# On Cyclic Delay Diversity OFDM Based Channels

by

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# Declaration

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners. I understand that my thesis may be made electronically available to the public.

# Abstract

Orthogonal Frequency Division Multiplexing, so called OFDM, has found a prominent place in various wireless systems and networks as a method of encoding data over multiple carrier frequencies. OFDM-based communication systems, however, lacking inherent diversity, are capable of benefiting from different spatial diversity schemes. One such scheme, Cyclic Delay Diversity (CDD) is a method to provide spatial diversity which can be also interpreted as a Space-Time Block Coding (STBC) step. The main idea is to add more transmit antennas at the transmitter side sending the same streams of data, though with differing time delays. In [1], the capacity of a point-to-point OFDM-based channel with CDD is derived for inputs with Gaussian and discrete constellations. In this dissertation, we use the same approach for an OFDM-based single-input single-output (SISO) two-user interference channel (IC). In our model, at the receiver side, the interference is treated as noise. Moreover, since the channel is time-varying (slow-fading), the Shannon capacity in the strict sense is not well-defined, so the expected value of the instantaneous capacity is calculated instead. Furthermore, the channel coefficients are unknown to the transmitters. Thus, in this setting, the probability of outage emerges as a reasonable performance measure. Adding an extra antenna in the transmitters, the SISO IC turns into an MISO IC, which results in increasing the diversity. Both the continuous and discrete inputs are studied and it turns out that decoding interference is helpful in some cases. The results of the simulations for discrete inputs indicate that there

are improvements in terms of outage capacity compared to the ICs with single-antenna transmitters.

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# Chapter 1

# Introduction and Background

An interference channel is a channel with N transmitters and N receivers. Each transmitter wants to send data to its corresponding receiver. So in each receiver there is one desire signal and N-1 interference signals and each communication gets interfered by the other communications [2]. The interference channel is an important channel in communication since many communication systems such as, cellular and ad-hoc networks can be modeled by that channel. An interference channel is one with N transmitters and N receivers. As each transmitter wants to send data to its corresponding receiver, there is one desire signal and N-1 interference signals in each receiver; hence, each communication is interfered by other communications [2]. The interference channel is an important channel in communication, since many communication systems, such as cellular and ad-hoc networks, can be modeled after that channel.

## 1.1 Introduction

In this section, we first define capacity in general and then discuss interference channel (IC) capacity in particular.

### 1.1.1 Capacity Definition

In a channel, capacity is the highest rate that information can be transmitted with arbitrarily low probability of error in terms of bits-per-channel use. Consider that in a discrete point-to-point channel $x$ is the space of the input signal, and $y$ is the space of the output signal. A channel is memoryless if the output depends on the input at that time. As well, it is independent from the previous channel inputs or outputs.

Capacity of a discrete memoryless channel is defined as

$$C = sup_{p(x)} I(X;Y) \tag{1.1}$$

where $I$ is the mutual information between input sequence $X$ and output sequence $Y$, and $p$ is the probability distribution of the input [3].

Now consider a discrete memoryless M-user interference channel. Finite sets of $x_1$, $x_2$, ..., $x_M$ are input alphabets and finite sets of $y_1$, $y_2$, ..., $y_M$ are output alphabets. $q(x)$ is the set of all joint probability distributions $p_i(x)$ $i = 1, ..., M$. The capacity is as follows:

$$C_i = sup_{q(x)} I(X_i; Y_i | X_1, ..., X_{i-1}, X_{i+1}, X_M) \tag{1.2}$$

### 1.1.2 Interference Channel Capacity

The full capacity region of an interference channel is still an open problem. The largest achievable rate for interference channel capacity is known as the Han-Kobayashi bound [4]. In this method, part of the noise is treated as noise, and part of it is decoded. Etkin et al. have shown that the Han-Kobayashi inner bound can achieve the Gaussian interference channel capacity within one bit, first as a symmetric case [5] and then for a general case [6]. There are also some results in [7, 8, 9] regarding the outer bound of capacity. Assume an interference channel with two transmitters and two receivers, as shown in Figure 1.

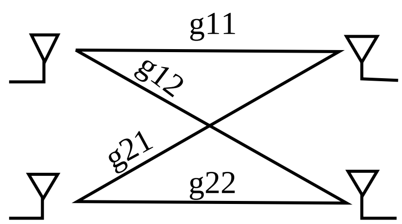


Figure 1.1: Two-user interference channel

Let $x_1$ be the input signal from transmitter 1 and $x_2$ from transmitter 2. Hence, the output signals are:

$$y_1 = g_{11}x_1 + g_{21}x_2 + n_1. \tag{1.3}$$

and

$$y_2 = g_{22}x_2 + g_{12}x_2 + n_2. \tag{1.4}$$

where $n_1$ and $n_2$ are additive Gaussian noise.

$$n_i = G(0, N_i), \quad for \;\; i = 1, 2. \tag{1.5}$$

Classifying interference channels with regards to the strength of the interference makes it easier to ascertain bounds on capacity. Such cases for discrete memoryless, two-user interference channel are defined as follows:

**Very strong interference-**

If

$$I(x1; y2) \geq I(x1; y1|x2) \tag{1.6}$$

$$I(x2; y1) \geq I(x2; y2|x1) \tag{1.7}$$

With regards to Gaussian input, and after applying some algebraic manipulation according to Figure 1, Equations (1.6) and (1.7) simplify as follows:

$$1+\left(\frac{g_{21}^2 P_1}{g_{22}^2 P_2+N_2}\right) \geq 1+\left(\frac{g_{11}^2 P_1}{N_1}\right) \tag{1.8}$$

$$1+\left(\frac{g_{12}^2 P_2}{g_{11}^2 P_1+N_1}\right) \geq 1+\left(\frac{g_{22}^2 P_2}{N_2}\right) \tag{1.9}$$

Hence,

$$(g_{21}^2 P_1 N_1) \geq (g_{11}^2 P_1)(g_{22}^2 P_2+N_2) \tag{1.10}$$

$$(g_{12}^2 P_2 N_2) \geq (g_{22}^2 P_2)(g_{11}^2 P_1+N_1) \tag{1.11}$$

Let us now normalize the channel coefficients and additive noises in such a way that the power of both Gaussian noise and forward gains become 1. In order to do that, we must first divide Equation (1.3) by $N_1$ and Equation (1.4) by $N_2$. We can then define the new channel coefficients as follows:

$$\alpha_{11}=1 \qquad \alpha_{22}=1 \tag{1.12}$$

$$\alpha_{12}=\left(\frac{g_{12}^2 N_2}{g_{22}^2 N_1}\right) \tag{1.13}$$

$$\alpha_{21}=\left(\frac{g_{21}^2 N_1}{g_{11}^2 N_2}\right) \tag{1.14}$$

Additionally, we must define new power constraints.

$$q_1=\frac{g_{11}^2}{N_1} P_1 \tag{1.15}$$

$$q_2=\frac{g_{22}^2}{N_2} P_2 \tag{1.16}$$

After inserting the normalized coefficients into Equations (1.10) and (1.11), the result is:

$$\alpha_{12} \geq 1+q_1 \tag{1.17}$$

and

$$\alpha_{21} \geq 1 + q_2 \tag{1.18}$$

So, if Equations (17) and (18) are satisfied, we are in a very strong interference region. Capacity in this region was obtained by Carleial and shows that interference under certain conditions does not reduce capacity in [10, 11]. In despite of what one might expect (i.e., that the capacity region decreases monotonically when interference increases), when the interference is too strong, the capacity region is the same as the capacity region with no interference. In this region without any loss in rates, interference can be canceled [12].

**Strong interference-**

If

$$I(x1; y2|x2) \geq I(x1; y1|x2) \tag{1.19}$$

$$I(x2; y1|x1) \geq I(x2; y2|x1) \tag{1.20}$$

These inequalities compare the information in the desired signal with the information in the undesired signal. After simplifying Equations (1.19) and (1.20), we have:

$$\frac{g_{21}^2 P_1}{N_2} \geq \frac{g_{11}^2 P_1}{N_1} \tag{1.21}$$

$$\frac{g_{12}^2 P_2}{N_1} \geq \frac{g_{22}^2 P_2}{N_2} \tag{1.22}$$

So if

$$\alpha_{12} \geq 1 \tag{1.23}$$

$$\alpha_{21} \geq 1 \tag{1.24}$$

and thus find ourselves in the strong interference region. Regarding Gaussian input, capacity was established by Sato in [13]. Shortly thereafter, Costa and Gamal in [14] obtained results on capacities of deterministic interference channels. They also established the capacity region of discrete memoryless interference channels in this region.

**Weak interference-**

If

$$I(x1; y2|x2) \leq I(x1; y1|x2) \tag{1.25}$$

$$I(x2; y1|x1) \leq I(x2; y2|x1) \tag{1.26}$$

**Very weak interference-**

If

$$I(x1; y2) \leq I(x1; y1|x2) \tag{1.27}$$

$$I(x2; y1) \leq I(x2; y2|x1) \tag{1.28}$$

Capacity for weak and very weak interference is unknown.

Another channel model that has been extensively studied is the cognitive interference channel (CIC). In CICs, one of transmitters knows the message of another transmitter non-causally [15]. CICs capacity in a very strong interference region has been proposed by Rini et al. in [16], who also obtained new results [17] on the inner and outer bounds of CIC.

## 1.2 Different strategies for dealing with interference

There are different ways to deal with interference. In each situation one of these methods or a combination of them is the optimum choice. There are various ways to deal with interference. In a given situation, any one of these methods or a combination of them could prove the optimum choice.

### 1.2.1 Treat interference as noise

One way to deal with interference is to treat it as noise. According to Figure 1, we would thus have:

$$R_1 \leq 0.5 \log(1 + \frac{g_{11}^2 P_1}{g_{21}^2 P_2 + N_1}) \tag{1.29}$$

$$R_2 \leq 0.5 \log(1 + \frac{g_{22}^2 P_2}{g_{12}^2 P_1 + N_2}) \tag{1.30}$$

### 1.2.2 Avoiding Interference

In this strategy, the available time interval or bandwidth is divided between the users. Hence, in the K-user channel, the rate per user becomes $\frac{1}{K} \log(1 + SNR) + o(\log(1 + SNR))$.

#### Time Division Multiplexing (TDM)

In time division multiplexing, each user sends a signal at a certain time interval, with no overlap between intervals (i.e., no users send data simultaneously). Public telephone networks and 2G mobile systems use this method. So, if $T$ is the whole time interval available and we assign $\gamma T$ to the first user and $(1-\gamma)T$ to the second user, we have:

$$R_1 \leq 0.5\gamma \log(1 + \frac{g_{11}^2 \frac{P_1}{\gamma}}{g_{21}^2 \frac{P_2}{1-\gamma} + N_1}) \tag{1.31}$$

$$R_2 \leq 0.5(1-\gamma) \log(1 + \frac{g_{22}^2 \frac{P_2}{1-\gamma}}{g_{12}^2 \frac{P_1}{\gamma} + N_1}) \tag{1.32}$$

#### Frequency Division Multiplexing (FDM)

Frequency division multiplexing is a process that divides the total available bandwidth into several non-overlapping frequency sub-bands. Each of these sub-bands is assigned to one of the communication channels.

### 1.2.3 Decoding Interference

In this method, both interference and message are decoded [18]. When interference is stronger, there is an alternative to decode it instead of the desired signal. After subtracting the decoded interference from the received signal, the result is the sum of the desired signal and AWGN. In this way, the desired signal can be decoded. However, as the implementation of this method is complicated, it is not commonly used [19]. As shown in [13], the decoding interference in a strong interference region is optimal.

Channels that have more than one antenna in their transmitters and receivers are called multiple-input multiple-output (MIMO) channels. Many studies have been carried out on capacity in MIMO interference channels. In [20], capacity of MIMO IC in a strong interference regime was established. It was shown that, in a very weak interference region, treating interference as noise is optimal. This result has been generalized for MIMO IC in [21].

