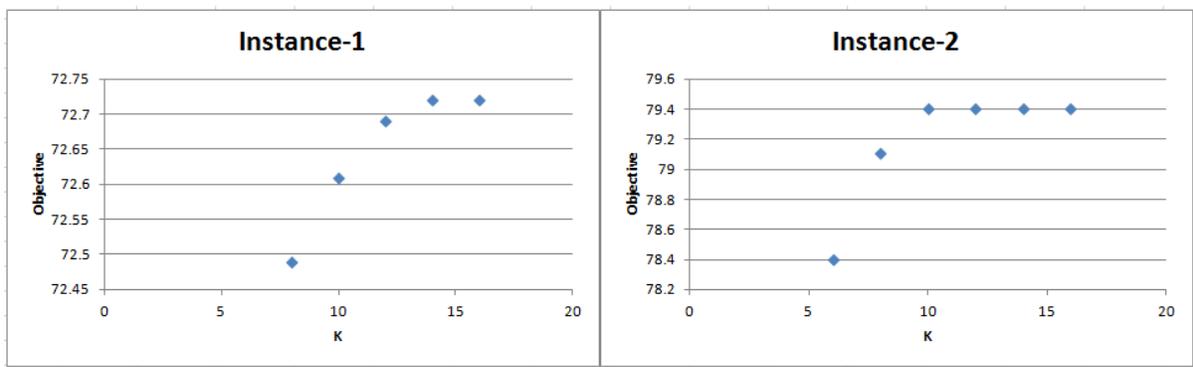


Figure 3.2: Trade off curves for instance-1

Instance-1		Instance-2	
k	Objective	k	Objective
6	72.40	6	78.4
8	72.49	8	79.11
10	72.61	10	79.4
12	72.69	12	79.4
14	72.72	14	79.4
16	72.72	16	79.4

Table 3.2: Varied objective with varied K for Model P2

$I=22, J=40$				$I=40, J=60$				$I=65, J=100$			
$\gamma$	$\sum r_{ij}b_{ij}$	RSs	CPU Time	$\gamma$	$\sum r_{ij}b_{ij}$	RSs	CPU Time	$\gamma$	$\sum r_{ij}b_{ij}$	RSs	CPU Time
0	72.73	16	0.11	0	79.4	12	0.31	0	75.98	15	0.73
0.01	67.96	4	6.34	0.01	74.03	4	12.85	0.01	72.15	3	78.24
0.05	64.54	1	2.23	0.05	70.55	1	8.12	0.05	69.14	1	53.43
0.1	64.54	1	2.44	0.1	70.55	1	6.42	0.1	69.14	1	54.03
0.5	64.54	1	2.2	0.5	70.55	1	3.3	0.5	69.14	1	54.4
1	64.54	1	1.33	1	70.55	1	3.22	1	69.14	1	56.93

Table 3.3: Result - Model P3

capacity decreases slightly at an average of 6%. As the weight on each CP is increased even more, we see slight decrease in number of RSs required and a stable capacity. Figure 3.3 shows the plot of achieved capacity versus the number of RSs required when the weight  $\gamma$  is varied between 0 to 1. In all instances, we observe that as  $\gamma$  increases, the models take more CPU time to solve, but they all solve within one minute.

