Financial Fraud:
A Game of Cat and Mouse

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

This thesis models rational criminals and regulators with flawed incentives. In it we develop a rational model of crime and regulation that we use to show the SEC's current incentive structure is ineffective at preventing fraud. Under our model, criminals balance the monetary rewards of larger frauds against an increased chance of being apprehended; and regulators design regulations to minimize either the damage caused by fraud or some other metric. We show that under this model, the SEC's focus on "stats" and "quick hits" leads to large frauds and a large social loss. We argue that regulators need to focus not just on successful prosecutions, but also on harm reduction and deterrence.

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Chapter 1

Introduction

"It's only when the tide goes out that you learn who's been swimming naked."

- Warren Buffet

In late 2008, Bernie Madoff was apprehended for running an enormous Ponzi scheme that defrauded investors of almost \$20 billion.[9] His victims include charities, pension funds, and individuals. This financial fraud shook the world's faith in investment management and in financial regulators. The damage this Ponzi scheme and other financial frauds have caused is clear. What is not clear is the best way to prevent this type of fraud from occurring.

Unfortunately, the current regulatory mechanisms appear to be ineffective. Most of the large frauds we look at were exposed not by regulatory fervor, but by the financial crisis and the receding tide of investor sentiment.

The recently released Investigator General's report on the US Security and Exchange Commission (SEC) highlights several flaws with the current enforcement model.[21] This thesis focuses on one of these: flawed regulatory incentives. We argue that the SEC's apparent preoccupation with "stats", or the number of successful prosecutions, exacerbated the damage these financial frauds caused.

Our analysis uses an economic model of crime where a von Neumann-Morgenstern utility maximizing fraudster chooses the level of fraud to commit. Using this framework, we derive first socially optimal regulations and then regulations that maximize the performance metrics the SEC used. We show that regulators whose incentives are to maximize

the number of cases prosecuted or the value of cases prosecuted make decisions that are far from socially optimal. It is possible that a similar model exists; however, our model was developed independently.

Chapter 2 discusses several notable frauds exposed during the recent financial crisis. We focus on how misaligned incentives may have contributed to these frauds, arguing that the SEC was overly focused on the number of successful prosecutions.

Our model of fraud and regulation is outlined in detail in the third chapter. We first analyze the decision making process of a financial fraudster under an economic model of crime. We then show how a rational regulator would create regulations to combat this fraudster when faced with budgets or other constraints. This model is applied in Chapters 4 and 5 to highlight how the SEC's incentive structure was flawed.

Chapter 4 applies our model to derive optimal regulations. We derive closed form solutions for socially optimal regulations and for regulations set by a regulator who maximizes the number of cases successfully prosecuted. We contrast these two sets of regulations to show how a regulator who works to maximize this type of metric makes flawed decisions.

Additional structure is used in Chapter 5 to confirm our results. We repeat our analysis for regulators who are less free to set regulations. We show that our conclusions still hold under this more restrictive framework.

We conclude with recommendations on how best to measure regulator performance. Our model suggests that regulators need to focus not just on successfully prosecuting cases, but also on harm reduction and on deterring future fraud.

Chapter 2

An Overview of Financial Fraud

The recent financial crisis has exposed several large frauds and made regulatory authorities a focus of criticism. This section provides a brief overview of some of the notable frauds of the last several years, and of the regulatory failures that may have enabled them.

Ponzi schemes are the first type of fraud we examine. We then briefly discuss other frauds such as insider trading or the sale of fictitious securities. The model outlined in Chapter 3 is general enough to apply to a variety of frauds.

The final section of this chapter discusses regulatory failure. We highlight how the SEC's incentives were flawed. This section motivates the results in Chapters 4 and 5.

2.1   Ponzi Schemes, Recent and Historic

The recently discovered Ponzi scheme run by Bernie Madoff was the largest financial fraud of all time. A Ponzi scheme is a fraudulent investment scheme that offers attractive returns to investors, but provides these returns not from actual investments, but by paying out the principal of other investors. The promoter usually offers abnormally attractive returns to entice investors and ensure a steady stream of deposits. Fresh deposits are used to fund the promoter's lifestyle and any withdrawals from the scheme. In order to keep the fraud going, the promoter needs an ever-increasing inflow of new deposits.

Ponzi schemes are named after Charles Ponzi, a now-notorious Italian swindler.[7] In 1920, he started one of the best known of these eponymous swindles. Ponzi's scheme

Table 2.1: Recent Financial Frauds Valued Over $500 million

Perpetrator	Type	Downfall	Dollar Value ($ million)	Duration (years)
Madoff B	Ponzi Scheme	Liquidity Crisis	18,000	21
Stanford A	Ponzi Scheme	Regulators	8,000	14
Petters T	Ponzi Scheme	Informant	3,650	?
Nami K	Ponzi Scheme	Liquidity Crisis	2,530	6
Dreier M	Fictitious Securities	Regulators	700	4
Greenwood P and Walsh S	Stole Investors' Money	Liquidity Crisis	667	4

Table 2.1: This table summarizes several recent large scale financial frauds. Most frauds were brought down not by regulators, but by liquidity concerns, or in the case of the Petters fraud, a coconspirator confessing. These frauds all went on for long periods of time without detection. Note that the media often exaggerates the financial loss resulting from Ponzi schemes by using the nominal fraud value (the fictitious value promised to investors) instead of the smaller and more accurate measure of the amount invested. We use the amount invested, where available.

defrauded investors of $15 million — a princely sum at that time. However, the idea is older even than Ponzi. It is likely that he was inspired by William Miller, who in 1899 ran a similar scheme.[7]

As mentioned, Bernie Madoff provides a more recent example of a massive Ponzi scheme. This New York financier defrauded a total of $18 billion from investors ranging from charities to professional investment managers. This massive twenty-year fraud was uncovered not by a vigilant regulator, but by Madoff's own inability to fund investor withdrawals.[9] Even more damning are the repeated letters from Harry Markopolos, at that point an accountant working for another fund, explaining that Madoff's fund was a fraud.[16] Markopolos, a a gave detailed reasons for his view, and in retrospect the SEC looks extremely foolish for ignoring his warnings. Several other sources also provided warnings and suggestions that the Madoff fund was likely a Ponzi scheme, and yet the SEC repeatedly failed to act.

Several factors may have made it difficult for the SEC to act on the Madoff case. The investigative staff were "relatively inexperienced" and "confused about certain critical and fundamental aspects of Madoff's operations".[13] Clearly these are undesirable features. Madoff was also politically connected and noted for his philanthropy. He had an excellent reputation and had relatives connected to the SEC's upper management. A young investigator certainly would not want to attract Madoff's animosity, and this may have been part of the reason the SEC staff blindly accepted what the SEC Investigator General describes as "evasive or contradictory answers".[13]

Texas billionaire and cricket fanatic Allen Stanford started another major Ponzi scheme. Like Madoff, Stanford was highly connected, and even knighted in Antigua where the fraud was based.[15] The international scope of this fraud was one of the factors that delayed action. A top Antigua regulator has in fact been charged with complicity in the fraud. However, there were also numerous internal failures within the SEC that allowed this fraud to continue. The SEC was aware that Stanford was running a Ponzi scheme for almost twelve years, but failed to take meaningful action.[21]

Two other notable Ponzi schemes are those run by Tom Petters, worth $3.65 billion, and Nicholas Cosmo, $370 million.[17] Both had elaborate setups that made detection difficult. Petters used forged documents to secure loans, and like Madoff, defrauded hedge funds and several large companies. He was eventually turned in by coconspirators.[10] Cosmo

claimed to use proceeds for construction loans, but, in reality, his fund was just a Ponzi scheme. Cosmo ran an investment company and an insurance company that managed the supposed loans.[22]

On an international note, Japan also recently suffered from a massive Ponzi scheme. This scheme was started by Kazutsugi Nami and based on a phantom currency known as Enten. Losses from this fraud have been estimated to be \$2.53 billion.[27]

The success of Ponzi schemes is in some ways a failure of investors to conduct due diligence. This is an especially damning criticism for the funds of funds that invested with Madoff — what value could these highly compensated managers possibly be offering? However, it is interesting to note that several of these Ponzi schemes were investigated by the SEC, without action being taken. Some of these frauds then used their clean bill of health to assure investors of their legitimacy.[13]

It is a mathematical certainty that Ponzi schemes will eventually collapse. But what triggers this collapse? Most recently, the financial crisis caused several of these frauds to unwind. The receding tide of investor sentiment causes withdrawals to increase and new investments to decline.[2] This prevents the Ponzi scheme promoter from raising enough money to keep up with the withdrawals, which causes the scheme to collapse under its own weight. Ideally, regulators would detect Ponzi schemes and other frauds before they become massive and before they collapse. Unfortunately, these frauds were not a priority for the SEC before the Madoff case.[21]

2.2   Other Types of Fraud

In addition to Ponzi schemes, the recent financial crisis has brought to light frauds ranging from insider trading to the sale of fictitious securities. Although the dollar losses are not as large as those seen with the Madoff or Stanford Ponzi schemes, preventing these other frauds is also an important mission for regulators.