### 1.2.4 Interference Alignment

This method aligns the interferences in space and reduces the dimensions of the interferences so that more spaces become available for the intended signals [22]. The alignment can be either in time or in frequency. By using interference alignment in a k user interference channel, it is possible to allocate roughly half of the space to the interference signals and the remaining half to the desired signals, with the sum capacity characterized as $C(SNR) = \frac{K}{2}\log(1 + SNR) + o(\log(1 + SNR))$ [19].

## 1.3 Orthogonal Frequency Division Multiplexing (OFDM)

Instead of using one wideband carrier, orthogonal frequency division multiplexing uses orthogonal narrowband multi-carriers (i.e., OFDM bandwidth divided into several narrower

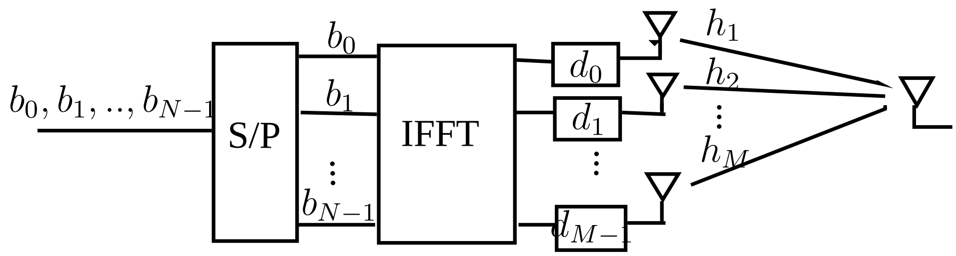


Figure 1.2: OFDM Block Diagram

bands). By choosing a sufficient number of carriers, OFDM converts a frequency-selective channel to flat sub-channels. OFDM can be seen as either a modulation or multiplexing technique. In communications, modulation is the variation of one or more parameters of a periodic waveform, which is called the carrier signal. These parameters are amplitude, phase, and frequency, and the information is mapped to the change of these properties. Multiplexing means sending multiple data streams over one signal through a shared channel. OFDM is a useful method, especially in wideband communications. The difference between FDM and OFDM is that, in OFDM, this division is done in such a way that the obtained sub-channels are orthogonal and thus do not interfere with each other. Since orthogonality is an important factor, a small degradation in frequency or phase causes inter-channel interference (ICI). One of the main reasons to use OFDM is to increase the robustness against frequency-selective fading or multipath environments. Also, in a mono-carrier system, a narrowband interferer can cause the entire link to fail, but in a multi-carrier system, just one or few of the sub-carriers are affected. Error correction coding can be used to correct for the few erroneous sub-carriers. In addition, it makes equalization easier. An OFDM block diagram is presented in Figure 2.

Suppose that we have a flat channel with a bandwidth of $W$. We divide the available bandwidth into N orthogonal sub-bands. As defined in Eq. (1.33), we can choose sinc pulses as the sub-carriers. In this case, the sub-carriers would be non-causal, and small errors in sampling time would result in inter-symbol interference (ISI). Accordingly, we can use a raised-cosine signal, which is a smoother pulse. However, it needs excess bandwidth for the same rate.

$$Sinc(\frac{t}{T}) = \frac{\frac{\sin(\pi t)}{T}}{\frac{\pi t}{T}} \tag{1.33}$$

$$Raised - cosine(t) = sinc(\frac{\pi}{T})\frac{\cos(\frac{\pi \beta t}{T})}{1 - \frac{4\pi^2 t^2}{T^2}} \tag{1.34}$$

where $\beta$ is the roll-off factor and the excess bandwidth is dependent on this parameter. The bandwidth of raised-cosine is $\frac{1}{2T}(\beta + 1)$.

For generating OFDM signals, we use an inverse fast Fourier transform (IFFT) block, as shown in Figure 2. FFT is an algorithm to compute discrete Fourier transform (DFT) in an efficient way. DFT is defined as

$$X_k = \sum_{n=0}^{N-1} x_n e^{(\frac{-2\pi ikn}{N})} \quad k = 0, ..., N-1 \tag{1.35}$$

and IDFT equation given by,

$$x_n = \sum_{k=0}^{N-1} X_k e^{(\frac{-2\pi ikn}{N})} \quad n = 0, ..., N-1 \tag{1.36}$$

## 1.4 Alamouti Space Time coding

The Alamouti technique uses two transmitting antennas and one receiving antenna [23]. In this method, by sending the same data from both antennas, diversity order two can be achieved for flat-fading channels and AWGN. If we have two channels, assume $h_1$ and $h_2$ are the channel coefficients with additive white Gaussian noises, $n_1$ and $n_2$. In two time-slots, signals $x_1, x_2^*$ and $x_2, -x_1^*$ from the first and second antenna are sent respectively.

X= $\begin{pmatrix} x_1 & x_2^* \\ x_2 & -x_1^* \end{pmatrix}$ , H= $\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$ and, N= $\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$.

So the received signal Y= $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ is $Y = XH + N$. We define an equivalent channel and rewrite the received signal as below.

$$Y' = \begin{pmatrix} y_1 \\ y_2^* \end{pmatrix} \begin{pmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix}.$$

The equivalent channel matrix is orthogonal. In [24] Rupp et al., the Alamouti scheme is extended in the case of $N = 2^K$ transmitting antennas and one receiving antenna with QPSK modulation. [25] develops space-time block coding (STBC) and presents it as a real constellation. This coding method achieves full diversity, while for complex constellations it achieves half of the maximum possible rate.

## 1.5 Diversity Scheme and Cyclic Delays

A diversity scheme is a technique that can improve the reliability of a message signal by using more than one channel. In this technique, each channel has different characteristics as multiple versions of the intended signal are sent, a redundancy that can actually help. For example, by using diversity methods in multipath propagation, we turn a slow-fading channel to a fast-fading one, or a flat channel to frequency-selective one.

Various types of diversity techniques are as follows:

### 1.5.1 Time Diversity

In this technique, different versions of the intended signal are transmitted in different time intervals, with the goal to increase time-selectivity.

### 1.5.2 Frequency Diversity

This method sends information on several frequency sub-channels. The available bandwidth is divided into intervals and the effect of fading and attenuation is not the same on different frequencies. Hence, the receivers choose the strongest signal.

### 1.5.3 spatial Diversity

One strategy to achieve spatial diversity is sending different versions of the desired signal from multiple transmitting antennas or along several different paths. This method is also called space-time coding (STC). Another strategy is having one transmit antenna and several receive antennas so that the receiver adds the data from these multiple antenna linearly and thereby results in diversity gain. This is also called reciprocity diversity [26] and relies on different channels having different fading characteristics. In [27, 28], Winters proposes transmitting diversity in Rayleighs fading channels. This method uses several transmit antennas, with messages sent from antennas at various times. In [29, 30], Wittneben implements a similar idea and introduces simulation diversity. The information in a flat fading environment is sent by different antennas with different modulations. This goal is achieved by utilizing finite impulse response filters. There are several techniques to implement spatial diversity in OFDM systems [31].

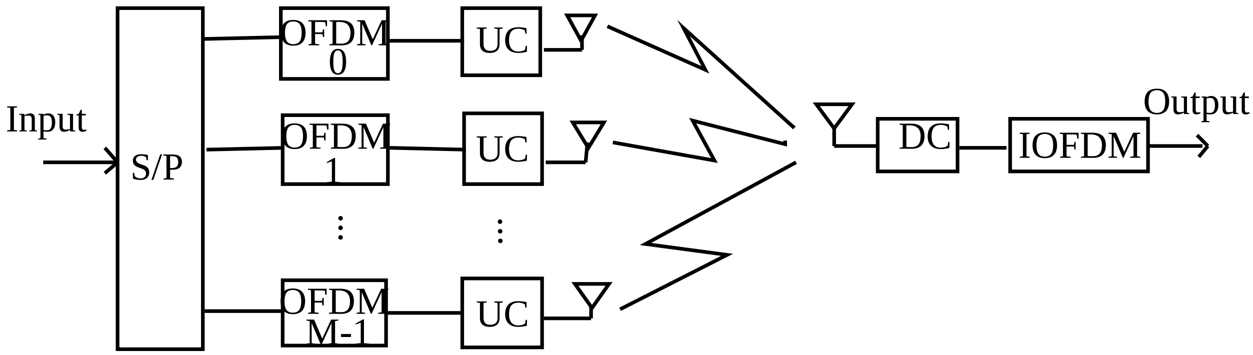


Figure 1.3: Subcarrier Diversity Block Diagram

**Subcarrier Diversity**

In each transmit antenna, there is an individual OFDM block. The subcarriers are divided into $M$ groups, and each group is used by one of the antennas, after which OFDM is applied. Choosing subcarriers that are spread over the entire bandwidth is a better option.

**Phase Diversity**

In each antenna, the signal is transmitted by different phase shifts, as shown in Figure. The equation below shows the applied phase shifts in $m^{th}$ antenna and $n^{th}$ subcarrier. N indicates the number of subcarriers.

$$\phi_{m,n} = 2\pi\frac{kmn}{N}, \quad k \geq 1,\ n = 1, ..., N,\ m = 1, ..., M-1 \tag{1.37}$$

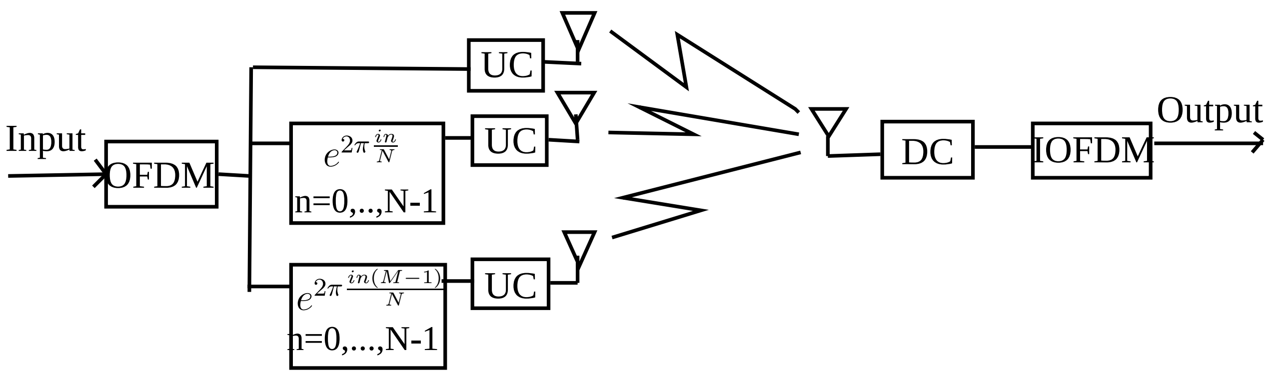


Figure 1.4: Subcarrier Diversity Block Diagram

**Time-Varient Phase Diversity**

This method works exactly like the previous one, except that the phase shifts change by time, as follows:

$$\phi_{m,n} = 2\pi\frac{kmn}{N} + 2\pi f_m t, \quad k \geq 1,\ n = 1, ..., N,\ m = 1, ..., M-1 \tag{1.38}$$

$f_m$ is the frequency shift of $m^{th}$ subcarrier.