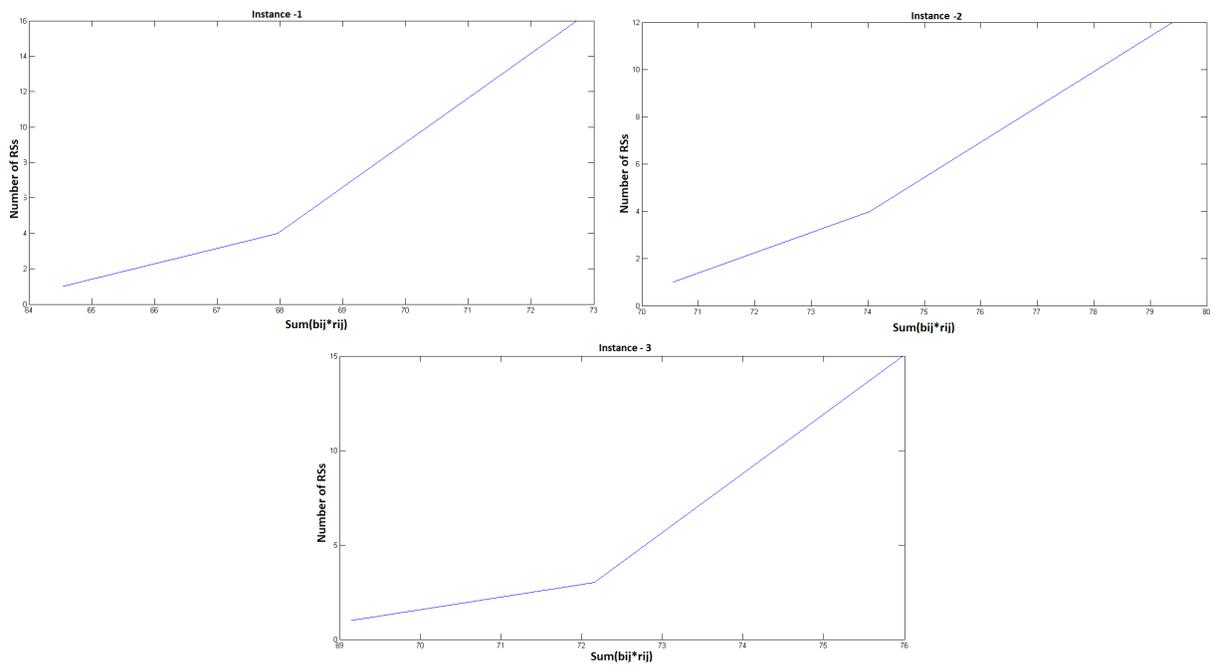


Figure 3.3: Trade off curves for instance 1, 2 and 3



## Chapter 4

# Two Level Planning: Base and Relay Station Placement

Now that we have successfully obtained the best relay placement locations in the network, we venture out for another possible extension to the model formulation. It is known that it is not possible to have only one stand-alone cell site. The entire telecommunication network consists of multiple cell sites and consequently multiple base stations. It is worth exploring the effects of BS locations on achieved capacity and on bandwidth allocation configuration. Also, a smaller number of BSs required to cover the entire cellular network means less planning complexity [5].

Given multiple potential locations for BSs and RSs, this formulation tries not only to find the best relay station placement, but also to find the best BS location and the best BS-RS-SS connection. We have increased the complexity of the model by including extra

BS locations and selecting a subset of them. This selection is entirely dependent upon the relay rates between BSs-RSs-SSs.

Let the set of BS potential locations be  $N_{BS}$  with  $L$  potential locations and indexed by  $l$ .

We modify the decision variables from Chapter 3 as:

**New Decision Variables:**

$$x_{ijl} = \begin{cases} 1 & \text{if SS } i \text{ is relayed via an RS at } CPj \text{ and BS } l, i \in N_{SS}, j \in N_{CP}, l \in N_{BS} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jl} = \begin{cases} 1 & \text{if an RS is placed at } CPj \text{ and connected to BS } l, j \in N_{CP}, l \in N_{BS} \\ 0 & \text{otherwise} \end{cases}$$

and introduce a binary BS location variable  $z_l$ :

$$z_l = \begin{cases} 1 & \text{if a BS is located at location } l \\ 0 & \text{otherwise} \end{cases}$$

$b_{ijl} = w_i$  if an  $SSi$  is relayed via  $CPj$  and allocated  $w_i$

An extension of Model P1 is:

$$(P4) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J \sum_{l=1}^L r_{ijl} b_{ijl} \quad (4.1)$$

subject to:

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} r_{ijl} b_{ijl} \geq \rho_i, \quad \forall i \in N_{SS} \quad (4.2)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} x_{ijl} = 1, \quad \forall i \in N_{SS} \quad (4.3)$$

$$x_{ijl} \leq y_{jl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.4)$$

$$x_{ijl} \leq z_l, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.5)$$

$$b_{ijl} \leq BW x_{ijl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.6)$$

$$b_{ijl} \leq BW(1 - x_{ijl}) + w_{il}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.7)$$

$$\sum_{i \in N_{SS}} w_{il} \leq BW z_l \quad \forall l \in N_{BS} \quad (4.8)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} y_{jl} = K_1 \quad (4.9)$$

$$\sum_{l \in N_{BS}} z_l = K_2 \quad (4.10)$$

$$b_{ijl} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.11)$$

$$x_{ijl} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.12)$$

$$y_{jl} \in \{0, 1\}, \quad \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.13)$$