There are frauds based on deception that are similar to Ponzi schemes. For example, Marc Dreier was apprehended for selling \$700 million worth of fictitious securities.[28] Paul Greenwood and Stephen Walsh ran an investment fund and simply stole investors' money.[14] If these fraudsters had tried to meet their obligations using money defrauded from new investors, then these frauds would have become Ponzi schemes.

An entirely different class of fraud is insider trading. This often involves smaller numbers than Ponzi schemes, but can result in substantial economic loss.[26] A recent example of insider trading is the Du Jun case, where a Morgan Stanley managing director earned a $4 million profit from insider trading on the Hong Kong Stock Exchange.

The model we describe in Chapter 3 works just as well for these types of fraud. In fact, it can be applied to any type of crime where it can be argued that the perpetrator makes a rational decision about the magnitude of the crime he is planning.

2.3   Regulatory Failure

This thesis compares effective and ineffective regulators. Why would we think financial regulators are ineffective? Because of their failure to detect and act on frauds. Take, for example, the Madoff and Stanford Ponzi schemes, both of which went on for over a decade and caused billions of dollars of damage. Madoff was identified as a fraud in great detail, yet the SEC gave him only a cursory examination. Stanford was known to be a Ponzi scheme by the examination division of the SEC, yet the enforcement division did nothing because it had other priorities.

Much of our analysis here focuses on the SEC. We do this both because it is the preeminent regulator and because we have excellent information on its internal failures from the SEC Investigator General's reports on the Stanford and Madoff cases. Our findings may have a broader application, because although the SEC has been under fire recently, it is likely other regulators face similar problems. See for example the Enten controversy in Japan[27] or the Haitian cooperative crisis,[23] both of which grew to massive proportions before being shut down by regulators.

Several factors make it difficult to track down and expose large frauds. These frauds often have elaborate setups and are run by politically connected fraudsters. Some are based in countries where there is limited access to information. Further, it can be difficult for the regulators to get staff with enough experience in finance, as people with the necessary skill-set may not be interested in working for the government. However, our model suggests that part of the problem may also be a flawed incentive structure within the SEC.

One reason for the SEC's failure to prevent financial frauds was an excessive focus on "actions brought" — the number of successfully prosecuted cases.[21] The SEC Investigator

General's report on the Stanford case states that "novel or complex cases were disfavored" in favor of "quick-hit" cases. This type of performance metric treats big cases and small cases equally, and in fact discouraged investigation of larger, more complex cases such as the Stanford Ponzi scheme.

This "actions brought" measure has several advantages. It is simple and easy for those within and outside the agency to understand. It is objective, simple to measure, quantitative, and easily comparable. These features may have lead to its widespread use within the SEC. However, we will show that this type of performance measure creates perverse incentives for regulators. Under our model, it leads to larger frauds, ineffective regulations, and much higher levels of economic loss. The SEC's new Enforcement Director, Robert Khuzami, also feels the incentive structure is flawed. He has stated that he plans to "de-emphasize the current quantitative metrics used to evaluate personnel and programs — the number of cases opened and the number of cases filed — in favor of a more qualitative standard, which includes concepts like timeliness, programmatic significance, and deterrent effect of a case."[20] Our analysis suggests that this is precisely the correct course of action.

As mentioned, complexity makes it difficult to investigate large frauds. These frauds may be based internationally, as Stanford's was; or be in the form of a multinational, as Petters' was. This complexity also means that it is difficult and costly to collect and process the evidence needed to get a conviction on a large fraud case.

Enforcement is also complicated by the political connectedness of many of these fraudsters. Madoff was well known and respected, and his niece was connected to SEC (although the SEC Investigator General has stated this connection did not affect the investigation). Stanford was a huge figure in Antigua and was even knighted. Madoff and several other Ponzi scheme managers were noted philanthropists and connected to the political elite. Because of these connections, a junior investigator assigned to investigate Bernie Madoff or Allen Stanford might be wary about pushing too hard.

All of these forces push regulators away from focusing on larger frauds. However, we will show that a failure to actively focus on large frauds can have disastrous consequences.

Chapter 3

A Game Theoretic Model of Fraud and Regulation

This chapter develops an expected utility maximization model of financial fraud that we use throughout this thesis. Here we outline the model; in Chapters 4 and 5 we apply it.

We start with a discussion of regulatory regimes and how to model them. Instead of focusing on the detection of a specific type of fraud, we use a general model that is equally applicable to several types of fraud.

In the second section we build on Becker's economic model of crime to model how a rational fraudster makes decisions.[3]

After discussing the motives of fraudsters, we analyze how regulators might create effective regulations. We build up a model of how performance metrics impact the regulatory decision making process.

Finally, we derive a measure of the social cost of fraud. We will later use this to look at the social loss resulting from misaligned regulator incentives.

3.1 Modeling Regulatory Structures

Much of this thesis focuses on the design of optimal regulations. But how can we mathematically model a regulatory structure?

Regulations take effect both through overarching policy decisions and through the low-level decisions made by the investigation and enforcement agents. The choices about which cases to investigate and prosecute are based both on the evidence and on the regulators' priorities.

Several papers have analyzed how regulators should make the low level decisions about which funds to investigate. For example, Bollen and Pool analyze the past returns of legitimate and fraudulent funds and propose a set of simple flags to identify funds that might be fraudulent.[6] Bernard and Boyle look at the split-strike-conversion strategy Madoff purported to use and show the returns he claimed were impossible.[4] Although making these low-level decisions effectively is crucial to effective regulation, such decisions are not the focus of this thesis.

We take a different approach: instead of looking at how to identify fraud, we focus on the optimal relationship between fraud size and probability of apprehension. Doing so allows us to us to better model regulatory incentives; however, it ignores much of the complexity of regulatory practice.

How should we model the probability of the fraudster being apprehended? We assume that the probability of apprehension is some function of the amount of damage done by the fraud. This has an intuitive appeal. Consider an individual who is considering illegally trading on insider information. The rewards of such a crime increase as the amount of money traded increases; however, the probability that the fraud fails and is detected also increases.

Definition 3.1.1 We use the term **regulation** to denote a function that maps levels of frauds to probabilities of failure:

$$\psi : [0, \infty) \to [0, 1].$$

Using this notation, $\psi(x) = q$ denotes that a fraud with value $\$x$ has a q probability of failure. We assume that non-involvement in fraud will not result in punishment, and so $\psi(0) = 0$ as there is a zero probability being apprehended.

This form is general enough to be applied to a variety of types of fraud. However, it ignores the specific structures of regulations and of fraud. A more realistic model might better take these structures into account.

For example, an extended version of this model could better match the mathematical characteristics of Ponzi schemes and the regulations that detect them. Such a model could be inspired by a variety of sources. For instance, Bhattacharya looks at how the possibility of a government bailout can lead to Ponzi scheme formation even among informed investors.[5] Artzrouni takes an extremely mathematical approach and models the conditions under which a Ponzi scheme will grow large or fail quickly.[1] Páscoa and Seghir look at how default penalties can induce Ponzi-like structures.[24] Our analysis goes beyond these frameworks in order to provide more insight into regulations and financial frauds.

Note that we have, for now, ignored the probability and costs of Type I error. We will talk later about how to incorporate this cost. Also note that we have used the terms failure and detection interchangeably. We ignore the distinction between these two to simplify analysis.

3.2   Inside the Mind of a Criminal

What drives the decision-making process of a white collar criminal? This section attempts to answer that by using an expected utility maximization framework based on the economic model of crime pioneered by Gary Becker.[3]

We assume that a fraudster makes his decision to commit fraud rationally, as an expected utility maximizer. This assumption of rationality has an empirical justification. Empirical work suggests that those who commit violent crimes and petty crimes respond to increases in the probability of apprehension and punishment by committing fewer crimes.[29] It seems very likely that white collar criminals would do the same. Several of the fraudsters we discussed were lawyers or successful businessmen. These professionals are likely to be better able to objectively judge the risks and rewards of crime.