**Delay Diversity**

Various delays are applied to the OFDM signals, as shown in Figure 4. The delayed versions of the message signal are transmitted from the antennas and have to satisfy the condition that the $BW$ is the bandwidth of the transmitted signal.

$$\delta_m = \frac{km}{BW} \quad k \geq 1,\ m = 1, ..., M-1 \tag{1.39}$$

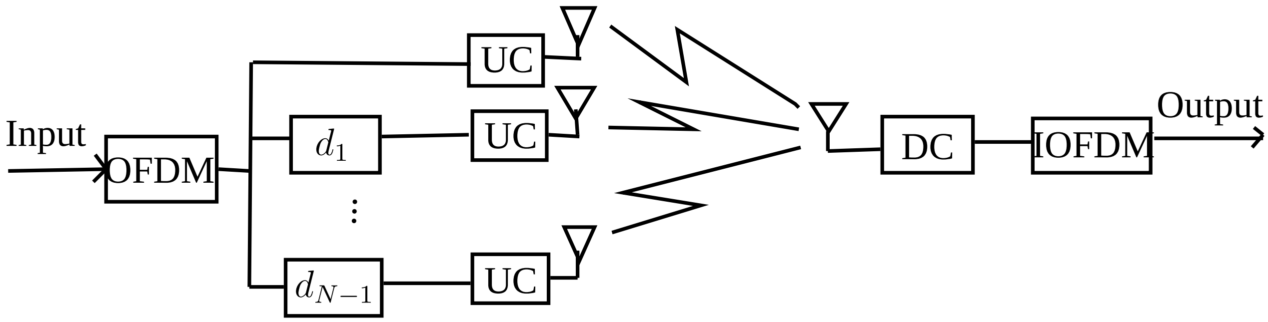


Figure 1.5: Phase Diversity Block Diagram

**Cylic Delay Diversity**

Cyclic Delay Diversity (CDD) is a special case of delay diversity. The delays in CDD are, as the name implies, cyclic. It can be shown that cyclic CDD is equal to PD. Consider a point-to-point channel with two antennas at the transmitter with an OFDM-based transmission scheme. Here, we also use cyclic delay diversity. In [32], simulation results show that applying this diversity method almost achieves the same diversity as the well-known Alamouti scheme. However, in the diversity method, and in contrast to the Alamouti scheme, there is no need to add complexity to the receiver. Also, in some cases, the optimal shifts in CDD have been calculated. The optimum cyclic delay, with phase shift key (PSK) modulation and the cardinality of the modulation alphabet $A$, is:

$$\Delta_{opt}^{N} = |\delta_1 - \delta 2| = \frac{N}{A} \tag{1.40}$$

where N is the number of tones of OFDM.

As mentioned previously, there are two options for applying diversity: M transmitting antennas and one receiving antenna (which is called transmitter-sided diversity) or one transmitting antenna and M receiver antenna (which is called receiver-sided diversity).

Maximum ratio combining (MRC) is a technique with one antenna in the transmitter and several antennas in the receiver, where all of the received signals are added together with their corresponding weighted factor. The weighted factor for each channel is proportional to the received signal-to-noise ratio in that antenna. In [23], Alamouti compares its scheme with MRC. The Alamouti scheme reaches the same order of diversity and has the same complexity level without requiring feedback from the transmitter to receiver and excess bandwidth. In [33], transmitter-sided CDD and receiver-sided MRC are combined and improve in terms of complexity and performance.

## 1.6 Outage Probability

As we know, capacity depends on SNR. Since, in slow-fading channels, SNR is not constant and varies over time, the channel rate also varies. In this situation, the capacity is compared to a threshold rate. Thus, when its value is below the threshold, an outage occurs [34]. Calculating the probability of outage gives us a parameter that shows the reliability of the link. The threshold rate should be chosen in such a way that the outage probability is less than the certain outage probability. Outage capacity is the largest rate achieved in a channel with a certain outage probability. The coding method that achieves it is called universal coding. However, fast-fading channel capacity can be calculated with an arbitrary small degree of error, and there is no need to use outage. When there is no knowledge about channel coefficients in the transmitters capacity of outage is a reasonable criterion for comparison [35].

## 1.7 Monte Carlo Algorithm

In the simulation part, we assume that the channels are time-varying and that channel coefficients for each symbol are independent of each other. In this situation, capacity is a random variable. The Monte Carlo algorithm provides solutions to the mathematical problems which need statistical simulations. This algorithm performs the simulations by using a sequence of randomized numbers and calculating an approximation of the answer. We choose one of the Monte Carlo methods on the basis of the problem [36].

# Chapter 2

# Main Contribution

As mentioned in Chapter 1, diversity increases the performance of wireless systems. In this chapter, spatial diversity, particularly the effect of adding one antenna to a transmitter, is studied. The same data stream with different time delays is sent from both antennas in each transmitter. We also consider a situation where the knowledge of channel coefficients is unknown and we only know their probability distribution, which is complex Gaussian. Hence, instead of capacity, we have to calculate outage probability. The Monte-Carlo method is applied in the simulations, and delays can be continuous or discrete, depending on the situation. Regarding continuous delays, infinite possible magnitude simulations have been done only for discrete cases. Nevertheless, in the following formulations, both cases are represented. Let us first investigate this situation in a PTP channel with continuous input.

## 2.1 On Delay Diversity of Point-to-point Channels in Continuous Time

There are two channels one between the first antenna in the transmitter and receiver, and the other between the added antenna and the receiver. Let us assume both channels are band-limited channels with bandwidth $W$, and that $c_1(\cdot)$, $c_2(\cdot)$ are channel impulse responses, respectively. In this part, we consider flat channels with constant gains $\alpha_1$ and $\alpha_2$. Using linear modulation signaling, the signal $\sum_{i=-\infty}^{\infty} a_i v(t-iT)$ is transmitted from the first antenna and a delayed version of this signal,$\sum_{i=-\infty}^{\infty} a_i v(t-\tau_1-iT)$ , is transmitted from the second antenna. Here, $v(\cdot)$ is a T-orthogonal signal, i.e., T is the smallest number $T_0$ such that $\int_{-\infty}^{\infty} v(t)v(t-T_0)=0$. Also, $(a_i)_{i\in\mathbb{Z}}$ is a stationary and ergodic sequence with power spectrum $S_a(e^{\mathrm{j}2\pi f})$. The processes $\sum_{i=-\infty}^{\infty} a_i v(t-iT)$ and $\sum_{i=-\infty}^{\infty} a_i v(t-\tau-iT)$ are cyclo-stationary with common power spectrum $\frac{1}{T}S_a(e^{\mathrm{j}2\pi fT})|V(f)|^2$. We impose the condition that

$$\frac{2}{T}\int_{-\infty}^{\infty} S_a(e^{\mathrm{j}2\pi fT})|V(f)|^2 df \leq P. \tag{2.1}$$

The receiver observes

$$y(t) = \sum_{i=-\infty}^{\infty} a_i(h(t-iT)+h(t-\tau-iT))+z(t) \tag{2.2}$$

where $h = c * v$ and $z(\cdot)$ in the AWGN process with the correlation function $\frac{N_0}{2}\delta(\cdot)$.

A set of sufficient statistics in the receiver is given by

$$\begin{aligned} y_j &= \int_{-\infty}^{\infty} y(t)(h(t-jT)+h(t-\tau-jT))dt \\ &= \sum_{i=-\infty}^{\infty} a_i g_{j-i} + z_j \\ &= (a*g)_j + z_j, \quad j\in\mathbb{Z}, \end{aligned} \tag{2.3}$$

where

$$g_n = \int_{-\infty}^{\infty} \widetilde{h}(t)\widetilde{h}(t - nT')dt, \quad n \in \mathbb{Z}, \tag{2.4}$$

$$z_n = \int_{-\infty}^{\infty} z(t)\widetilde{h}(t - nT')dt, \quad n \in \mathbb{Z} \tag{2.5}$$

and $\widetilde{h}(\cdot)$ is defined as

$$\widetilde{h}(t) = h(t) + h(t - \tau). \tag{2.6}$$

An application of the Maximum Entropy Lemma shows that, for any $N \in \mathbb{N}$, $I((a_i)_{i=-N}^{N}; (y_j)_{j=-N}^{N})$ is maximized if $(a_i)_{i\in\mathbb{Z}}$ is a Gaussian sequence. In this case[1],

$$\lim_{N\to\infty} \frac{I((a_i)_{i=-N}^{N}; (y_j)_{j=-N}^{N})}{NT} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{S_{a*g}(e^{\mathrm{j}\omega})}{S_z(e^{\mathrm{j}\omega})}\right) d\omega. \tag{2.7}$$

It is thus easy to see that

$$S_{a*g}(e^{\mathrm{j}\omega}) = S_a(e^{\mathrm{j}\omega})|G(e^{\mathrm{j}\omega})|^2 \tag{2.8}$$

and

$$S_z(e^{\mathrm{j}\omega}) = \frac{N_0}{2}G(e^{\mathrm{j}\omega}). \tag{2.9}$$

Noting that $G(\cdot)$ is a real function,

$$\lim_{N\to\infty} \frac{I((a_i)_{i=-N}^{N}; (y_j)_{j=-N}^{N})}{NT} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{2S_a(e^{\mathrm{j}\omega})G(e^{\mathrm{j}\omega})}{N_0}\right) d\omega. \tag{2.10}$$

let us define

$$u(t) = \widetilde{h}(t) * \widetilde{h}(-t). \tag{2.11}$$

Then, $g_n = u(nT)$, and hence,

$$G(e^{\mathrm{j}\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} U\left(\frac{1}{T}\left(\frac{\omega}{2\pi} + k\right)\right) \tag{2.12}$$

where $U(f) = |\widetilde{H}(f)|^2 = \left|1 + e^{-j2\pi\tau f}\right|^2 |H(f)|^2 = \left|1 + e^{-j2\pi\tau f}\right|^2 |V(f)|^2|C(f)|^2$ is the Fourier transform of $u(\cdot)$.

[1]The entropy rate of a stationary Gaussian process with spectrum $S(e^{\mathrm{j}\omega})$ is $\frac{1}{2\pi}\int_0^{\pi} \log S(e^{\mathrm{j}\omega})d\omega$.

Let us consider a situation where $C(\cdot)$ is some constant gain $\alpha$ in $[-W, W]$, $V(\cdot)$ is flat and equal to unity in $[-W, W]$ and $(a_i)_{i\in\mathbb{Z}}$ is an i.i.d. sequence, i.e., $S_a(e^{j\omega}) = \sigma^2$. Then, letting $\int_{-\infty}^{\infty} |V(f)|^2 df = 1$, we get

$$\sigma^2 \leq PT. \tag{2.13}$$

Hence,

$$\text{Rate in bits/sec/hz} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{2\sigma^2|G(e^{j\omega})|}{N_0}\right) d\omega. \tag{2.14}$$

where

$$G(e^{j\omega}) = \frac{\alpha^2}{T}\sum_{k=-\infty}^{\infty} \left|1 + e^{-\frac{j2\pi\tau}{T}\left(\frac{\omega}{2\pi}+k\right)}\right|^2 \tag{2.15}$$

## 2.2 On Delay Diversity of Interference Channels in Continuous Time

Let us now implement the same strategy for an interference channel. $c_1(\cdot)$, $c_2(\cdot)$, $e_1(\cdot)$, and $e_2(\cdot)$ are channel impulse responses from the first and second antennas in transmitter one to receivers one and two, respectively. $d_1(\cdot)$, $d_2(\cdot)$, $f_1(\cdot)$, and $f_2(\cdot)$ are channel impulse responses from the first and second antennas in transmitter two to receivers two and one, respectively. As above, by using linear modulation signaling, the signal $\sum_{i=-\infty}^{\infty} a_i v(t - iT)$ in the first transmitter is transmitted from the first antenna and a delayed version of this signal, $\sum_{i=-\infty}^{\infty} a_i v(t - \tau_1 - iT)$, is transmitted from the second antenna. Similarly, in the second transmitter, the signal $\sum_{i=-\infty}^{\infty} a_i u(t-iT)$ is transmitted from the first antenna and a delayed version of this signal, $\sum_{i=-\infty}^{\infty} a_i u(t-\tau_2-iT)$, is transmitted from the second antenna. $v(\cdot)$ and $u(\cdot)$ are T-orthogonal signals. Also, $(a_i)_{i\in\mathbb{Z}}$ and $(b_i)_{i\in\mathbb{Z}}$ are stationary and ergodic sequences with a power spectrum $S_a(e^{j2\pi f})$ and $S_b(e^{j2\pi f})$. The processes $\sum_{i=-\infty}^{\infty} a_i v(t-iT)$ and $\sum_{i=-\infty}^{\infty} a_i v(t-\tau_1-iT)$ are cyclo-stationary with common power spectrum $\frac{1}{T}S_a(e^{j2\pi fT})|V(f)|^2$.