$$z_l \in \{0, 1\}, \quad \forall l \in N_{BS} \quad (4.14)$$

The objective function (4.1) maximizes total capacity. Constraint (4.2) ensures that the throughput of each SS is not less than its minimum traffic load. Constraint (4.3) makes sure that a SS is serviced by exactly one RS and one BS. Constraint (4.4) enforces an RS to be placed at CP  $j$  if SS  $i$  is associated with it. Constraint (4.5) enforces a BS to be placed at BS  $l$  if SS  $i$  is associated with it. Constraints (4.6) and (4.7) define the decision

variables  $b_{ijl}$  and  $w_i$ . Constraint (4.9) fixes the number of RSs in place to  $K_1$ . Constraint (4.10) is used to fix the number of BSs located to  $K_2$ . Constraint (4.8) is the bandwidth capacity constraint for each cell site. Constraints (4.11) are the nonnegativity constraints on  $b_{ijl}$  and  $w_i$ . Constraints (4.12), (4.13) and (4.14) are binary constraints on  $x_{ijl}$ ,  $y_{jl}$  and  $z_l$  respectively.

Similarly, an extension of Model P2 is:

$$(P5) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J \sum_{l=1}^L r_{ijl} b_{ijl} \quad (4.15)$$

subject to:

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} r_{ijl} b_{ijl} \geq \rho_i, \quad \forall i \in N_{SS} \quad (4.16)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} x_{ijl} = 1, \quad \forall i \in N_{SS} \quad (4.17)$$

$$x_{ijl} \leq y_{jl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.18)$$

$$x_{ijl} \leq z_l, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.19)$$

$$b_{ijl} \leq BW x_{ijl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.20)$$

$$\sum_{i \in N_{CP}} \sum_{j \in N_{CP}} b_{ijl} \leq BW z_l \quad \forall l \in N_{BS} \quad (4.21)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} y_{jl} = K_1 \quad (4.22)$$

$$\sum_{l \in N_{BS}} z_l = K_2 \quad (4.23)$$

$$b_{ijl} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.24)$$

$$x_{ijl} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.25)$$

$$y_{jl} \in \{0, 1\}, \quad \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.26)$$

$$z_l \in \{0, 1\}, \quad \forall l \in N_{BS} \quad (4.27)$$

Similar to [P2],  $w_i$  is dropped in [P5] and constraints (4.6), (4.7) and (4.8) are replaced by (4.20) and (4.21).

## 4.1 Weighted Formulation

After observing the advantages of Model P3 in Section 3.3, we were motivated to utilize the same approach for Model P5. As earlier, given the fixed cost  $f_j$  and weight  $\gamma$  of all candidate positions for RS and fixed cost  $f_l$  and weight  $\beta$  of all BS positions, the purpose of this formulation is to find the best BS-RS placement in the network. Constraints (4.22) and (4.23) which fix the number of RS and BS are dropped and two weighted terms are added to the objective function.

$$(P6) \quad \text{maximize} \quad C = \sum_{i=1}^I \sum_{j=1}^J r_{ij} b_{ij} - \sum_{j=1}^J \sum_{l=1}^L \gamma f_j y_{jl} - \sum_{l=1}^L \beta f_l z_l \quad (4.28)$$

subject to:

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} r_{ijl} b_{ijl} \geq \rho_i, \quad \forall i \in N_{SS} \quad (4.29)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} x_{ijl} = 1, \quad \forall i \in N_{SS} \quad (4.30)$$

$$x_{ijl} \leq y_{jl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.31)$$

$$x_{ijl} \leq z_l, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.32)$$

$$b_{ijl} \leq BW x_{ijl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.33)$$

$$\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} b_{ijl} \leq BW z_l \quad \forall l \in N_{BS} \quad (4.34)$$

$$b_{ijl} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.35)$$

$$x_{ijl} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.36)$$

$$y_{jl} \in \{0, 1\}, \quad \forall j \in N_{CP}, \forall l \in N_{BS} \quad (4.37)$$

$$z_l \in \{0, 1\}, \quad \forall l \in N_{BS} \quad (4.38)$$

## 4.2 Numerical Results

This section covers the computational results for Model P5 and P6. Model P4 does not solve within one hour of CPU time. Since we are considering multiple BSs, there needs to be an associated increase in number of CPs and SSs. Table 4.1 provides a summary for the two scenarios used as well as the results obtained by Model P5. Figure 4.1 shows us the network configuration selected by Model P5 for instance 1. Table 4.2 shows how the objective function varies with varying values of  $K_1$  and  $K_2$  for Model P5.