Under our framework, a potential financial fraudster is aware of the probability of success for different levels of fraud. He uses this information to make a decision about the amount of fraud to commit. Faced with some regulation ψ, the potential fraudster can choose either to remain honest, or to take some positive amount of money from investors. If the fraudster attempts to take \$$x$ from investors, he has a $\psi(x)$ chance of failure.

In line with the rational model of crime, we assume that the fraudster works to maximize his expected utility. We use $u_S(x)$ to denote the utility associated with a successful fraud

of value x. We write the utility associated with remaining honest as $u_S(0)$. We assume that the fraudster prefers a larger successful fraud to a smaller successful fraud, in other words that $u_S'(x) \geq 0$. This assumption is very plausible for an unsatiated criminal and it helps create the cost benefit tradeoff that we use in the economic model of crime.

We have discussed the utility resulting from a successful fraud, but it is clear that not every fraud is successful. Because we assume that the fraudster is aware of the likelihood of failure, he must also be aware of the penalties associated with being apprehended. These could include loss of reputation, isolation, fines, or even a prison term. Some form of punishment is a very real possibility and likely carries a substantial utility penalty. We use $u_F(x)$ to denote the utility associated with being apprehended while attempting to commit a fraud of value x. We make two assumptions about this punishment function. First, we assume that being caught and punished is undesirable and always worse than either a successful fraud or remaining uninvolved, so that

$$u_S(x) > u_F(y) \quad \forall x \geq 0, y > 0.$$

Additionally, we assume that being caught for a larger fraud is at least as undesirable as being caught for a smaller fraud. Again, this seems reasonable as prison sentences and other punishments appear to be an increasing function of fraud size. We write this as

$$u_F(x) \geq u_F(y) \quad \forall y \geq x > 0.$$

For convenience, we will often use a constant negative utility of $-p$ as the utility of being apprehended and punished, where p is some positive real constant. As we assume p is positive, the level of utility $-p$ is always negative. If we do not use constant utility for a failed fraud, we will usually use a utility function that is convex over fraud size. There are two simple justifications for this:

First, punishments do not directly scale with fraud size. Consider Bernie Madoff, who defrauded investors of \$18 billion and was sentenced to 150 years in prison. The 72-year-old financier is unlikely to serve 30 years of that sentence. If he had defrauded 'only' \$1 billion it is unlikely that his sentence would be a proportionally smaller 20 months. It is also unlikely that the damage to his reputation, assets, or status would scale proportionally.

Secondly, empirical work has suggested people are risk seeking over utility loss.[18][12] Thus, we feel justified in assuming that $u_F'' \leq 0$.

Given the knowledge of these two utility functions, we can write out the expected utility derived by a fraudster who chooses to commit a fraud worth \$$x$. This will allow us to model the fraudster's decision-making process. We write the expected utility as the sum of two parts:

1) The fraudster's utility if he is not caught, denoted $u_S(x)$, multiplied by the probability of not being caught, equal to $1 - \psi(x)$;

- plus -

2) The fraudster's utility if he is caught, equal to $u_F(x)$, multiplied by the probability of being caught, $\psi(x)$.

Thus the expected utility of a fraudster who chooses to commit a fraud worth x is equal to the following expression:

$$u_S(x) \times \big(1 - \psi(x)\big) + u_F(x) \times \psi(x). \tag{3.1}$$

We can model the fraudster's decision process using the above derivation of the fraudster's expected level of utility and our assumption that the fraudster is an expected utility maximizer. Thus we can determine how a fraudster will behave when faced with some given regulation. The fraudster's decision problem is to balance the benefits of larger frauds against the increased possibility of apprehension and perhaps increased punishment.

We write the fraudster's decision problem as maximizing (3.1) over the level of fraud. For a fixed regulation ψ, the fraudster would defraud \$$x$ that solves the following maximization problem

$$\max_{x \geq 0} \; u(x) \times \big(1 - \psi(x)\big) + u_F(x) \times \psi(x). \tag{3.2}$$

We use $x(\psi)$ to denote the solution to this maximization problem — the amount of fraud induced by regulation ψ. Note that this notation can be ambiguous as in some cases multiple values of x solve the maximization problem (3.2). We will work to avoid this ambiguity, but for convenience we assume that in ambiguous situations the fraudster chooses the lowest x value that solves the maximization problem.

As Becker notes, under an economic model of crime we can easily deter fraud by increasing the punishments for failed frauds.[3] However, in most developed countries crimi-

nal punishments for financial fraud are determined by the judiciary and not by regulators. As our primary focus is on large frauds how regulator's make decisions, we assume that punishments cannot be changed by the regulator.

Our model of fraudster decision making can be implemented in a straightforward manner. To illustrate this, we include the following example of how a potential fraudster might make decisions.

Example 3.2.1 This example shows how a potential fraudster might weigh the costs and benefits of committing a financial fraud. Consider an individual who is risk neutral over the gains from a successful frauds, $u_S(x) = x$, and faces a penalty of apprehension that is constant across fraud size $u_F(x) = -p < 0$. Suppose that this individual is faced with a regulation of the form $\psi(x) = 1 - \eta_0 e^{-\eta_1 x}$ for some $\eta_0 \in (0, 1]$ and $\eta_1 > 0$. This type of regulation has a simple form that ensures larger frauds are more likely to be detected.

Suppose this individual commits a fraud. Given this, we can use (3.2) to write the fraudster's utility maximization problem as

$$\max_{x>0} \; x \times \eta_0 e^{-\eta_1 x} - p \times \left(1 - \eta_0 e^{-\eta_1 x}\right).$$

This optimization problem shows the conflict between increasing monetary rewards and decreasing success probability. As the amount of fraud increases, the utility resulting from a successful fraud increases but so does the likelihood of failure. Using the first order condition, we can solve this problem and find the level of fraud that maximizes fraudster utility:

$$0 = \frac{\partial}{\partial x} x \times \eta_0 e^{-\eta_1 x} - p \times \left(1 - \eta_0 e^{-\eta_1 x}\right) = \left(1 - p\,\eta_1 - x\,\eta_1\right)\eta_0\; e^{-\eta_1 x}.$$

From this first order condition, we get that $x = \frac{1}{\eta_1} - p$.[1]

The above considers the optimal level of fraud if a fraud is committed, but what about the option of not committing fraud? Clearly $u_S(0) = 0$ and so the utility of not committing fraud is zero. We can compare this to the utility of committing a fraud.

From our solution and (3.1), we have the utility the fraudster derives from the optimal level of fraud is

$$\frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} - p.$$

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[1] A quick check of the signs of the derivatives shows that $\frac{\partial}{\partial x}\; x \times \eta_0 e^{-\eta_1 x} - p \times \left(1 - \eta_0 e^{-\eta_1 x}\right) = \left(1 - p\,\eta_1 - x\,\eta_1\right)\eta_0\; e^{-\eta_1 x}$ is positive for $x < \frac{1}{\eta_1} - p$ and negative for $x > \frac{1}{\eta_1} - p$, provided $\eta_0, \eta_1 > 0$.

If $\frac{1}{\eta_1} - p < 0$, then this utility level is less than 0 and so the interior solution is the preferred strategy for $\frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} > p$, and honesty is the preferred strategy for $\frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} \leq p$.

Using these derivations, we can write the fraudster's optimal strategy as follows:

$$
x(\psi) = \begin{cases} \frac{1}{\eta_1} - p & \text{for } \frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} > p \\ 0 & \text{otherwise.} \end{cases}
$$

Unfortunately, even with this stylized regulation structure, we are left with two cases. However, this solution has a simple and intuitive interpretation. The potential fraudster stays honest if the penalty is large enough that the fraud is not worthwhile, if $p \geq \frac{\eta_0}{\eta_1} e^{\eta_1 p - 1}$. If the potential fraudster does choose to commit a fraud, the amount of the fraud increases as the following parameters decrease: p, the penalty if the fraudster is apprehended, and η_1, the rate at which the likelihood of detection increases as the level of fraud increases.

We illustrate this decision problem with Figure 3.1.

The utility maximization framework outlined in this thesis is general enough to apply to several types of financial fraud. For Ponzi schemes, insider trading, and the sale of fictitious securities, there is a clear relationship between the amount of damage done and the amount of monetary gain for the perpetrator. Further, it is likely that the manager of one of these frauds has a good deal of control over the amount of money taken in. It is also reasonable to speculate that increasing the fraud size would bring additional exposure, which increases the risk of detection for this type of fraud.