So $\sum_{i=-\infty}^{\infty} b_i u(t-iT)$, $\sum_{i=-\infty}^{\infty} b_i u(t-\tau_2-iT)$ are cyclo-stationary with common power spectrum $\frac{1}{T}S_b(e^{\mathrm{j}2\pi fT})|U(f)|^2$. We impose two conditions that

$$\frac{2}{T}\int_{-\infty}^{\infty} S_a(e^{\mathrm{j}2\pi fT})|V(f)|^2 df \leq P_1. \tag{2.16}$$

and

$$\frac{2}{T}\int_{-\infty}^{\infty} S_b(e^{\mathrm{j}2\pi fT})|U(f)|^2 df \leq P_2. \tag{2.17}$$

At the first and second receiver we observe $y_1(t)$ and $y_2(t)$ respectively,

$$y_1(t) = \sum_{i=-\infty}^{\infty} a_i(h_1(t-iT)+h_2(t-\tau_1-iT)) + b_i(g_1(t-iT)+g_2(t-\tau_2-iT)) + z_1(t) \tag{2.18}$$

$$y_2(t) = \sum_{i=-\infty}^{\infty} b_i(h_3(t-iT)+h_4(t-\tau_2-iT)) + a_i(g_3(t-iT)+g_4(t-\tau_1-iT)) + z_2(t). \tag{2.19}$$

where $h_i = c_i * v,\ \ i=1,2$, $h_i = d_{i-2} * u,\ \ i=3,4$, $g_i = f_i * u,\ \ i=1,2$, $g_i = e_{l-2} * v,\ \ i=3,4$ and $z_i(\cdot),\ i=1,2$ are both additive white Gaussian noises (AWGN) with the power of $\frac{N_0}{2}\delta(\cdot)$.

A set of sufficient statistics at the receiver 1 and 2 are given by,

$$\begin{aligned} y_1(j) &= \int_{-\infty}^{\infty} y_1(t)(h_1(t-jT)+h_2(t-\tau_1-jT))dt \\ &= \sum_{i=-\infty}^{\infty} a_i h^{(1)}_{j-i} + b_i g^{(1)}_{j-i} + z_1(j) \\ &= (a * h^{(1)})_j + (b * g^{(1)})_j + z_1(j), \quad j \in \mathbb{Z}, \end{aligned} \tag{2.20}$$

and

$$\begin{aligned} y_2(j) &= \int_{-\infty}^{\infty} y_2(t)(h_3(t-jT)+h_4(t-\tau_2-jT))dt \\ &= \sum_{i=-\infty}^{\infty} b_i h^{(2)}_{j-i} + a_i g^{(2)}_{j-i} + z_2(j) \\ &= (b * h^{(2)})_j + (a * g^{(2)})_j + z_2(j) \quad j \in \mathbb{Z}, \end{aligned} \tag{2.21}$$

where,

$$h^{(i)}_n = \int_{-\infty}^{\infty} \widetilde{h^{(i)}}(t)\widetilde{h^{(i)}}(t-nT)dt, \quad n \in \mathbb{Z}, \tag{2.22}$$

and

$$g_n^{(i)} = \int_{-\infty}^{\infty} \widetilde{g^{(i)}}(t)\widetilde{h^{(i)}}(t-nT)dt, \quad n \in \mathbb{Z}, \tag{2.23}$$

$$z_i(n) = \int_{-\infty}^{\infty} z_i(t)\widetilde{h^{(i)}}(t-nT)dt, \quad n \in \mathbb{Z} \tag{2.24}$$

for $i = 1, 2$. and,

$$\widetilde{h^{(1)}}(t) = h_1(t) + h_2(t-\tau_1). \tag{2.25}$$

$$\widetilde{h^{(2)}}(t) = h_3(t) + h_4(t-\tau_2). \tag{2.26}$$

$$\widetilde{g^{(1)}}(t) = g_1(t) + g_2(t-\tau_2). \tag{2.27}$$

$$\widetilde{g^{(2)}}(t) = g_3(t) + g_4(t-\tau_1). \tag{2.28}$$

An application of the Maximum Entropy Lemma shows that, for any $N \in \mathbb{N}$, $I((a_i)_{i=-N}^{N}; (y_j)_{j=-N}^{N})$ is maximized if $(a_i)_{i\in\mathbb{Z}}$ is a Gaussian sequence. In a similar way, $(b_i)_{i\in\mathbb{Z}}$ must be a Gaussian sequence.

In this case[2],

$$\lim_{N\to\infty} \frac{I((a_i)_{i=-N}^{N}; (y_1(j))_{j=-N}^{N})}{NT} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{S_{a*h^{(1)}}(e^{j\omega})}{S_{z_1}(e^{j\omega}) + S_{b*g^{(1)}}(e^{j\omega})}\right)d\omega. \tag{2.29}$$

and

$$\lim_{N\to\infty} \frac{I((b_i)_{i=-N}^{N}; (y_2(j))_{j=-N}^{N})}{NT} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{S_{b*h^{(2)}}(e^{j\omega})}{S_{z_2}(e^{j\omega}) + S_{a*g^{(2)}}(e^{j\omega})}\right)d\omega. \tag{2.30}$$

It is easy to see that

$$S_{a*h^{(1)}}(e^{j\omega}) = S_a(e^{j\omega})|H^{(1)}(e^{j\omega})|^2 \tag{2.31}$$

$$S_{b*g^{(1)}}(e^{j\omega}) = S_b(e^{j\omega})|G^{(1)}(e^{j\omega})|^2 \tag{2.32}$$

$$S_{a*g^{(2)}}(e^{j\omega}) = S_a(e^{j\omega})|G^{(2)}(e^{j\omega})|^2 \tag{2.33}$$

[2]The entropy rate of a stationary Gaussian process with spectrum $S(e^{j\omega})$ is $\frac{1}{2\pi}\int_0^{\pi}\log S(e^{j\omega})d\omega$.

$$S_{b*h^{(2)}}(e^{j\omega}) = S_b(e^{j\omega})|H^{(2)}(e^{j\omega})|^2 \tag{2.34}$$

and

$$S_{z_1}(e^{j\omega}) = \frac{N_0}{2}H^{(2)}(e^{j\omega}). \tag{2.35}$$

$$S_{z_2}(e^{j\omega}) = \frac{N_0}{2}H^{(2)}(e^{j\omega}). \tag{2.36}$$

Note that $H^{(1)}(\cdot)$, $H^{(2)}(\cdot)$, $G^{(1)}(\cdot)$, and $G^{(2)}(\cdot)$ are all real functions. Let us now define

$$r_1(t) = \widetilde{h^{(1)}}(t) * \widetilde{h^{(1)}}(-t) \tag{2.37}$$

$$r_2(t) = \widetilde{h^{(2)}}(t) * \widetilde{h^{(2)}}(-t) \tag{2.38}$$

$$r_3(t) = \widetilde{g^{(1)}}(t) * \widetilde{h^{(1)}}(-t) \tag{2.39}$$

and,

$$r_4(t) = \widetilde{g^{(2)}}(t) * \widetilde{g^{(2)}}(-t) \tag{2.40}$$

So

$$h_n^{(1)} = r_1(nT), \tag{2.41}$$

$$h_n^{(2)} = r_2(nT), \tag{2.42}$$

$$g_n^{(1)} = r_3(nT), \tag{2.43}$$

$$g_n^{(2)} = r_4(nT), \tag{2.44}$$

After applying Fourier transform, we have

$$H^{(1)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} R_1\left(\frac{1}{T}\left(\frac{\omega}{2\pi}+k\right)\right) \tag{2.45}$$

$$H^{(2)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} R_2\left(\frac{1}{T}\left(\frac{\omega}{2\pi}+k\right)\right) \tag{2.46}$$

$$G^{(1)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} R_3\left(\frac{1}{T}\left(\frac{\omega}{2\pi}+k\right)\right) \tag{2.47}$$

$$G^{(2)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} R_4\left(\frac{1}{T}\left(\frac{\omega}{2\pi}+k\right)\right) \tag{2.48}$$

where

$$\begin{aligned} R_1(f) &= |\widetilde{H^{(1)}}(f)|^2 \\ &= \left|H_1(f) + e^{-j2\pi\tau_1 f} H_2(f)\right|^2 \\ &= \left|C_1(f) + e^{-j2\pi\tau_1 f} C_2(f)\right|^2 |V(f)|^2 \end{aligned} \tag{2.49}$$

$$\begin{aligned} R_2(f) &= |\widetilde{H^{(2)}}(f)|^2 \\ &= \left|H_3(f) + e^{-j2\pi\tau_2 f} H_4(f)\right|^2 \\ &= \left|D_1(f) + e^{-j2\pi\tau_2 f} D_2(f)\right|^2 |U(f)|^2 \end{aligned} \tag{2.50}$$

$$\begin{aligned} R_3(f) &= \widetilde{G^{(1)}}(f)\ \widetilde{H^{(1)}}(f)^* \\ &= \left(G_1(f) + e^{-j2\pi\tau_2 f} G_2(f)\right)\left(H_1(f) + e^{-j2\pi\tau_1 f} H_2(f)\right) \\ &= \left(F_1(f) + e^{-j2\pi\tau_2 f} F_2(f)\right)\left(C_1(f) + e^{-j2\pi\tau_1 f} C_2(f)\right)^* U(f)V^*(f) \end{aligned} \tag{2.51}$$

$$\begin{aligned} R_4(f) &= \widetilde{G^{(2)}}(f)\ \widetilde{H^{(2)}}(f)^* \\ &= \left(G_3(f) + e^{-j2\pi\tau_1 f} G_4(f)\right)\left(H_3(f) + e^{-j2\pi\tau_2 f} H_4(f)\right)^* \\ &= \left(E_1(f) + e^{-j2\pi\tau_1 f} E_2(f)\right)\left(D_1(f) + e^{-j2\pi\tau_2 f} D_2(f)\right)^* V(f)U(f)^* \end{aligned} \tag{2.52}$$

Here, interference is treated as noise. Let us consider a situation where $C_i(\cdot)=\alpha_i$, $d_i(\cdot)=\beta_i$, $e_i(\cdot)=\gamma_i$, and $f_i(\cdot)=\delta_i$ are constant gains in $[-W, W]$ for $i = 1, 2$, $V(\cdot)$ and $U(\cdot)$ are flat and equal to unity in $[-W, W]$, and both that $(a_i)_{i\in\mathbb{Z}}$ and $(a_i)_{i\in\mathbb{Z}}$ are i.i.d. sequences, i.e.,

$S_a(e^{j\omega}) = \sigma_a^2$ and $S_b(e^{j\omega}) = \sigma_b^2$. Then, letting $\int_{-\infty}^{\infty} |V(f)|^2 df = 1$ and $\int_{-\infty}^{\infty} |U(f)|^2 df = 1$, we get

$$\sigma_a^2 \le PT \tag{2.53}$$

$$\sigma_b^2 \le PT \tag{2.54}$$

Hence,

$$\text{Rate1 in bits/sec/hz} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{2\sigma_a^2 |H^{(1)}(e^{j\omega})|^2}{2\sigma_b^2 |G^{(1)}(e^{j\omega})|^2 + N_0 H^{(1)}(e^{j\omega})}\right) d\omega. \tag{2.55}$$

where

$$H^{(1)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \left|\alpha_1 + \alpha_2 e^{-\frac{j2\pi\tau_1}{T}\left(\frac{\omega}{2\pi}+k\right)}\right|^2. \tag{2.56}$$

$$G^{(1)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \left(\alpha_1 + \alpha_2 e^{-\frac{j2\pi\tau_1}{T}\left(\frac{\omega}{2\pi}+k\right)}\right)\left(\delta_1 + \delta_2 e^{-\frac{j2\pi\tau_2}{T}\left(\frac{\omega}{2\pi}+k\right)}\right)^*. \tag{2.57}$$

$$\text{Rate2 in bits/sec/hz} = \frac{1}{2\pi T}\int_0^{\pi} \log\left(1 + \frac{2\sigma_b^2 |H^{(2)}(e^{j\omega})|^2}{2\sigma_a^2 |G^{(2)}(e^{j\omega})|^2 + N_0 H^{(2)}(e^{j\omega})}\right) d\omega. \tag{2.58}$$

where

$$H^{(2)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \left|\beta_1 + \beta_2 e^{-\frac{j2\pi\tau_2}{T}\left(\frac{\omega}{2\pi}+k\right)}\right|^2. \tag{2.59}$$

$$G^{(2)}(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \left(\gamma_1 + \gamma_2 e^{-\frac{j2\pi\tau_1}{T}\left(\frac{\omega}{2\pi}+k\right)}\right)\left(\beta_1 + \beta_2 e^{-\frac{j2\pi\tau_2}{T}\left(\frac{\omega}{2\pi}+k\right)}\right)^*. \tag{2.60}$$

## 2.3 On Delay Diversity of Point-to-point channels in Discrete time

In this part, a PTP channel with two transmit antennas in its transmitter has been studied. The modulation scheme at the transmitter is based on OFDM. Assume that the number of tones of OFDM is N. The delay can be applied in either a linear or a circular way. However, as mentioned in 1.4, cyclic delay diversity (CDD) has an advantage over linear delay diversity (LDD) (see [1] for different PSK/QAM transmit symbols). Here, Gaussian input is applied.