$L$	$I$	$J$	$K_1$	$K_2$	No. of variables	No. of constraints	CPU time
3	60	30	12	2	10,893	21,725	17.36
5	100	50	15	3	50,255	100,207	69.64

Table 4.1: Result - Model P5

Tables 4.3 and 4.4 show the computational results for Model P6. Results obtained are similar to the results obtained in Section 3.3.2; the number of RSs required decreases substantially as soon as we assign weight to a CP at an average of 50%. As more weight is applied, the number of RSs and BSs required drop further. Figure 4.2 compares results obtained via Model P2 and Model P5. Model P5 requires more number of RSs to cover the same area, but provides a 10% better overall capacity than combined capacities of Model P2.

K1	K2	Objective
10	1	418.42
12	1	421.6
14	1	421.92
10	2	424.8
12	2	425.22
14	2	425.22
10	3	425.22
12	3	425.22
14	3	425.22

Table 4.2: Varied objective with varied K1 and K2 for Instance-1, Model P5

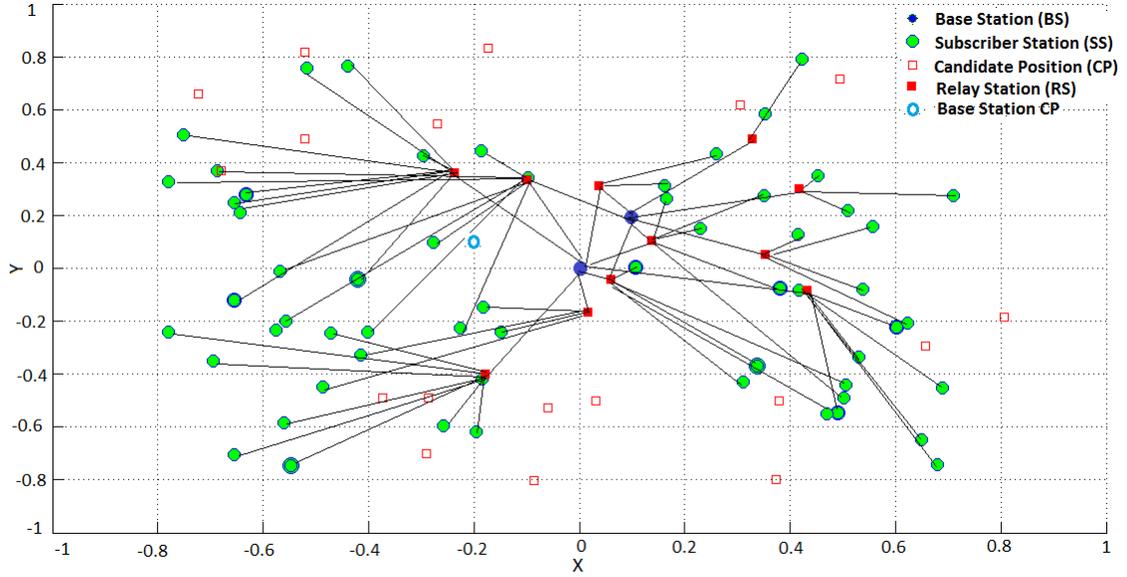


Figure 4.1: Network Configuration given by Model P5 for  $I=60$ ,  $J=30$ ,  $L=3$

$\gamma$	$\gamma'$	$\sum r_{ijl}b_{ijl}$	RSs	BSs	CPU Time
0	0	452.22	11	3	0.37
0.01	0	417.14	6	3	41.74
0.05	0	399.55	3	3	32.33
0.1	0	399.55	3	3	28.83
0.5	0	399.55	3	3	33.21
1	0	399.55	3	3	514.8
1	0.01	399.55	3	3	468
1	0.05	399.55	3	3	492.42
1	0.1	382.42	2	2	191.7
1	0.5	382.42	2	2	215
1	1	382.42	2	2	213