3.3   Fraudsters and Regulators

A perfect regulatory structure would detect all fraud at no cost and without instituting costs on legitimate funds. Unfortunately, we are a long way from such a structure. The recent financial crisis has shown that it is extremely difficult to detect financial fraud. In this section we assume that a regulator sets a regulation ψ in order to minimize the damage caused by financial fraud or to maximize some other metric.

How can we design regulations that minimize the damage caused by financial fraud? As before, we look at regulations as simply functions that assign a certain possibility of

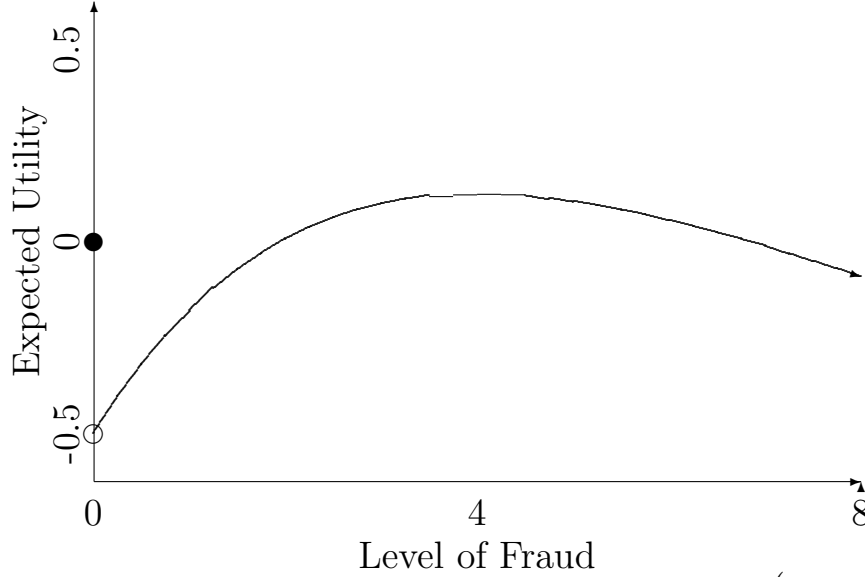Figure 3.1: The Fraudster's Utility Maximization Problem

Figure 3.1: This chart shows how a fraudster's expected utility $\big($as given by (3.1)$\big)$ varies with the amount of fraud committed for a sample parameterization. Low levels of fraud result in a low probability of being apprehended but yield low monetary rewards. Higher levels of fraud work in the opposite way. The above chart shows this problem using a probability of detection of $1 - \frac{1}{2}e^{-0.2 \times x}$ and a fraudster that is risk neutral over successful frauds but faces a constant penalty of apprehension. We characterize such a fraudster using the utility functions $u_S(x) = x$ and $u_F(x) = -1$ where x is the level of fraud. Under this, the fraudster's expected utility is zero if a fraud is not committed and $x \times \frac{1}{2}e^{-0.2 \times x} - (1 - \frac{1}{2}e^{-0.2 \times x})$ if a fraud is committed. By solving the first order condition, we get $x = 4$ as the level of fraud that maximizes the fraudster's expected utility.

capture to fraud of a certain value. Thus, under our framework the regulator's task is to determine the relative priority to be placed on large and small frauds. This setup lets us focus on the allegation we make in Section 2.3: that the SEC's incentive structure concentrated the agency's attention on the wrong types of fraud.

What types of damage measures might a regulator try to optimize? To provide full generality, we use two measures, one for failed (discovered) frauds and one for successful (undiscovered) frauds. We write v_S as a utility function that maps the value of a successful fraud (or rather, a fraud that does damage) to regulator utility; and v_F for a similar function that maps the value of an apprehended fraud to regulator utility.

We will consider a variety of plausible regulator utility functions. Much of Chapters 4 and 5 contrast a risk neutral regulator with a regulator who has skewed incentives. We do restrict our consideration to continuous functions in order to avoid technicalities in Chapter 4. For example, a regulator who myopically focuses on successfully apprehending as many fraudsters as possible, thus has $v_S(x) = 1$ and $v_F(x) = 0$. We will show that such a regulator produces regulations that are far from optimal, and yet this type of utility function is very close to what the SEC encouraged by its emphasis on the number of successful prosecutions.

Using the fraudster's decision process, we can write out the regulator's utility for a given regulation. As before, we use $x(\psi)$ to denote the amount of fraud that is committed under regulation ψ. From this, we write $v_F(x(\psi))$ to denote the regulator utility associated with a failed fraud of the amount induced by ψ. Similarly, we use $v_S(x(\psi))$ to denote the amount of regulator utility associated with a successful fraud of value $x(\psi)$.

Using this, we can write out the regulator's expected utility in a similar way to the previous section, as the sum of two parts:

1) The regulator's utility if a fraud of value $x(\psi)$ is not detected, $v_F(x(\psi))$, multiplied by $1 - \psi(x(\psi))$, the probability that a fraudster committing that amount of fraud is not caught;

- plus -

2) The regulator's utility if a fraudster committing a fraud of $x(\psi)$ is apprehended , $v_F(x(\psi))$, multiplied by the probability of that fraudster being apprehended, $\psi(x(\psi))$.

Using this, we can succinctly write the regulator's expected utility under regulation ψ

as

$$v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big). \tag{3.3}$$

If the regulator is rational and has perfect knowledge of the fraudster's decision-making process, it will work to maximize this expected utility. This is equivalent to solving the following optimization problem:

$$\max_{\psi \in A} v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big), \tag{3.4}$$

where A is some set of possible regulations. We will address what we consider plausible regulations in the next section.

We first note that we have assumed that the regulator has complete knowledge of the fraudster's decision-making process. This assumption is not entirely reasonable, but in Chapter 5 we will show how to relax it. If the fraudster's utility function is unknown, we can write the regulator's expected utility as

$$\mathbb{E}\left[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\right] \tag{3.5}$$

and the regulator's minimization problem as

$$\max_{\psi \in A} E\left[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\right] \tag{3.6}$$

where $x(\psi)$ is a random variable. This formulation will be useful later on.

We also note that we have assumed the fraudster is aware of the regulation that is in place and the associated detection probabilities. In reality, this may be far from true — one would hope potential criminals have less than perfect information about the behavior of regulators. However, we will continue to assume that fraudsters have an unbiased picture of detection probabilities. Since information about many of the regulatory actions is released, this is plausible. For example, the SEC releases information on its enforcement actions on its website at www.sec.gov.

An additional reason for assuming that the fraudster has unbiased information is that it is difficult to model other cases. For example, suppose the regulator can credibly state it will institute regulatory regime ψ_1 while actually following regime ψ_2. If the regulator can deceive fraudsters in such a manner, it is trivial to create regulatory regimes that both push fraud to an arbitrary level and also catch all frauds.

For example, consider a regulator that claims to institute a regulation

$$\psi_1(x) = \begin{cases} 1 & \text{for } x = y \\ 0 & \text{otherwise} \end{cases} \text{ but actually institutes } \psi_2(x) = 1 - \psi_1(x) = \begin{cases} 1 & \text{for } x \neq y \\ 0 & \text{otherwise} \end{cases}.$$

Unfortunately for would-be criminals, this strategy catches all financial frauds. It should also be close to costless, as legitimate cash flows are unlikely to take a value precisely equal to y. So although our assumption of fraudster knowledge is imperfect, it suffices for now.

This section described the regulator's decision-making process: maximize some metric based on the amount of failed and successful fraud that occurs, with the optimization occurring across some set of regulations A. The following section discusses what sets of regulations we consider and contains a brief example of the regulator's decision-making process.

3.4   Limitations to Regulation

In the previous section we discuss the optimization problem that the regulator faces, and note that it is reasonable to assume that not all regulatory structures are possible. In this section we will discuss possible constraints.

We will use two type of constraints on the regulator's optimization. The first type is used throughout, and is a budget constraint. The second type is a complexity constraint, and is used in Chapter 5.

We will usually define our budget constraints in the general form $c(\psi) \leq \kappa$ where we define c as follows:

Definition 3.4.1 We use c to denote the cost of a regulation. This maps from the set of regulations to the nonnegative real numbers

$$c : F((0, \infty), [0, 1]) \to [0, \infty) : \psi \to \int_0^\infty g(y, \psi(y)) dy$$

for some fixed function $g : \mathbb{R} \times \mathbb{R} \to [0, \infty)$.