Let $(\boldsymbol{s}_i[n])_{0\leq n\leq N-1}$ be a vector of size N representing N consecutive symbols in a codeword of user i.

## 2.4 On Delay Diversity of Interference Channels in Discrete time

Let us generalize the equations in the previous section for an interference channel with $n_T$ antennas at each transmitter and $n_R$ antennas at each receiver. The following equations are derived for any channel impulse responses and Gaussian input. We can then simplify the equations for two antennas at the transmitters and receivers. The impulse response from antenna n in the transmitter i to receive antenna m in the receiver i (in other words, the forward channels) is given by

$$h_i^{(mn)} = [h_i^{(mn)}(0), h_i^{(mn)}(1), ..., h_i^{(mn)}(D), 0, ..., 0] \tag{2.61}$$

and the impulse response of cross channels are

$$g_i^{(mn)} = [g_i^{(mn)}(0), g_i^{(mn)}(1), ..., g_i^{(mn)}(D), 0, ..., 0] \tag{2.62}$$

For $n = 1, ..., n_T$, $m = 1, ..., n_R$ and $i = 1, 2$. D is the maximum length of channel memories. All channel coefficients are independent complex Gaussian random variables. Additive white Gaussian noise with variance $\sigma^2 = N_0/2$ per real dimension is added at each receive antenna. The output symbols in transmitters 1 and 2 before applying cyclic delay diversity are $x_1[n]$ and $x_2[n]$, $n = 0, 1, ..., N-1$. After applying cyclic delay diversity, $\Delta_1^{(j)}$, $j = 1, ..., n_T$, delays in transmitter 1 and $\Delta_2^{(j)}$, $j = 1, ..., n_T$, delays in transmitter 2. Hence,

$$x_i^{(j)}[n] = x_i^{(j)}[((n - \Delta_i^{(j)}))_N] \quad for\ \ i = 1, 2 \quad j = 1, ..., N_T,\ and\ \ n = 1, ..., N. \tag{2.63}$$

are data sent from transmitter 1 and 2, respectively.

It is possible to replace this channel with an equivalent SIMO channel with the channel impulse responses, as below:

$$\boldsymbol{h_i^{(n)}} = [\boldsymbol{h_i^{(n)}(0)}, ..., \boldsymbol{h_i^{(n)}(N-1)}] \tag{2.64}$$

where,

$$\boldsymbol{h_i^{(n)}(d)} = \sum_{n=1}^{n_T} h_i^{(nm)}((d - \Delta_i^{(n)})\ mod\ N), \quad m = 1, ..., n_R\ and\ \ i = 1, 2. \tag{2.65}$$

In the same way,

$$\boldsymbol{g_i^{(n)}} = [\boldsymbol{g_i^{(n)}(0)}, ..., \boldsymbol{g_i^{(n)}(N-1)}] \tag{2.66}$$

where,

$$\boldsymbol{g_i^{(n)}(d)} = \sum_{n=1}^{n_T} g_i^{(nm)}((d - \Delta_i^{(n)})\ mod\ N), \quad m = 1, ..., n_R\ and\ \ i = 1, 2. \tag{2.67}$$

After applying discrete Fourier transform to Equations (2.64) and (2.66),

$$\boldsymbol{H_i^{(n)}} = [H_i^{(n)}(0), ..., H_i^{(n)}(N-1)], \quad n = 1, ..., n_R\ and\ \ i = 1, 2 \tag{2.68}$$

and,

$$\boldsymbol{G_i^{(n)}} = [G_i^{(n)}(0), ..., G_i^{(n)}(N-1)] \quad n = 1, ..., n_R\ and\ \ i = 1, 2 \tag{2.69}$$

where,

$$\boldsymbol{H_i}^{(n)}(d) = \sum_{n=1}^{n_T} H_i^{(n)}(d) e^{(2\pi d \Delta_i^{(n)})}/N, \quad d = 0, ..., N-1, \ \ n = 1, ..., n_R \tag{2.70}$$

$$\boldsymbol{G_i}^{(n)}(d) = \sum_{n=1}^{n_T} G_i^{(n)}(d) e^{(2\pi d \Delta_i^{(n)})}/N, \quad d = 0, ..., N-1, \ \ n = 1, ..., n_R \tag{2.71}$$

As in the previous sections, the input distribution here is Gaussian and interference is treated as noise. Assume a sufficiently long gaurd interval of length $G \geq D$ with no channel state information (CSI) at the transmitters but perfect CSI at the receivers. $p_i$ is power per tone in transmitter i, for i=1,2. Hence,

$$Rate1 = \frac{1}{N+G} \sum_{d=0}^{N-1} log_2(det(\boldsymbol{I}_{n_R} + \frac{p_1 \boldsymbol{H_1}(d) \boldsymbol{H_1}(d)^H}{N_0 + p_2 \boldsymbol{G_1}(d) \boldsymbol{G_1}(d)^H})) \tag{2.72}$$

$$Rate2 = \frac{1}{N+G}\sum_{d=0}^{N-1} log_2 det(\boldsymbol{I}_{n_R} + \frac{p_2\boldsymbol{H_2}(d)\boldsymbol{H_2}(d)^H}{N_0 + p_1\boldsymbol{G_2}(d)\boldsymbol{G_2}(d)^H}) \tag{2.73}$$

As discussed in the previous chapter, there are several methods to deal with interference. One method is treating interference as noise, which is a popular choice especially when interference is weak. We also talked about diversity in the previous chapter, ascertaining that a diversity scheme improves the reliability of a message signal. The diversity method that we use here is antenna diversity or space diversity. Specifically, we use more than one antenna in each transmitter and send the same data streams from antennas one and two, but with different time delays.

Now consider a two-user interference channel where each transmitter is equipped with two transmitting antennas. The modulation scheme at each transmitter is based on OFDM together with cyclic delays in the second antenna, as shown in Figure 6. Here, let $(\boldsymbol{s}_i[n])_{0\le n\le N-1}$ be a vector of size N representing N consecutive symbols in a codeword of user i. For i=1,2 let

$$(\boldsymbol{x}_i^{(1)}[n])_{0\le n\le N-1} = \mathrm{IDFT}((\boldsymbol{s}_i[n])_{0\le n\le N-1}), \tag{2.74}$$

and

$$(\boldsymbol{x}_i^{(2)}[n])_{0\le n\le N-1} = (\boldsymbol{x}_i^{(1)}[((n-\delta_i))_N])_{0\le n\le N-1}, \tag{2.75}$$

where $((a))_b$ indicates $a \mod b$. Note that we can also write

$$(\boldsymbol{x}_i^{(2)}[n])_{0\le n\le N-1} = \mathrm{IDFT}((W_N^{n\delta_i}\boldsymbol{s}_i[n])_{0\le n\le N-1}), \tag{2.76}$$

where

$$W_N = e^{-\frac{\hat{j}2\pi}{N}}. \tag{2.77}$$

Then, $(\boldsymbol{x}_i^{(1)}[n])_{0\le n\le N-1}$ and $(\boldsymbol{x}_i^{(2)}[n])_{0\le n\le N-1}$ are transmitted from the first and second antennas of user i, respectively, in N consecutive transmitted slot. Let us denote the channel gain from transmitting antenna k of user i to the receiving antenna of user j by $h_{i,j}^{(k)}$. Denoting the N received symbols of user i by $(\boldsymbol{y}_i[n])_{0\le n\le N-1}$,

$$(\boldsymbol{y}_1[n])_{0\le n\le N-1} = \sum_{k=1}^{2} h_{1,1}^{(k)}(\boldsymbol{x}_1^{(k)}[n])_{0\le n\le N-1} + \sum_{k=1}^{2} h_{2,1}^{(k)}(\boldsymbol{x}_2^{(k)}[n])_{0\le n\le N-1} + (\boldsymbol{z}_1[n])_{0\le n\le N-1}, \tag{2.78}$$

$$(\boldsymbol{y}_2[n])_{0\le n\le N-1} = \sum_{k=1}^{2} h_{2,2}^{(k)}(\boldsymbol{x}_2^{(k)}[n])_{0\le n\le N-1} + \sum_{k=1}^{2} h_{1,2}^{(k)}(\boldsymbol{x}_1^{(k)}[n])_{0\le n\le N-1} + (\boldsymbol{z}_2[n])_{0\le n\le N-1}, \tag{2.79}$$

where $(\boldsymbol{z}_1[n])_{0\le n\le N-1}$ and $(\boldsymbol{z}_2[n])_{0\le n\le N-1}$ are the ambient noise symbols at the receiver of user 1 modeled as i.i.d. $\mathcal{CN}(0,1)$ random variables. One can also write (2.78) and (2.79) as

$$\vec{\boldsymbol{y}}_1 = A_1\vec{\boldsymbol{s}}_1 + A_2\vec{\boldsymbol{s}}_2 + \vec{\boldsymbol{z}}_1, \tag{2.80}$$

$$\vec{\boldsymbol{y}}_2 = A_3\vec{\boldsymbol{s}}_1 + A_4\vec{\boldsymbol{s}}_2 + \vec{\boldsymbol{z}}_2, \tag{2.81}$$

where

$$A_1 = F^{-1}(h_{1,1}^{(1)} I_N + h_{1,1}^{(2)} D_1), \tag{2.82}$$

$$A_2 = F^{-1}(h_{2,1}^{(1)} I_N + h_{2,1}^{(2)} D_2), \tag{2.83}$$

$$A_3 = F^{-1}(h_{2,2}^{(1)} I_N + h_{2,2}^{(2)} D_2), \tag{2.84}$$

$$A_4 = F^{-1}(h_{1,2}^{(1)} I_N + h_{1,2}^{(2)} D_1), \tag{2.85}$$

$$F = (W_N^{mn})_{0\le m,n\le N-1}, \tag{2.86}$$

is the DFT matrix of size $N \times N$ and

$$D_i = \mathrm{diag}((W_N^{n\delta_i})_{0\le n\le N-1}), \quad i = 1, 2. \tag{2.87}$$

Let us denote the noise plus interference as

$$\vec{\boldsymbol{w}}_1 = A_2\vec{\boldsymbol{s}}_2 + \vec{\boldsymbol{z}}_1. \tag{2.88}$$

Assuming both users employ random Gaussian codewords, the transmission rate of user 1 and user 2, $R_1(\delta_1, \delta_2, C_1, C_2, \mathcal{H}_1)$ and $R_2(\delta_1, \delta_2, C_1, C_2, \mathcal{H}_2)$ respectively, are:

$$\begin{aligned} R_1(\delta_1, \delta_2, C_1, C_2, \mathcal{H}_1) &= \frac{I(\vec{\boldsymbol{s}}_1; \vec{\boldsymbol{y}}_1)}{N} \\ &= \frac{1}{N} \log \frac{\det \operatorname{cov}(\vec{\boldsymbol{y}}_1)}{\det \operatorname{cov}(\vec{\boldsymbol{w}}_1)} \\ &= \frac{1}{N} \log \frac{\det(A_1 C_1 A_1^H + A_2 C_2 A_2^H + I_N)}{\det(A_2 C_2 A_2^H + I_N)}, \end{aligned} \tag{2.89}$$

$$\begin{aligned} R_2(\delta_1, \delta_2, C_1, C_2, \mathcal{H}_2) &= \frac{I(\vec{\boldsymbol{s}}_2; \vec{\boldsymbol{y}}_2)}{N} \\ &= \frac{1}{N} \log \frac{\det \operatorname{cov}(\vec{\boldsymbol{y}}_2)}{\det \operatorname{cov}(\vec{\boldsymbol{w}}_2)} \\ &= \frac{1}{N} \log \frac{\det(A_3 C_1 A_3^H + A_4 C_2 A_4^H + I_N)}{\det(A_3 C_1 A_3^H + I_N)}, \end{aligned} \tag{2.90}$$

where

$$C_i = \operatorname{cov}(\vec{\boldsymbol{s}}_i), \;\; i = 1, 2, \tag{2.91}$$

and $\mathcal{H}_i$ denotes the set of all channel gains that are involved in the expression of $R_i$ for i=1,2. Note that user i has a transmission power constraint

$$E\left\{ \|\vec{\boldsymbol{x}}_i^{(1)}\|^2 + \|\vec{\boldsymbol{x}}_i^{(2)}\|^2 \right\} \leq N P_i. \tag{2.92}$$