Table 4.3: Result - Model P6,  $I=60$ ,  $J=30$ ,  $L=3$ ,  $f=100$ ,  $f'=1000$

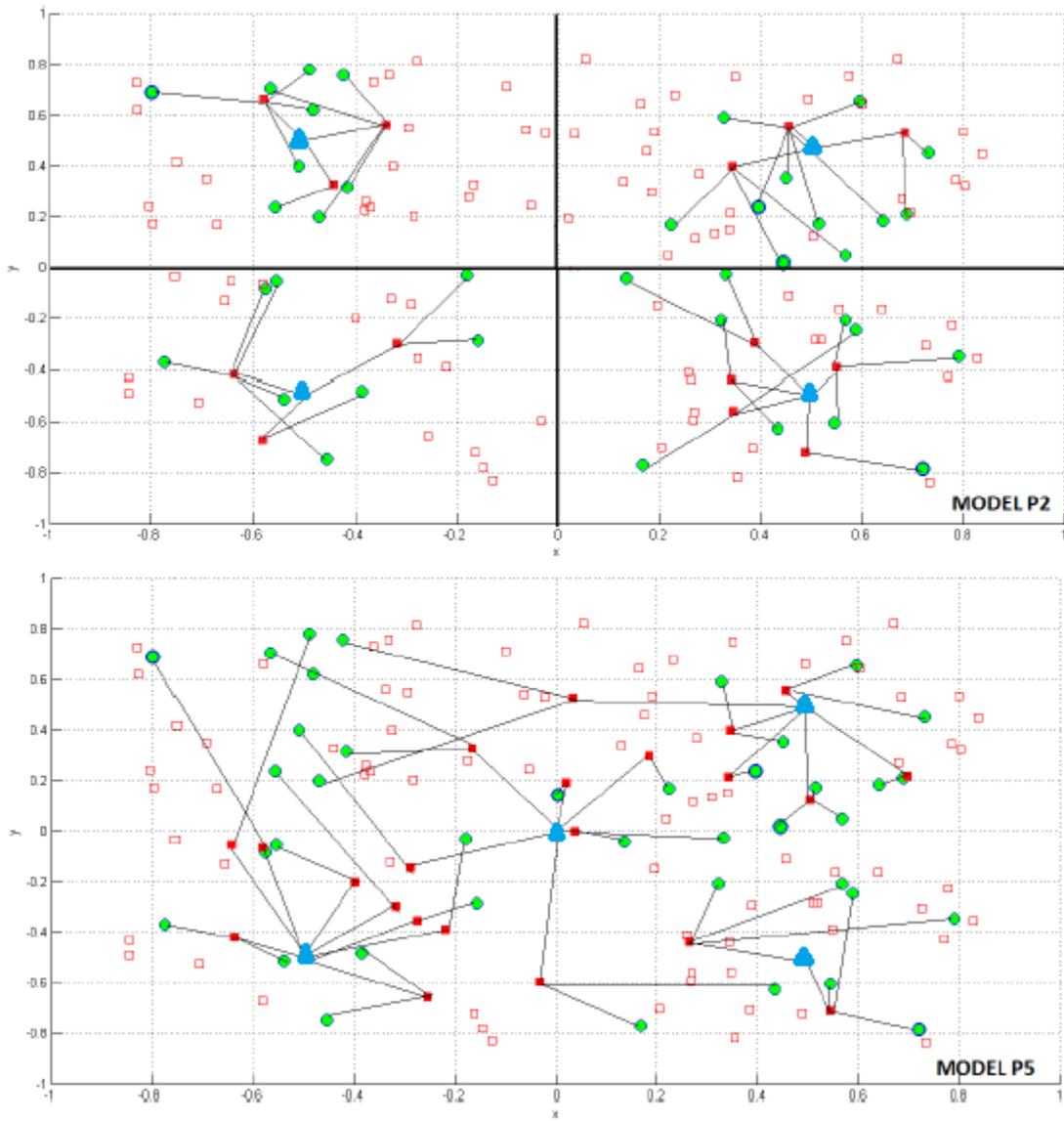


Figure 4.2: Comparison between Model P2 and Model P5

$\gamma$	$\gamma'$	$\sum r_{ijl}b_{ijl}$	RSs	BSs	CPU Time
0	0	560.49	16	5	15.64
0.01	0	550.54	7	5	1820
0.05	0	533.08	6	5	1592
0.1	0	533.08	6	5	1630
0.5	0	515.75	5	5	2210
1	0	515.75	5	5	2289
1	0.01	515.75	5	5	2156
1	0.05	515.75	5	5	2362
1	0.1	515.75	5	5	2278
1	0.5	515.75	5	5	2220
1	1	5 515.75		5	2015

Table 4.4: Result - Model P6,  $I=100$ ,  $J=50$ ,  $L=5$ ,  $f=100$ ,  $f'=1000$

# Chapter 5

## Maxi-min Objective

It was observed that even though Model P2 increases the overall system capacity, it was just fulfilling the demands of all SSs and giving rest of the available BW to the SS with the best relay rate. We saw this as system dysfunctionality because that is wastage of resources. It makes sense to assign some part of extra BW to each of the SS in order to cope up with future increase in user demand rather than assigning it to one SS. Hence, the models from Chapter 3 and 4 are modified with a maxi-min objective to achieve more balanced capacity assignments.