The above structure allows for a large degree of flexibility, as g can be modified to suit a variety of structures.

We can implement intuitive notions of cost using this structure. For example, suppose that a regulator is looking for a small number of fraudulent cash flows among a large number n of legitimate cash flows.

Suppose that there are a large number of cash flows $x_1, \ldots, x_n$, some of which may be fraudulent. We will assume that checking any cash flow has a fixed cost of d and that a fraudulent cash flow will be detected if, and only if, it is checked. Thus, in order to detect an average of $\psi(x)$ of the fraudulent cash flows of level x, the regulator must check $\psi(x)$ of these cash flows. So the expected cost of a regulation that detects $\psi(x)$ of the fraudulent cash flows of value x will be equal to $\sum_{i=1}^{n} d \times \psi(x_i)$. Suppose n is large and x_i follow a distribution $f(x)$ for $i = 1, 2, \ldots, n$. Under any checking scheme that resulted in detection probabilities given by ψ, a randomly selected cash flow of value x would be checked with $\psi(x)$ probability. So the expected cost for a random cash flow is equal to $E[\psi(X)]$ where X has distribution f. But then the cost of checking all of the cash flows in this way equals $n \times d \times \int_0^\infty \psi(y) \times f(y) dy$. This form corresponds to $g(y, z) = z \times f(y)$.

We can easily extend this to a model where the cost of detecting fraud increases with fraud size. Suppose, for example, that the cost of checking a cash flow of size x is $j(x)$ for some $j : [0, \infty) \to [0, \infty)$. Then repeating the above analysis we get that the cost of regulation ψ is approximately $n \times d \times \int_0^\infty j(y) \times \psi(y) v f(y) dy$, so $g(y, z) = z \times j(y) \times f(y)$.

Another way to make the model my realistic is by using an increasing marginal cost of fraud detection, so that, for example, the cost of detecting 20% of frauds was more than twice the cost of detecting 10% of frauds. We could easily set up a g that followed this structure, for example setting $g(y, z) = z^\gamma \times \lambda e^{-\lambda y}$ for some $\gamma > 1$ would make it so that the cost of catching additional fraudsters would increase with the level of regulation already in place.

Note that we assume that g is increasing with respect to its second argument. This ensures that the cost of catching more fraudsters is higher than the cost of catching fewer. This assumption is extremely plausible and helps our analysis.

We could also use this structure to take into account the cost of Type I errors; that is the cost of prosecuting innocent people. This certainly has high social costs, but it also has direct costs to the regulator in the form of court fees and employee time. Such errors would

presumably be an increasing function of $\psi(y)$, and so this cost could also be captured by the increasing marginal cost setup described above.

In addition to budget constraints, we also consider constraints of scope. Given the way in which regulators operate, it would be impossible to create regulations of arbitrarily complex form. This follows both from the fact that it is impossible to know the precise scope of a fraud before the investigation is complete and that multiple layers of regulation and administration distort any incentive scheme.

To take this into account, in Chapter 5 we implement constraints that restrict our regulation types further. We consider only regulations of certain, parameterized forms. For example, we assume regulations must be of the form

$$\psi^{\eta_0, \eta_1}(x) = 1 - \eta_0 \times e^{-\eta_1 \times x}$$

for some $0 < \eta_0 \leq 1$ and $0 < \eta_1$.

This type of two parameter model is simple while still providing the regulator an extra degree of freedom. The regulator can control the relative amount of attention given to large and small frauds by manipulating the parameters. In the case above, a simple view of these two parameters is of basic checks and checks of exceptional events. The regulator strikes a balance between the level of scrutiny given to small potential frauds and the level of scrutiny given to larger potential frauds.

This structure also avoids the problem of deriving a closed-form solution for a truly optimal regulation. This is unfortunately beyond our reach in many cases, as it is a nonconvex optimization problem over a function space.[2]

To illustrate how these constraints affect the regulator's decision problem, we include another simple example.

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[2] We show by counterexample that the problem is nonconvex. Let $\mathbb{I}$ equal the indicator function and consider a world where the financial fraudsters have utility functions of the form $u_S(x) = x$ and $u_F(x) = -p$ where $p = 1$ is a penalty function. We will show that for the two regulations $\psi_1(x) = 0.8\mathbb{I}_{[x > 0]} + 0.2\mathbb{I}_{[x > 2]}$ and $\psi_2(x) = \mathbb{I}[x > 2]$ their convex combination, $\psi_c(x) = 0.4\mathbb{I}_{[x > 0]} + 0.6\mathbb{I}_{[x > 2]}$, results in less regulator utility than either regulation. Under ψ_1, the fraudster chooses not to commit fraud, for an expected loss of 0. Under ψ_2, the fraudster chooses $x = 2$ and is detected 0% of the time for an expected loss of 2. But under ψ_c, the fraudster chooses $x = 2$ and is detected 40% of the time, for an expected loss of $1.2 > \frac{0+2}{2}$. This shows that the regulator's optimization problem is nonconvex.

Example 3.4.2 This example builds on our previous example to show how a regulator might choose between different regulatory schemes. Suppose that the regulator is risk neutral and wants to minimize the losses from fraud, we use $v_S(x) = -x$ and $v_F(x) = -c \times x$ for some $c \in (0, 1)$. In the next section, we will argue that this type of regulation is socially optimal. Further assume a constant cost of checking cash flows and legitimate cash flows that are exponentially distributed with parameter λ. So, as we outlined above, we get $g(y, z) = z \times \lambda e^{-\lambda y}$ for our cost constraint. We also implement a constraint of scope, so that the regulator can implement only those regulations of the form $\psi(x) = 1 - \eta_0 \times e^{-\eta_1 x}$ for $\eta_0 \in [0, 1], \eta_1 > 0$. Again, this gives the regulator basic control over the balance between examining small and large frauds.

Now suppose, as in our previous example, that the fraudster is risk neutral over successful frauds, $u_S(x) = x$, and faces a constant penalty for failed frauds, $u_F(x) = -p < 0$. As shown in Example 3.2.1, when faced with a regulation of the form $\psi(x) = 1 - \eta_0 \times e^{-\eta_1 x}$ this type of fraudster chooses a level of fraud that depends on the model parameters:

$$x(\psi) = \begin{cases} \frac{1}{\eta_1} - p & \text{for } \frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} > p \\ 0 & \text{otherwise.} \end{cases}$$

Faced with such a fraudster, our socially optimal regulator would seek to solve its optimization problem (3.4):

$$\max_{\psi \in A} v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi)\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big).$$

If the regulator can set a regulation ψ such that $x(\psi) = 0$, then the regulator can induce the fraudster to remain honest. Clearly this is the best solution, as the regulator's utility will be zero, which is its least upper bound. The regulator can deter fraud if it can set η_0, η_1 such that $\frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} < p$ where $\psi^{\eta_0, \eta_1} \in A$. This will be impossible if the budget constraint is too low.

If it is not possible, then from (3.4) the regulator's problem is to solve

$$\max_{\psi^{\eta_0, \eta_1} \in A} -x(\psi^{\eta_0, \eta_1}) \times \left(1 - \psi^{\eta_0, \eta_1}\big(x(\psi^{\eta_0, \eta_1})\big)\right) - c \times x(\psi^{\eta_0, \eta_1}) \times \psi^{\eta_0, \eta_1}\big(x(\psi^{\eta_0, \eta_1})\big)$$

$$\Leftrightarrow \min_{\psi^{\eta_0, \eta_1} \in A} x(\psi^{\eta_0, \eta_1}) \times \left((1 + c) \times \left(1 - \psi^{\eta_0, \eta_1}\big(x(\psi^{\eta_0, \eta_1})\big)\right) + c\right)$$

$$\Leftrightarrow \min_{\psi^{\eta_0, \eta_1} \in A} \frac{1 - p\eta_1}{\eta_1} \times \big((1 + c) \times \eta_0 \times e^{p\eta_1 - 1} + c\big).$$

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The minimand is an increasing function of η_0. For any value of $\eta_1 > 0$ our minimand will be minimized by choosing the smallest value of η_0 that satisfies the budget constraint. We simplify the budget constraint:

$$A = \left\{ (\eta_0, \eta_1) \text{ such that } \kappa \geq \int_0^\infty (1 - \eta_0 e^{-\eta_1 y}) \lambda e^{-\lambda y} dy \text{ and } \eta_0 \in (0, 1], \eta_1 > 0 \right\}$$

$$= \left\{ (\eta_0, \eta_1) \text{ such that } \kappa \geq 1 - \frac{\eta_0 \lambda}{\eta_1 + \lambda} \text{ and } \eta_0 \in (0, 1], \eta_1 > 0 \right\}.$$