It is clear that this is equivalent to

$$\operatorname{tr}(C_i) \leq \frac{N P_i}{2}, \;\; i = 1, 2. \tag{2.93}$$

Since in our case we do not know the channel channels, i.e., $\mathcal{H}_i$ we have to write the capacity formulas in terms of these variables and for comparing different configurations we have to consider outage probability. For a certain selection of $C_1$ and $C_2$, we are interested in the following design criteria

However, as we do not know the channel channels (i.e., $\mathcal{H}_i$ we have to write the capacity formulas in terms of these variables and when comparing different configurations), we have to consider outage probability. For a certain selection of $C_1$ and $C_2$, we are interested in the following design criteria

$$\hat{\delta}_1, \hat{\delta}_2 = \arg\max_{\delta_1,\delta_2} \mathrm{C}_{\text{outage}}(\varepsilon), \tag{2.94}$$

where $\mathrm{C}_{\text{outage}}(\varepsilon)$ is $\varepsilon$-outage *sum-capacity* given by

$$\mathrm{C}_{\text{outage}}(\varepsilon) = \sup\left\{ r_1 + r_2 : \Pr\left\{ \bigcup_{i=1}^{2} \left\{ R_i(\delta_1, \delta_2, C_1, C_2, \boldsymbol{\mathcal{H}}_i) < r_i \right\} \right\} < \varepsilon \right\}. \tag{2.95}$$

Note that,

$$\begin{aligned} \Pr\left\{ \bigcup_{i=1}^{2} \left\{ R_i(\delta_1, \delta_2, C_1, C_2, \boldsymbol{\mathcal{H}}_i) < r_i \right\} \right\} &= \sum_{i=1}^{2} Pr\left\{ R_i(\delta_1, \delta_2, C_1, C_2, \boldsymbol{\mathcal{H}}_i) < r_i \right\} \\ -\prod_{i=1}^{2} Pr\left\{ R_i(\delta_1, \delta_2, C_1, C_2, \boldsymbol{\mathcal{H}}_i) < r_i \right\}. \end{aligned} \tag{2.96}$$

as $\boldsymbol{\mathcal{H}}_1$ and $\boldsymbol{\mathcal{H}}_2$ are independent.

### 2.4.1 Simulation Results

In this section, we simulate the derived equations in section 2.4 using a Matlab program [37]. Since we do not know the channel coefficients, we perform the simulations using the Monte-Carlo algorithm; the channels are Rayleigh fading. Therefore, instead of capacity, outage capacity is calculated.

According to Equations 2.95 and 2.96 rates, probability and capacity of outage are dependent on these parameters: delays $\delta_1$ and $\delta_2$, covariance matrices $C_1$ and $C_2$, channel matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, and N number of tones in OFDM.

With the assumption of complex Gaussian channels coefficients, there are two options: channel gains are constant for all realizations of channels (frequency-flat channels), or channel gains are different for each realization (frequency-selective channels). As mentioned in [1], simulation results are presented for point-to-point channels, with the input of Gaussian, BPSK, and QPSK for both cases of frequency-flat and frequency-selective channels. Here, simulations are done for two-user interference channel with Gaussian inputs for frequency-flat and frequency-selective channels assumptions, when interference is treated as noise.

. Since power allocated to transmitter antennas is equal, covariance matrix elements $C_i = [\rho_i(m,n)]$ for $m \neq n$ must also be the same. This is due to symmetry; in other words, if we exchange two antennas, we should have the same results. Thus, $\rho_i(m,n) = \rho_i$ for i=1,2. For the PTP channel, [32] and [1] assume that input symbols are independent. Our simulations prove both of these results. Note that valid values for $\rho_1$ and $\rho_1$ are between 0 and 1. In the simulation figures we have shown the result curves for only few values including the best one in terms of rate.

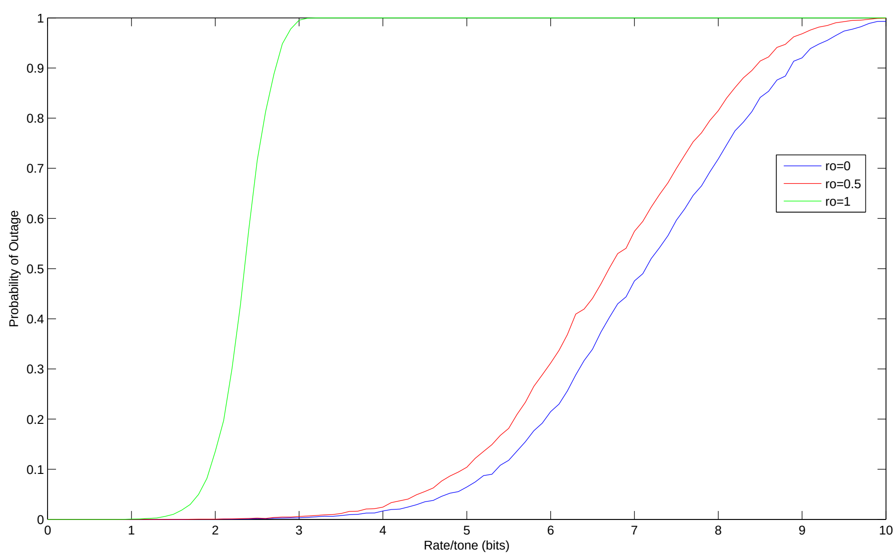


Figure 2.1: point to point channel N=4 and SNR=20dB

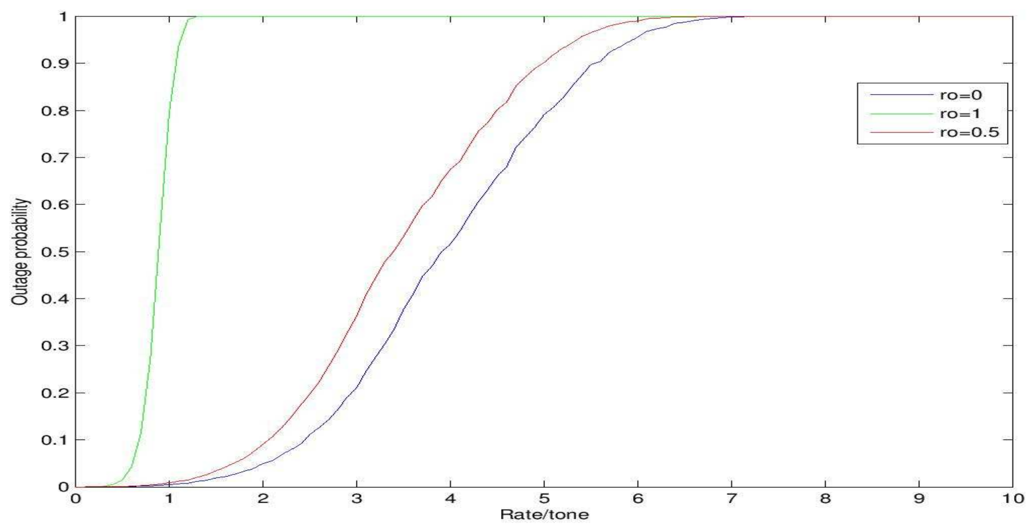


Figure 2.2: point to point channel N=8 and SNR=10dB

As stated previously, since the channels are Rayleigh fading, capacity is the average of a large number of capacities calculated for various random channel gains. Simulations are first done for the frequency-flat channel, N=8, SNR=30 dB, and independent inputs. In this simulation, capacity of outage is fixed at $C_{outage} = 0.4$, and probabilities of outage vs. delays are calculated as presented in Table 1.

| | $\delta_1 = 0$ | $\delta_1 = 1$ | $\delta_1 = 2$ | $\delta_1 = 3$ | $\delta_1 = 4$ | $\delta_1 = 5$ | $\delta_1 = 7$ | $\delta_1 = 7$ |
|---|---|---|---|---|---|---|---|---|
| $\delta_2 = 0$ | 0.4252 | 0.2905 | 0.2959 | 0.2915 | 0.3143 | 0.2908 | 0.2946 | 0.2897 |
| $\delta_2 = 1$ | 0.2932 | 0.1553 | 0.1450 | 0.1432 | 0.1651 | 0.1415 | 0.1475 | 0.1540 |
| $\delta_2 = 2$ | 0.2937 | 0.1469 | 0.1632 | 0.1454 | 0.1698 | 0.1453 | 0.1642 | 0.1482 |
| $\delta_2 = 3$ | 0.2904 | 0.1429 | 0.1488 | 0.1555 | 0.1633 | 0.1547 | 0.1458 | 0.1433 |
| $\delta_2 = 4$ | 0.3147 | 0.1628 | 0.1710 | 0.1645 | 0.2035 | 0.1632 | 0.1683 | 0.1633 |
| $\delta_2 = 5$ | 0.2896 | 0.1427 | 0.1441 | 0.1540 | 0.1630 | 0.1542 | 0.1468 | 0.1424 |
| $\delta_2 = 6$ | 0.2948 | 0.1458 | 0.1627 | 0.1490 | 0.1706 | 0.1462 | 0.1595 | 0.1461 |
| $\delta_2 = 7$ | 0.2896 | 0.1544 | 0.1462 | 0.1428 | 0.1629 | 0.1426 | 0.1490 | 0.1521 |

Figure 2.3: Probability of Outage vs. delays

As we can see from Table 1, some delays result in lower outage, indicating that choosing non-zero delays helps achieve higher rates. Let us now find out if the assumption of independent input symbols is optimal or not. To reach this goal, different values of $\rho_1$ and $\rho_2$ outage probability vs rate per tone are depicted for a specified number of tones and four different SNRs in the figures below. Note that each graph is sketched for its corresponding optimum delays.