The following is a modification of Model P2:

$$(P11) \quad \text{maximize} \quad Z \quad (6.1)$$

subject to:

$$Z \leq \sum_{j \in N_{CP}} b_{ij} r_{ij} - \rho_i, \quad \forall i \in N_{SS} \quad (6.2)$$

$$\sum_{j \in N_{CP}} b_{ij} r_{ij} \geq \rho_i, \quad \forall i \in N_{SS} \quad (6.3)$$

$$\sum_{j \in N_{CP}} x_{ij} = 1, \quad \forall i \in N_{SS} \quad (6.4)$$

$$x_{ij} \leq y_j, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (6.5)$$

$$b_{ij} \leq BW x_{ij}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (6.6)$$

$$\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} b_{ij} \leq BW, \quad (6.7)$$

$$\sum_{j \in N_{CP}} y_j = K \quad (6.8)$$

$$Z \geq 0, \quad (6.9)$$

$$b_{ij} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (6.10)$$

$$x_{ij} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP} \quad (6.11)$$

$$y_j \in \{0, 1\}, \quad \forall j \in N_{CP} \quad (6.12)$$

Constraints (6.3) - (6.12) work similar to constraints (3.13) - (3.21). Constraints (6.1) and (6.2) maximize the difference between the bandwidth allocated to each SS and the individual demand. This ensures that the total excess BW is distributed evenly among all SSs. Constraint (6.9) is the nonnegativity constraint on  $Z$ .

We utilize the same approach to find a more balanced solution for the two-level location RS-BS Model P5.

$$(P12) \quad \text{maximize} \quad Z \quad (4.15)$$

subject to:

$$Z \leq \sum_{j \in N_{CP}} \sum_{l \in N_{BS}} r_{ijl} b_{ijl} - \rho_i, \quad \forall i \in N_{SS} \quad (6.13)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} r_{ijl} b_{ijl} \geq \rho_i, \quad \forall i \in N_{SS} \quad (6.14)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} x_{ijl} = 1, \quad \forall i \in N_{SS} \quad (6.15)$$

$$x_{ijl} \leq y_{jl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.16)$$

$$x_{ijl} \leq z_l, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.17)$$

$$b_{ijl} \leq BW x_{ijl}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.18)$$

$$\sum_{i \in N_{SS}} \sum_{j \in N_{CP}} b_{ijl} \leq BW z_l \quad \forall l \in N_{BS} \quad (6.19)$$

$$\sum_{j \in N_{CP}} \sum_{l \in N_{BS}} y_{jl} = K_1 \quad (6.20)$$

$$\sum_{l \in N_{BS}} z_l = K_2 \quad (6.21)$$

$$b_{ijl} \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.22)$$

$$Z \geq 0, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.23)$$

$$x_{ijl} \in \{0, 1\}, \quad \forall i \in N_{SS}, \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.24)$$

$$y_{jl} \in \{0, 1\}, \quad \forall j \in N_{CP}, \forall l \in N_{BS} \quad (6.25)$$

$$z_l \in \{0, 1\}, \quad \forall l \in N_{BS} \quad (6.26)$$

## 5.1 Numerical Results

Table 5.1 gives us an overview of the demand at each SS and the capacity allocation of each under Model P2 and Model P11 for Instance-1 with  $I=22$ ,  $J=40$ . Table 5.2 compares the results obtained from both the models. It is observed that even though the overall system capacity decreases by an average of 14%, the BW allocated to each SS increases by an average of 35.18% which makes it a more practical formulation as compared to Model P2. This approach might be useful in network configurations where the demands vary over time, which is the case for wireless networks. With excess bandwidth allocated at each SS, a larger number of users can be serviced without the need for more BW. Also, higher QoS factors can be met such as better audio quality, grade-of-service, etc. Figure 5.1 gives the trade-off curves for Model P11. The result is similar to results obtained in Section 3.3.2. The values of  $K$  to reach threshold limits are the same; for instance-1,  $K=14$  and for instance-2,  $K=10$ . Table 5.3 provides numerical results to support figure 5.1.