So from the budget constraint, we get

$$\kappa \geq 1 - \frac{\eta_0 \lambda}{\eta_1 + \lambda} \Rightarrow \eta_0 \geq \frac{(1 - \kappa)(\eta_1 + \lambda)}{\lambda} \text{ for } \eta_1 \leq \frac{\lambda \kappa}{1 - \kappa}$$

Thus, we recast our optimization problem in terms of η_1:

$$\min_{\eta_1 \leq \frac{\lambda \kappa}{1 - \kappa}} \frac{1 - p\eta_1}{\eta_1} \times \left((1 + c) \times \frac{(1 - \kappa)(\eta_1 + \lambda)}{\lambda} \times e^{p\eta_1 - 1} + c \right).$$

The partial derivative of the minimand is always negative:

$$\frac{\partial}{\partial \eta_1} \frac{1 - p\eta_1}{\eta_1} \times \left((1 + c) \times \frac{(1 - \kappa)(\eta_1 + \lambda)}{\lambda} \times e^{p\eta_1 - 1} + c \right)$$

$$= -\frac{1}{\eta_1^2} \left(c - (1 + c) \frac{(1 - \kappa)}{\lambda} \times e^{p\eta_1 - 1} \left(p^2 \eta_1^3 + \lambda(1 - \eta_1 p + p^2 \eta_1^2) \right) \right).$$

Thus, it is always optimal to set η_1 to its maximal value of $\frac{\lambda \kappa}{1 - \kappa}$ and η_0 to 1. This means that in this scenario it is socially optimal for the regulator to put a high weight on large frauds.

3.5   A True Measure of Loss

We have discussed how a regulator might behave when trying to maximize an arbitrary metric, but what are the true losses associated with a fraud of value $\$x$? We argue that these losses are proportional to the amount of fraud, and so it would be optimal for regulators to act as risk neutral fraud minimizers.

Clearly society's production is well diversified, and the losses from any one fraud, no matter how large, are unlikely to significantly change GDP or overall consumption levels. This suggests that loss is best measured in a risk neutral perspective: thus, the loss from two $100,000 frauds is equal to the loss from one $200,000 fraud. We can characterize such a risk neutral regulator using $v_S(x) = -x$ and $v_F(x) = -c \times x$, where c is a constant that allows us to change the relative value of discovered and undiscovered fraud.

As we have hinted, the next question is how to weight the loss from a discovered fraud versus an undiscovered fraud. We first assert that both undiscovered and discovered frauds do damage, and so $c > 0$. The next question is whether damage from a discovered fraud is less than the damage from an undiscovered fraud. We argue that undiscovered frauds are more damaging not only because they can continue to defraud investors, but also because investors who have been defrauded would prefer to discover their losses sooner rather than later. Thus a discovered fraud does less damage than an undiscovered fraud and should be weighted less. We have not included the cost of administering justice and investigating the fraud. These costs can be substantial, for example, Stanford's legal defense cost $20–30 million.[8] However, these costs will very likely be incurred eventually as most frauds are eventually discovered, and so a fraud remaining undiscovered does not avoid these costs, but only pushes them farther into the future. Therefore, we argue that $0 < c < 1$ - that a discovered fraud is better than an undiscovered fraud but worse than no fraud. We will use these assumptions for the remainder of this thesis.

In a similar manner to the previous section, we can write out the expected social cost associated with a given regulation:

1) The size of fraud induced by regulation ψ, $x(\psi)$, multiplied by $1 - \psi(x(\psi))$, the probability that a fraudster committing that amount of fraud is not caught;

- plus -

2) The size of fraud induced by ψ, $x(\psi)$, multiplied by the factor by which to discount frauds that are captured, c, multiplied by the probability of the fraudster being caught, $\psi(x(\psi))$.

Using this formulation, we write the expected social loss associated with regulation ψ as

$$- x(\psi) \times \big(1 - \psi(x(\psi))\big) - c \times x(\psi) \times \psi\big(x(\psi)\big). \tag{3.7}$$

Clearly this is just a special case of (3.3), and we can extend it in a similar way. For the regulator, there is a loss minimization problem associated with this function. We write this problem as

$$\max_{\psi \in A} \; -x(\psi) \times \big(1 - \psi(x(\psi))\big) - c \times x(\psi) \times \psi\big(x(\psi)\big). \tag{3.8}$$

Chapters 4 and 5 compare the regulations set by regulators looking to either minimize social loss or to maximize some skewed internal metric. This structure will allow us to analyze the costs of various inefficient performance measurement structures that could be applied to regulators.

One weakness of our approach was the our assumption of risk neutrality. This is reasonable for the US, but perhaps less so for smaller countries. For example, the Haitian cooperative crisis cost Haiti a huge portion of its GDP.[23] However, most of our analysis holds even without the assumption of risk neutrality. If social utility as a function of fraud is any decreasing function of $x \times \psi(x) + c \times x(\psi) \times \psi(x)$, then the socially optimal regulator faces the same maximization problem as (3.8).

Another question is how to discount fraud. In states of the world where the economy performs poorly, more frauds are likely to be discovered, as we have seen with the large number of frauds that came to light during the recent financial crisis. This would suggest that the cost of fraud is more than would be expected when looking at its real world probability. However, the money that is invested into fraudulent schemes is likely invested primarily in states of the world when the economy has performed well — irrational exuberance is the fuel of Ponzi schemes. This suggests a higher degree of discounting. The answer to this question is beyond the scope of this model; however we highlight this question as being worth further discussion.

Building on this, one interesting aspect of financial fraud is that it can be difficult to define what loss really means. For example, consider an investor who invests $10 into a Ponzi scheme or other financial fraud in 2005 and sees their reported balance increase to $30 by 2009. If the scheme collapses at that point and they receive $5 back, what was their loss? Do we use the fraudulent numbers and report the difference between the amount returned and the fictitious value, $25? Or do we ignore the opportunity cost of investing the money and use the difference between the initial investment and the amount returned, $5? In the Madoff case, investors lost not only all of the fictitious investment returns they believed they had earned, but also most of their initial investment.

We feel the preceding analysis is best understood by using the second definition of loss: the amount of cash taken in by the fraudster. There are two reasons for this. First, this is the view that has been taken by regulators.[11] Secondly, the cash lost by the investor is what the fraudster gains. The fraudster gains nothing directly by awarding fictitious returns, and that makes it more difficult to set up a structure around them. Using cash invested as a measure of loss simplifies the problem, as it reduces it in some ways to a simple minimax problem: the regulator works to minimize the loss, while the fraudster works to maximize that same loss. That is our cat-and-mouse game.

Chapter 4

Utility Based Regulations

This chapter applies the model of crime and regulation outlined in Chapter 3 in order to examine the effect of regulatory incentives. We show how a regulator operating under our model would make decisions about what regulations to implement. We use this to argue that flawed regulatory incentives lead to ineffective regulatory regimes.

Our first section derives optimal solutions for the regulator and fraudster described in Chapter 3 — we solve the cat and mouse game.

Next, we apply this model to fraudsters with constant relative risk aversion (CRRA). We compare the regulations intended to maximize the number of frauds apprehended with those designed to minimize social loss. Our results show that incentive schemes are ineffective when they only reward the number or dollar value of successful cases. These incentive schemes produce regulations that are ineffective at preventing large frauds.

The final section generalizes these results beyond the case of CRRA fraudsters. By making plausible assumptions about fraudster utility, we confirm our previous results that regulators focused on maximizing the number of cases brought will set ineffective regulatory schemes.

We find that a key component of effective regulations under this model is a "harm reduction" approach that discourages fraudsters from choosing to commit larger frauds. This approach is not practiced by regulators seeking to maximize discovered fraud. The SEC's focus on "actions brought" appears to create an incentive structure that suffers from this flaw.

The analysis in this chapter is based on regulators with perfect information. This assumption is relaxed in Chapter 5, where scope constraints are used to provide more general solutions.

4.1   Optimal Regulations Given A Budget Constraint

This section shows how a rational regulator would make the decision about what regulation to implement. We reduce this problem to optimizing across the levels of fraud and associated detection probabilities that are possible given the budget constraint. Theorem 4.1.1 and Corollary 4.1.2 show which regulatory outcomes are unattainable. Theorem 4.1.3 then shows how we can construct regulations that come arbitrarily close to those regulatory outcomes that are attainable. Finally, we show how a utility maximizing regulator would optimize across these attainable regulatory outcomes.