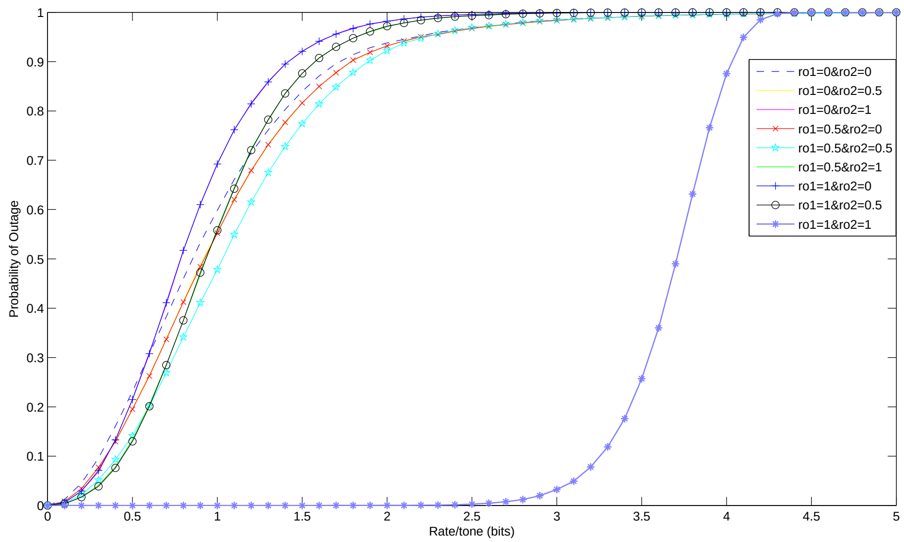


Figure 2.4: Two-user Gaussian interference channel with N=4 and SNR=40dB (Frequency-flat)

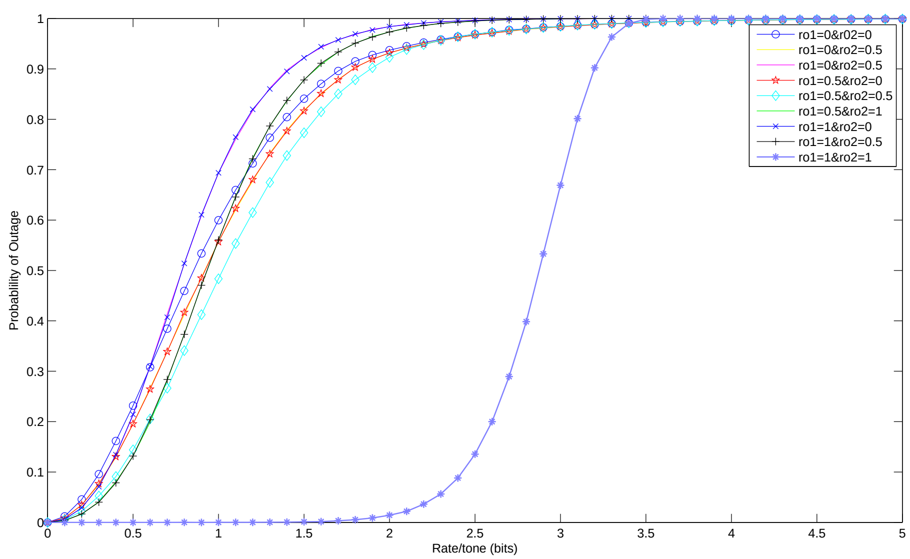


Figure 2.5: Two-user Gaussian interference channel with N=4 and SNR=30dB (Frequency-flat)

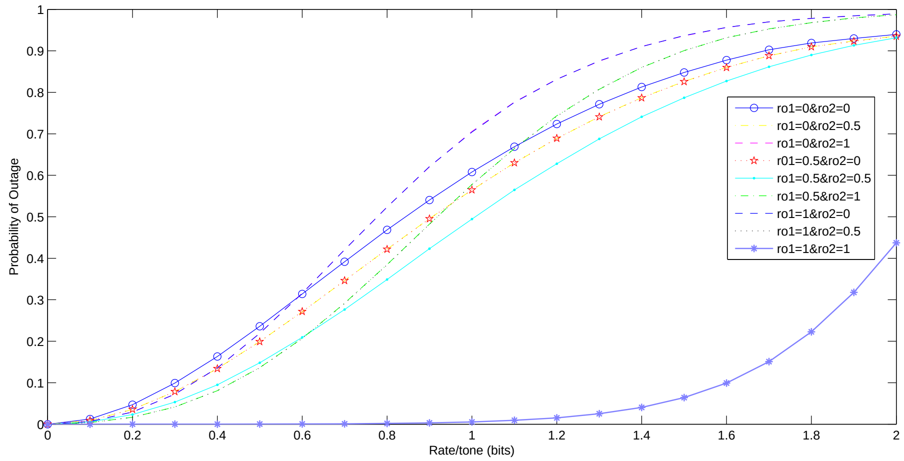


Figure 2.6: Two-user Gaussian interference channel with N=4 and SNR=20dB (Frequency-flat)

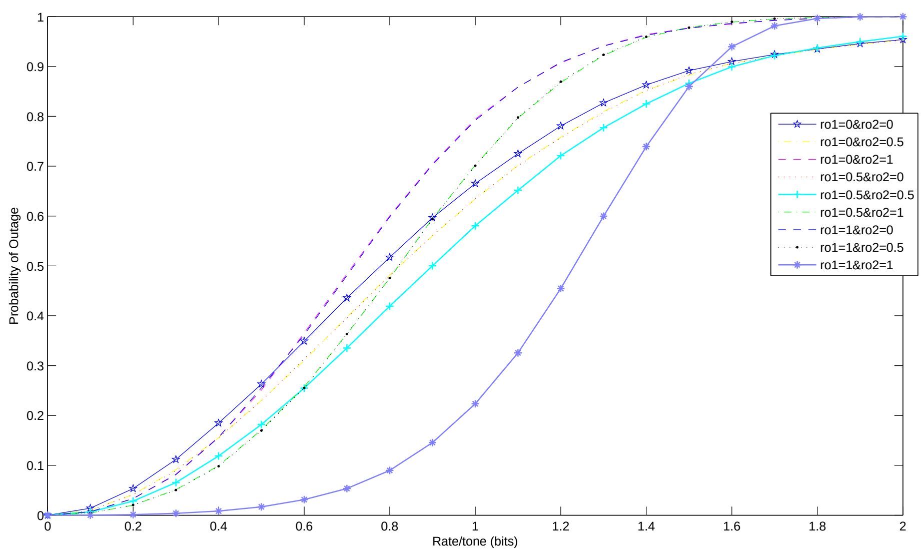


Figure 2.7: Two-user Gaussian interference channel with N=4 and SNR=10dB (Frequency-flat)

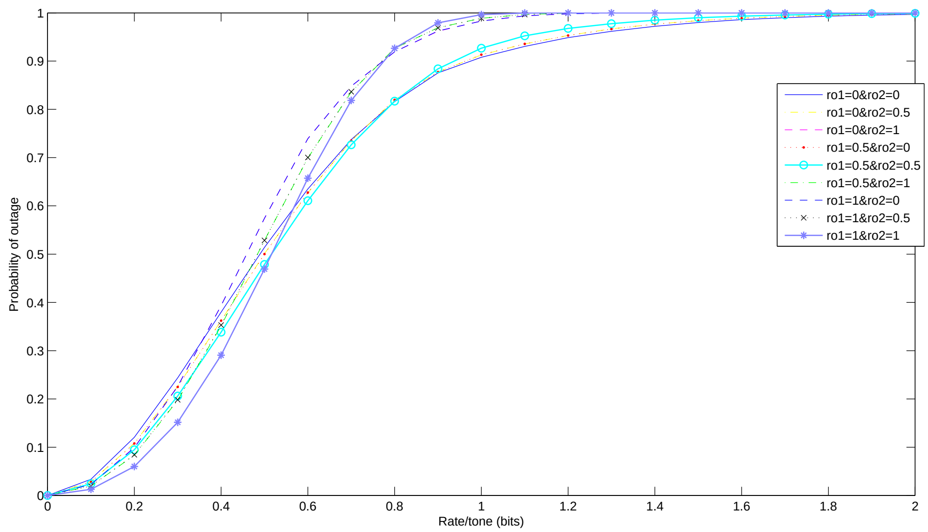


Figure 2.8: Two-user Gaussian interference channel with N=4 and SNR=0dB (Frequency-flat)

The above figures show that, for SNR= 0 dB to 40 dB and N=4 around the outage probability of 0.05 (which is a reasonable value), the optimum value of $\rho_1$ and $\rho_2$ is one. For other SNRs and numbers of tones, we achieve the same result. Figure (2.4.1) provides yet another example.

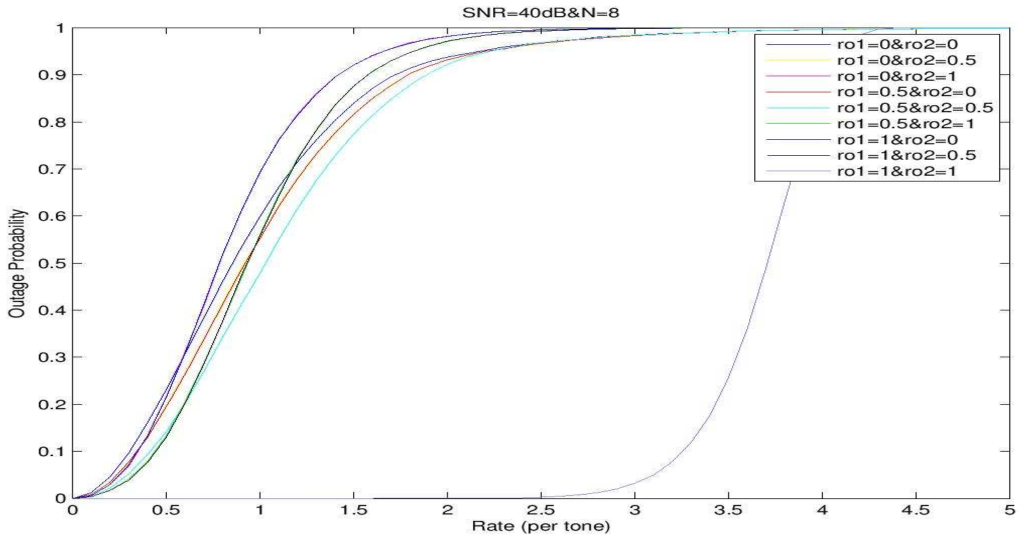


Figure 2.9: Two-user Gaussian interference channel with N=8 and SNR=40dB

Similar simulations are done for frequency-selective interference channels. The difference between frequency-flat and frequency-selective channels is that, in the former, channels are constant when sending OFDM tones, while in the latter channel gains are different for each OFDM symbol. In [1], Bauch has carried out simulations for both frequency-flat and frequency-selective point-to-point channels.

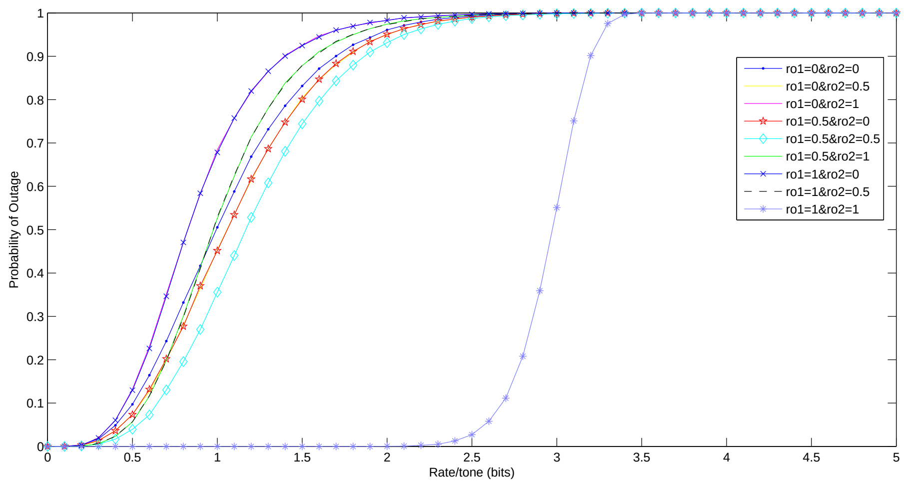


Figure 2.10: Two-user Gaussian interference channel with N=4 and SNR=30dB (Frequency-selective)

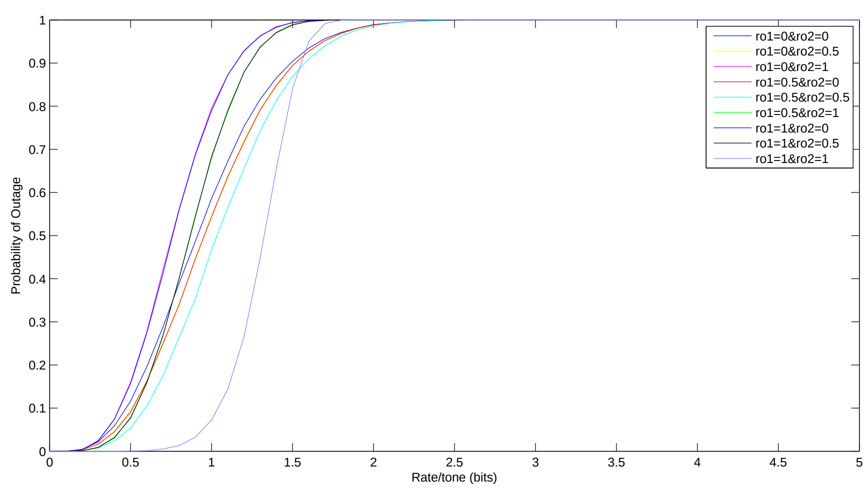


Figure 2.11: Two-user Gaussian interference channel with N=4 and SNR=10dB (Frequency-selective)

As mentioned another way to deal with interference is decoding interference. So by adding this ability receiver chooses between two options: treating interference as noise or decoding interference. This flexibility helps in terms of rate and in a similar situation achieving rate is equal or higher.