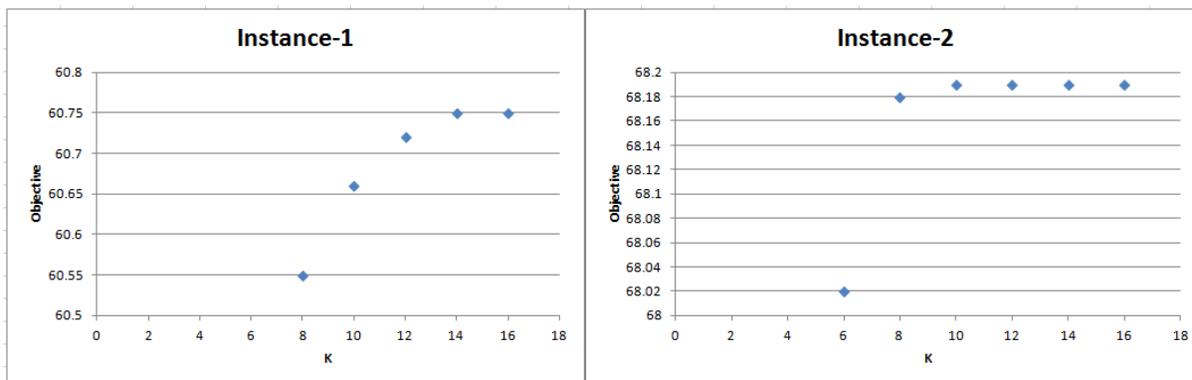


Figure 5.1: Trade off curves for Instance-1, Model P11

Table 5.5 gives a snapshot of the the results obtained by Model P5 and Model P12 for

SS	$\rho_i$	$r_{ij}b_{ij}$ based on P2	$r_{ij}b_{ij}$ based on P11
1	2.2	2.2	3.1285
2	1.3	1.3	2.2285
3	1	1	1.9285
4	3	3	3.9285
5	1.6	1.6	2.5285
6	3.2	3.2	4.1285
7	4	4	4.9285
8	1.2	1.2	2.1285
9	1.1	1.1	2.0285
10	3	3	3.9285
11	1.1	1.1	2.0285
12	2.1	2.1	3.0285
13	2.3	2.3	3.2285
14	1	1	1.9285
15	2.2	2.2	3.1285
16	1	1	1.9285
17	2	2	2.9285
18	1	33.3985	1.9285
19	1	1	1.9285
20	2	2	2.9285
21	2	2	2.9285
22	1	1	1.9285

Table 5.1: Comparison of bandwidth assignment based on P2 and P11

$L=3, I=60, J=30$ . It is observed that Model P5 assigns BW to each SS exactly equal to its demand and the BW is distributed between SS 29 and SS 42. On the other hand, Model P12 distributes the excess BW to all SSs. Table 5.4 compares the results for the two scenarios for Model P5 and Model P12. It is observed that Model P12 is also able to increase the BW at each SS by an average of 64.17% for the first instance and by 38.96% for the second instance as compared to Model P5.

Scenario	I	J	K	Model P2		Model P11		
				$\sum r_{ij}b_{ij}$	CPU Time	$\sum r_{ij}b_{ij}$	CPU Time	Average increase in BW at each SS
1	22	40	12	72.69	2.08	60.72	1.62	36.58%
2	40	60	16	79.4	4.35	68.19	6.23	26.03%
3	65	100	32	75.98	8.56	66.38	21.65	42.94%

Table 5.2: Comparison of achieved capacity between Model P2 and Model P11

Instance-1		Instance-2	
k	Objective	k	Objective
6	60.48	6	68.02
8	60.55	8	68.18
10	60.66	10	68.19
12	60.72	12	68.19
14	60.75	14	68.19
16	60.75	16	68.19

Table 5.3: Varied objective with varied K for Model P11

Scenario	L	I	J	$K_1$	$K_2$	Model P5		Model P12		
						$\sum r_{ij}b_{ij}$	CPU Time	$\sum r_{ij}b_{ij}$	CPU Time	Average increase in BW at each SS
1	3	60	30	12	2	425.22	17.48	213.01	35.09	64.17%
2	5	100	50	15	3	139.86	72.48	84.61	408.4	38.69%