We first provide a lower bound on the cost of a regulation that induces a given level of fraud at a given detection likelihood:

Theorem 4.1.1 Fix a level of fraud $x^\star \in [0, \infty)$ and a detection rate $d \in [0, 1]$. Then any regulation ψ that induces $x^\star$ amount of fraud with a detection rate at least as high as d must have a cost satisfying the following lower bound:

$$c(\psi) \geq \int_0^\infty g\Big(y, \Big(\frac{u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d}{u_S(y) - u_F(y)}\Big)^+\Big) dy$$

where g is as in the cost function.

Thus, such a fraud level and detection rate pair $(x^\star, d)$ is not attainable with a budget constraint below this lower bound.

Proof

Let $U^\star$ equal the fraudster's maximum expected utility under the regulation ψ. From (3.1), we write this as

$$U^\star = u_S(x^\star) \times \big(1 - \psi(x^\star)\big) + u_F(x^\star) \times \psi(x^\star) = u_S(x^\star) \times (1 - d) + u_F(x^\star) \times d.$$

Now we know $x^\star$ is the solution to the fraudster's utility maximization problem (3.2). So we must have that

$$U^\star \geq u_S(y) \times \big(1 - \psi(y)\big) + u_F(y) \times \psi(y) \quad \forall y \geq 0.$$

If this inequality failed to hold for some $y \geq 0$, the fraudster would choose to defraud y instead of $x^\star$, which would contradict our assumptions.

We rewrite the inequality in order to better isolate $\psi(y)$:

$$U^\star \geq u_S(y) + \big(u_F(y) - u_S(y)\big) \times \psi(y) \quad \forall y \geq 0.$$

From our assumptions about fraudster utility, we know that $u_F(y) - u_S(y) \leq 0$. Thus, our inequality can again be rewritten, this time to

$$\psi(y) \geq \frac{u_S(y) - U^\star}{u_S(y) - u_F(y)}.$$

But this is sufficient to derive the lower bound on the cost. Using the definition of the cost function c and our assumption that g is monotonic nondecreasing in its second argument, we get that

$$c(\psi) = \int_0^\infty g(y, \psi(y)) dy \geq \int_0^\infty g\left(y, \frac{u_S(y) - U^\star}{u_S(y) - u_F(y)}\right) dy.$$

Expanding $U^\star$ completes the proof. ∎

The preceding theorem shows that some regulatory outcomes are unattainable without a sufficiently generous budget. Corollary 4.1.2 provides what is effectively the dual of this result: the unattainable regulatory outcomes for a given budget.

Corollary 4.1.2 For every budget constraint $\kappa \geq c(\psi)$, there exists a U_κ such that any regulation ψ satisfying the budget constraint must also satisfy

1. $x(\psi) \geq u_S^{-1}(U_\kappa)$; and

2. $\psi(x(\psi)) \leq \frac{u_S(x(\psi)) - U_\kappa}{u_S(x(\psi)) - u_F(x(\psi))}.$

We write this U_κ as $U_\kappa = \inf\left\{U \text{ such that } \kappa \geq \int_0^\infty g\left(y, \left(\frac{u_S(y)-U}{u_S(y)-u_F(y)}\right)^+\right)dy\right\}$. Note that U_κ is equivalent to the lowest expected utility the regulator can force the fraudster to accept, where the fraudster's expected utility is given by (3.1).

Proof

Consider a regulation ψ such that $\kappa \geq c(\psi)$. By Theorem 4.1.1 we have that

$$\kappa \geq c(\psi) \geq \int_0^\infty g\left(y, \left(\frac{u_S(y) - u_S(x(\psi)) + \big(u_S(x(\psi)) - u_F(x(\psi))\big) \times \psi(x(\psi))}{u_S(y) - u_F(y)}\right)^+\right)dy.$$

From this inequality and the infimum definition of U_κ it follows that

$$U_\kappa \leq u_S(x(\psi)) - \big(u_S(x(\psi)) - u_F(x(\psi))\big) \times \psi(x(\psi)).$$

This inequality must hold for all $y \geq 0$, as it failing to hold would contradict our assertion that U_κ is an infimum.

Statement 2 can be shown simply by reorganizing this inequality, while noting that $u_S(x(\psi)) - u_F(x(\psi) > 0$.

Statement 1 follows from the fact that $\psi \geq 0$. Because of this, we must have $U_\kappa \leq u_S(x(\psi))$ and thus $x \geq u_S^{-1}(U_\kappa)$ which is precisely our first statement. This inverse is well defined because of our assumption that u_S is an increasing function.

∎

This corollary shows what regulatory outcomes are unattainable. But what about the fraud level detection rate pairs that Corollary 4.1.2 does not show are unattainable? Theorem 4.1.3 shows that these other outcomes are effectively attainable, by explicitly constructing a sequence of regulations that comes arbitrarily close to the limits imposed by Corollary 4.1.2.

Theorem 4.1.3 Consider the following sequence of regulations

$$\phi_n^{x^\star,d}(y) = \left(\frac{u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\mathbb{I}[y = x^\star]}{u_S(y) - u_F(y)}\right)^+$$

for $n = 1, 2, 3, \ldots$ where $\mathbb{I}$ is the indicator function.

This sequence of regulations has the following three properties:

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1. Each regulation meets the cost lower bound in Theorem 4.1.1 for the given d and $x^\star$;

2. We have $x(\phi_n^{x^\star,d}) = x^\star$ for all n; and

3. As $n \to \infty$ we get $\phi_n^{x^\star,d}(x^\star) \to d$.

It is clear from Theorem 4.1.1 that this sequence is in a sense optimal. If a lower cost regulation induces $x^\star$ fraud, there must be some n such that the $\phi_n^{x^\star,d}$ induces the same level of fraud while providing a higher detection rate.

Proof We prove our assertions in order.

1. For our first statement, consider the cost of the regulation $\phi_n^{x^\star,d}$. We have that

$$c(\phi_n^{x^\star,d}) = \int_0^\infty g\big(y, \phi_n^{x^\star,d}(y)\big)$$

$$= \int_0^\infty g\Big(y, \Big(\frac{u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\mathbb{I}[y = x^\star]}{u_S(y) - u_F(y)}\Big)^+\Big)dy$$

$$= \int_0^\infty g\Big(y, \Big(\frac{u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d}{u_S(y) - u_F(y)}\Big)^+\Big)dy,$$

with the third equality holding because the set $[y = x^\star]$ has zero measure. This shows that the cost of $\phi_n^{x^\star,d}$ is precisely the cost lower bound.

2. Next, consider the level of fraud induced by $\phi_n^{x^\star,d}$. In order to show the level is equal to $x^\star$, from (3.2) we must show that

$$u_S(x^\star) \times \big(1 - \phi_n^{x^\star,d}(x^\star)\big) + u_F(x^\star) \times \phi_n^{x^\star,d}(x^\star) \geq u_S(y) \times \big(1 - \phi_n^{x^\star,d}(y)\big) + u_F(y) \times \phi_n^{x^\star,d}(y)$$

with the inequality holding strictly for $y < x^\star$.

We rewrite this inequality, expanding $\phi_n^{x^\star,d}$ and reorganizing:

$$\Big(u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d\Big)^+ \geq u_S(y) - u_S(x^\star) + \Big(\big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\Big)^+.$$

We then show our result using two cases:

- If $y > x^\star$, we have $u_S(y) > u_S(x^\star)$, and the inequality simplifies to

$$\big(u_S(x^\star) - u_F(x^\star)\big) \times d \geq \Big(\big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\Big)^+$$

which clearly holds as $u_S(x^\star) \geq u_F(x^\star)$.

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- If $y < x^\star$, we have $u_S(y) < u_S(x^\star)$ and the inequality holds strictly:

  For $u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d < 0$, we have

  $$0 > u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}$$

  $$\geq u_S(y) - u_S(x^\star) + \left(\big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\right)^+.$$

  For $u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d \geq 0$, we have

  $$u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d > u_S(y) - u_S(x^\star) + \left(\big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\right)^+$$

  $$\Leftrightarrow \big(u_S(x^\star) - u_F(x^\star)\big) \times d > \left(\big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\right)^+$$

  which holds as $u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d \geq 0$ implies $\big(u_S(x^\star) - u_F(x^\star)\big) \times d \geq 0$.