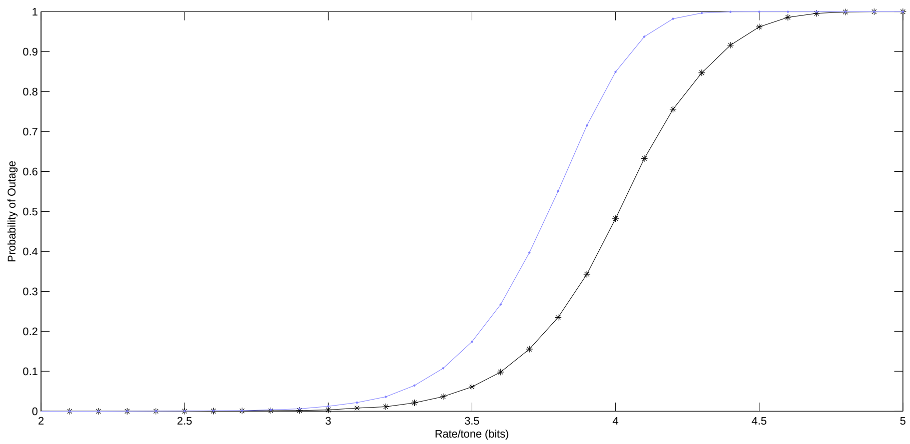


Figure 2.12: Two-user Gaussian interference with and without decoding interference option

### 2.4.2 Mathematical Derivations for Simulation Results

As we know from Equations (2.89) and (2.90), both $R_1$ and $R_2$ are dependent on covariance matrices $C_1$ and $C_2$. In our situation, since we do not know channel coefficients, we must therefore consider outage probability and maximize the sum-rate $r_1 + r_2$. However, finding the optimum matrices by considering probability of outage is not a straightforward task. Moreover, as increasing $R_1$ and $R_2$ increases the sum-rate, we instead must find the matrices that maximize $R_1$ and $R_2$.

As we can see from simulation results the studied interference channel in terms of covariance matrices is symmetry so we suppose that $C_1 = C_2 = C$ and that $P_1 = P_2 = P$ . Hence, the optimization problem becomes:

$$\max_{C} \ \log \ \det \frac{\{A_1 C A_1^H + A_2 C A_2^H + I_N\}}{\{A_2 C A_2^H + I_N\}} \tag{2.97}$$

subject to

$$tr(C) \leq NP \tag{2.98}$$

Since this problem is not convex, solution can not be found with common methods of optimization. In this section we simplify equation (2.97) but closed form solution is still open. Before attempting to solve the problem, let us review the details of this optimization.

**Optimization with inequality constraints (The Kuhn-Tucker conditions)**

Consider a problem of the form

$$\max_x \; u(x) \quad \textit{subject to} \quad g_i(x) \leq c_i \quad i = 1, ..., n \tag{2.99}$$

The problem model in the above is general. All other optimization problems (i.e., either minimization problems or problems with equality constraints) can be converted to a maximization problem with inequality constraints. To solve this problem, we need to define the Lagrangean function, as below:

$$L(x) = u(x) - \lambda_i(g_i(x) - c_i) \tag{2.100}$$

It has been proved that, if $g(x*) = c$, we have $\lambda \geq 0$, and if $g(x*) < c$ the value of $\lambda$ does not matter. In this case, we can choose any value, and so select $\lambda = 0$. Under this assumption, we have $u^{'}(x) = L^{'}(x)$ and, therefore, we have $L^{'}(x*) = 0$. In the first case, we have $g(x*) = c$ and in the second case $\lambda = 0$.

The inequalities $\lambda \geq 0$ and $g(x*) \leq c$ are called complementary slackness conditions. We know that $L(x)$ is maximum at $\bar{x}$ if

$$DL(\bar{x}, x - \bar{x}) = \lim_{t \to 0} \frac{L(\bar{x} + t(x - \bar{x})) - L(x)}{t} \leq 0 \tag{2.101}$$

where D is a directional derivation. The directional derivative of a given function f at $x$ with increment d is defined by

$$Df(x; d) = \lim_{t \to 0} \frac{f(x + td) - f(x)}{t} \tag{2.102}$$

If this limit exists for all of d, the function is a Gateaux differentiable at x. A Gateaux derivative is a generalization of the concept of directional derivative. Here, our objective function is(2.97). We also know that derivation is a linear function, i.e., $D(a+b) = D(a) + D(b)$. Thus, we first need to calculate the directional derivation of our objective function. Let

$$D(\log(\det(A_1CA_1^H + A_2CA_2^H + I_N)) - \log(\det(A_2CA_2^H + I_N))) =$$
$$D(\log(\det(A_1CA_1^H + A_2CA_2^H + I_N))) - D(\log(\det(A_2CA_2^H + I_N))) \qquad (2.103)$$

Now we want to find the optimum covariance matrix $C$. We start with the second part of the expression above and try to calculate it. From (2.83) we have

$$\det\{A_2CA_2^H + I_N\} = \det\{F^{-1}(h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2)C(F^{-1}(h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2))^H\} =$$
$$\det\{F^{-1}(h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2)C((h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2))^H F^{-H}\} =$$
$$\det\{F^{-1}\}\det\{(h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2)C((h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2))^H\}\det\{F^{-H}\} \qquad (2.104)$$

because we know that $\det(AB) = \det(A)\det(B)$. According to the definition of matrix $F$ (2.86) $\det\{F^{-1}\} = \det\{F^{-H}\} = 1$. Hence if we define

$$E_2 = h_{2,1}^{(1)}I_N + h_{2,1}^{(2)}D_2 \qquad (2.105)$$

then (2.104) equals to $\det\{E_2CE_2^H + I_N\}$.

Let C= $\begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,N} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ c_{N,1} & c_{N,2} & \cdots & c_{N,N} \end{pmatrix}$

$$DL(C) = D\{\log(\det(E_2CE_2^H + I_N))\} = \nabla_C\{\log(\det(E_2CE_2^H + I_N))\} \qquad (2.106)$$

$$DL(C) = \begin{pmatrix} \frac{\partial L}{\partial c_{1,1}} & \frac{\partial L}{\partial c_{1,2}} & \cdots & \frac{\partial L}{\partial c_{1,N}} \\ \frac{\partial L}{\partial c_{2,1}} & \frac{\partial L}{\partial c_{2,2}} & \cdots & \frac{\partial L}{\partial c_{2,N}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial L}{\partial c_{1,N}} & \frac{\partial L}{\partial c_{N,2}} & \cdots & \frac{\partial L}{\partial c_{N,N}} \end{pmatrix} \tag{2.107}$$

$$\frac{\partial L}{\partial c_{i,j}} = \frac{\partial \log\{\det(E_2 C E_2^H + I_N)\}}{\partial c_{i,j}} = \frac{\frac{\partial \det(E_2 C E_2^H + I_N)}{\partial c_{i,j}}}{\det(E_2 C E_2^H) + I_N} \tag{2.108}$$

Based on the definition of determinant of a matrix [38] we have

$$\det\{A\} = \sum_{j=1}^{N} (-1)^{i+j} a_{i,j} M_{i,j} \tag{2.109}$$

$\{M_{i,j}\}_{j=1}^{N}$ are the minors of the matrix $A$. The $M_{i,j}$ is the determinant of the $(N1)(N1)$ matrix resulting from removing the $i - th$ row and the $j - th$ column of matrix $A$ and $(-1)^{i+j} M_{i,j}$ are called co-factors.

We define a new matrix $R = E_2 C E_2^H + I_N$ in order to simplify the equations. $r_{i,j}$ is the element of the matrix $R$ in the $i^{th}$ row and the $j^{th}$ column.

$$\frac{\partial \det\{R\}}{\partial c_{i,j}} = \frac{\partial \sum_{n=1}^{N} (-1)^{i+j} r_{i,j} M_{i,j}}{\partial c_{i,j}} = \sum_{n=1}^{N} (-1)^{i+n} \{M_{i,n} \frac{\partial r_{i,n}}{\partial c_{i,j}} + r_{i,n} \frac{\partial M_{i,n}}{\partial c_{i,j}}\} \tag{2.110}$$

As indicated above, $M_{i,n}$ is the matrix resulting from omitting row $i$ and column $n$ so $M_{i,n}$ is independent from $c_{i,j}$. Hence,

$$\frac{\partial M_{i,n}}{\partial c_{i,j}} = 0 \tag{2.111}$$

According to (2.105), $D_2$ and $I_N$ are diagonal matrices $E_2$ is also a diagonal matrix. thus, let $e_i$ be the $i^{th}$ element of the diameter of $E_2$. The element in row $i$ and column $n$ of matrix $R$ is now

$$r_{i,n} = e_i e_n^* c_{i,n} + \delta_{i,n} \tag{2.112}$$

where

$$\delta_{i,n} = \begin{cases} 0 & \text{if } n \neq i \\ 1 & \text{if } n = i \end{cases}$$

Hence,

$$\frac{\partial r_{i,n}}{\partial c_{i,j}} = \begin{cases} 0 & \text{if } n \neq i \\ e_i e_n^* & \text{if } n = i \end{cases}$$

Consequently,

$$\frac{\partial \log\{\det(E_2 C E_2^H + I_N)\}}{\partial c_{i,j}} = \frac{e_i e_j^* (-1)^{i+j} M_{i,j}}{\det(E_2 C E_2^H + I_N)} \tag{2.113}$$

Let $A$ be an $N \times N$ matrix. The adjugate matrix of $A$ is an $N \times N$ matrix and is defined as below:

$$adj(A)_{i,j} = (-1)^{(i+j)} M_{j,i} \tag{2.114}$$

Hence,

$$A\, adj(A) = \det(A) I_N \tag{2.115}$$

So

$$D\{\log(\det(E_2 C E_2^H + I_N))\} = e_i e_j^* (E_2 C E_2^H + I_N)^{-H} = E_2 (E_2 C E_2^H + I_N)^{-H} E_2^H \tag{2.116}$$

In the same way we have,

$$D\{\log(\det((1_2 C E_1^H + E_2 C E_2^H + I_N))\} = E_1 (E_1 C E_1^H + I_N)^{-H} E_2^H + E_2 (E_2 C E_2^H + I_N)^{-H} E_2^H \tag{2.117}$$

# Chapter 3

# Conclusion and Future Work

## 3.1 Conclusion

In this thesis, we have shown that we can increase achievable rates by using antenna diversity in OFDM-based interference channels. This method can also be considered as space-time block coding. The channels are time-varying and unknown at transmitters, so outage capacity and probability are the criterion. Similar work has been done for point-to-point channels by Bossert et al. and Bauch in [32, 1], respectively, who have shown that adding extra antennas to PTP channel transmitters increases channel rates. Shifted versions of data are sent from transmit antennas, with delays done in a cyclic way, causing no restriction to the applied delays by guard interval. In this method, in contrast to other spatial diversity methods such as the Alamouti scheme, there is no need to change the receiver in comparison to one antenna case. We have implemented the same strategy on a two-user SISO interference channel while treating interference as noise. In addition to obtaining optimum delays, we found optimum covariance matrices. Independent OFDM symbols are not ideal for interference channels, as they must be completely correlated. We also investigated another scenario which decodes interference, with results showing that, in this case, optimum covariance matrices are not equal anymore.

## 3.2 Future Work

While we have established results for SISO ICs, the same can also be done for MIMO ICs. The proposed method can be generalized for k-user channels, and more than one extra antenna can be added to the transmitters. As well, it can be compared to other space-time block coding in order to ascertain its advantages and disadvantages, and simulations can be done for channels with memory (D-tap impulse response channel where $D \geq 2$). In addition to Gaussian input, we can use other modulations such as BPSK, QPSK, and, PAM.

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