Table 5.4: Comparison of achieved capacity between Model P5 and Model P12

SS	$\rho_i$	$r_{ij}b_{ij}$ based on P5	$r_{ij}b_{ij}$ based on P12	SS	$\rho_i$	$r_{ij}b_{ij}$ based on P5	$r_{ij}b_{ij}$ based on P12
1	0.4388	0.4388	2.601	31	2.5765	80.9079	4.7387
2	2.3308	2.3308	4.493	32	0.7335	0.7335	2.8957
3	0.6311	0.6311	2.7933	33	0.719	0.719	2.8812
4	1.0184	1.0184	3.1805	34	1.4729	1.4729	3.635
5	0.1445	0.1445	2.3067	35	1.1246	1.1246	3.2868
6	1.6237	1.6237	3.7859	36	1.0564	1.0564	3.2186
7	0.5717	0.5717	2.7339	37	1.0423	1.0423	3.2045
8	1.8794	1.8794	4.0416	38	0.3239	0.3239	2.4861
9	1.7738	1.7738	3.936	39	0.3282	0.3282	2.4904
10	1.417	1.417	3.5792	40	2.932	2.932	5.0942
11	0.7096	0.7096	2.8717	41	0.776	0.776	2.9381
12	1.7347	1.7347	3.8969	42	2.1556	230.2981	4.3178
13	2.0173	2.0173	4.1795	43	1.2195	1.2195	3.3817
14	2.6561	2.6561	4.8183	44	2.9933	2.9933	5.1555
15	1.8922	1.8922	4.0544	45	1.6132	1.6132	3.7754
16	2.1203	2.1203	4.2825	46	2.9988	2.9988	5.161
17	2.7393	2.7393	4.9015	47	1.5757	1.5757	3.7378
18	1.9775	1.9775	4.1397	48	1.2298	1.2298	3.392
19	1.6873	1.6873	3.8495	49	0.2153	0.2153	2.3774
20	0.8212	0.8212	2.9834	50	1.9642	1.9642	4.1264
21	1.9058	1.9058	4.068	51	0.2782	0.2782	2.4404
22	1.5291	1.5291	3.6913	52	1.8473	1.8473	4.0095
23	2.3001	2.3001	4.4623	53	1.3926	1.3926	3.5548
24	1.9664	1.9664	4.1286	54	0.2616	0.2616	2.4238
25	0.6804	0.6804	2.8426	55	0.0826	0.0826	2.2448
26	1.3456	1.3456	3.5077	56	0.2345	0.2345	2.3967
27	1.18	1.18	3.3422	57	0.7125	0.7125	2.8747
28	2.0314	2.0314	4.1935	58	0.4238	0.4238	2.586
29	1.6749	37.1423	3.837	59	2.4534	2.4534	4.6156
30	0.389	0.389	2.5512	60	1.3556	1.3556	3.5177

Table 5.5: Comparison of bandwidth assignment based on P5 and P12



# Chapter 6

## Conclusion

This thesis considers the problem of BS and RS placement in broadband wireless networks. Mixed integer programming formulations are provided. For the first step, an improved MILP formulation is provided for the RS placement which finds the optimal solution in a matter of seconds as compared to the model given in [7]. A weighted objective formulation is also provided which shows the trade-off between the number of RSs required and the achieved cell capacity. It is observed that the capacity of the network decreases by 0.06% with a 73% decrease in number of RSs required when a weight of 0.01 is assigned to the candidate positions.

The formulation is extended to find the joint BS-RS placement for a multi cell-site scenario. The formulation provides an average 10% higher network capacity as compared to the earlier formulation for each cell-site. This increase is achieved at the cost of increase in the number of RSs required. At this point, it was noticed that our formulations were

just fulfilling the user demand at each subscriber station and assigning excess bandwidth to the user with maximum relay rate. This motivated us to present our last formulation, maxi-min formulation. With this, we were able to distribute the excess BW to all the subscriber stations and achieve at an average 35.18% increase in the bandwidth allocated to each subscriber station.

Numerical testing on different formulations and instances reveal the efficiency of our proposed formulations. In terms of resource utilization, weighted objective BS-RS formulation was most effective due to fewer number of RSs required to fulfill the user demand.

For future research, using mobile subscriber stations which can move between cell-sites and their effect on the network configuration is worth exploring. Also, mobile RSs and RSs that are active for a fixed period of time are worth exploring.

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