3. Finally, consider the behaviour of $\phi_n^{x^\star, d}(x^\star)$ as $n \to \infty$. We have

   $$\phi_n^{x^\star, d}(x^\star) = \left(d - \frac{1}{n\big(u_S(x^\star) - u_F(x^\star)\big)}\right)^+$$

   and so clearly as $n \to \infty$ we have $\phi_n^{x^\star, d}(x^\star) \to d$.

∎

We will use these results to construct optimal regulations. However, at this stage it is useful to summarize what we have shown. For a regulator with budget constraint κ, the following hold:

- The regulator cannot induce the fraudster to commit frauds with values below $u_S^{-1}(U_\kappa)$. This means that for every budget, there is a lower limit on the amount of fraud the regulator can induce.

- Detection rates above $\frac{u_S(x^\star) - U_\kappa}{u_S(x^\star) - u_F(x^\star)}$ are unattainable. Due to budget constraints, it is impossible to design regulations that catch all fraud.

- Detection rates arbitrarily close to $\frac{u_S(x^\star)-U_\kappa}{u_S(x^\star)-u_F(x^\star)}$ are attainable using a sequence of regulations $\phi_n^{x,d}$. This gives an explicit construction for any fraud level, detection rate pair that is not unattainable.

From formula (3.3), it is clear that the regulator's expected utility only depends on the regulation through the level of fraud induced and the associated detection rate. We will refer to fraud level, detection rate pairs as regulatory outcomes. Consider the space of all such regulatory outcomes, $(x^\star, d) \in [0, \infty) \times [0, 1]$. Corollary 4.1.2 showed that many of these outcomes are unattainable, while Theorem 4.1.3 allows us to get arbitrarily close to those outcomes that are attainable. We use these two results to partition the space of regulatory outcomes into those that are attainable and those that are not.

Figure 4.1 illustrates this partition. Attainable regulatory outcomes are fraud level, detection rate pairs $(x^\star, d)$ such that $x^\star > u_S^{-1}(U_\kappa)$ and $d \leq \frac{u_S(x^\star)-U_\kappa}{u_S(x^\star)-u_F(x^\star)}$. The regulator can construct regulations that are arbitrarily close to any of these outcomes, whereas other outcomes are unattainable.

For the purposes of finding an optimal regulation, only the curve of highest detection rates matters. We can see this by looking at the regulators expected utility function (3.3):

$$v_S\big(x(\psi)\big) - \big(v_S\big(x(\psi)\big) - v_F\big(x(\psi)\big)\big) \times \psi\big(x(\psi)\big).$$

As $v_S > v_F$, this expected utility is increasing in the detection rate. Thus, we need only consider those regulatory outcomes that maximize detection rate for the level of fraud they induce. This corresponds to the solid curve on Figure 4.1. This is intuitive: all else equal, the regulator would prefer detecting more fraud to detecting less fraud. The regulator may not be able to achieve all of the regulatory outcomes on that curve. However, as the regulator can induce a solution arbitrarily close to any of these points, this is not a concern (due to our assumption of continuous regulator utility functions).

So we will restrict our consideration to only those points on the border of attainable and unattainable. We can construct a series of regulations that deliver expected utility arbitrarily close to what these outcomes provide. The border between these two regions is the set of nonnegative frauds x such that $x \geq u_S^{-1}(U_\kappa)$ and the levels of detection $d = \frac{u_S(x)-U_\kappa}{u_S(x)-u_F(x)}$.
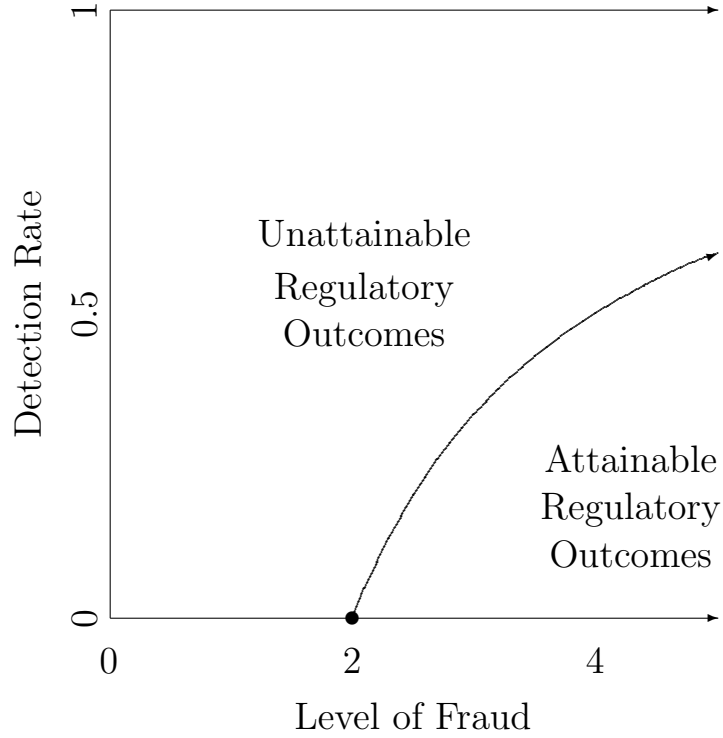
Figure 4.1: Attainable Regulatory Outcomes

Figure 4.1: Under our model, the regulator's expected utility depends only on the size of the fraud that occurs and the probability that the fraudster is apprehended. The above chart shows how we can divide the set of all possible (fraud level, detection rate) pairs into those that are attainable given the regulator's budget constraint, and those that are not. Attainable regulatory outcomes are those points (x, d) where the regulator can set a regulation ψ that induces x level of fraud with a detection rate arbitrarily close to d. Unattainable outcomes are where this is not possible. The depicted example is for a fraudster who is risk neutral over expected gains and perceives a constant penalty of apprehension, thus has $u_S(x) = x$ and $u_F(x) = -1$. We construct the regions using a regulator with enough budget to force the fraudster to accept 2 as an expected utility.

By considering only these points, we can rewrite the regulator's utility maximization problem as a simple search for the 'optimal' level of fraud. We recast equation (3.4) as

$$\max_{x \geq u_S^{-1}(U_\kappa) \text{ and } x \geq 0} v_S(x) - \big(v_S(x) - v_F(x)\big) \times \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)}, \tag{4.1}$$

where U_κ is as formulated in Corollary 4.1.2.

This problem can often be easily solved. After this solution is known, an optimal or nearly optimal regulation can be constructed as described as above. Namely, the sequence of regulations

$$\phi_n^{x^\star, d}(y) = \left(\frac{u_S(y) - u_S(x^\star) + \big(u_S(x^\star) - u_F(x^\star)\big) \times d - \frac{1}{n}\mathbb{I}[y = x^\star]}{u_S(y) - u_F(y)} \right)^+$$

for $n = 1, 2, 3, \ldots$ will approach the regulator's optimal utility. In the following section we will use this result to examine how skewed regulator incentives can result in suboptimal regulations.

4.2   A Constant Relative Risk Aversion Fraudster

This section contrasts effective and ineffective regulatory incentives by applying the results derived in Section 4.1. We consider a fraudster with constant relative risk aversion preferences.

We assume that the fraudster exhibits such risk aversion over both successful and failed frauds, so that $u_S(x) = x^\alpha \geq 0$ and $u_F(x) = -p - q \times x^\beta < 0$ for some $\alpha, \beta, p, q > 0$. This structure provides a good level of generality while remaining fairly simple in form.

The derivations of the previous section allow us to consider the regulator's maximization problem directly. From (4.1), we write this as

$$\max_{x \geq \sqrt[\alpha]{U_\kappa}} v_S(x) - \big(v_S(x) - v_F(x)\big) \times \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}.$$

What types of regulator utility function should we use? We first consider socially optimal regulations. In Section 3.5 we argue that a socially optimal regulator would have

utility functions of the form $v_S(x) = -x$ and $v_F(x) = -c \times x$ for $0 < c < 1$. Using this, we can rewrite our optimization problem as

$$\min_{x \geq \sqrt[\alpha]{U_\kappa}} \frac{x^{\alpha+1} + x \times \frac{p+(1-c)\times U_\kappa}{c}}{x^\alpha + q \times x^\beta + p}.$$

To shed some light on what an optimal solution might be, we consider the first derivative of the minimand:[1]

$$\frac{\partial}{\partial x} \frac{x^{\alpha+1} + x \times \frac{p+(1-c)\times U_\kappa}{c}}{x^\alpha + q \times x^\beta + p}$$

$$= \frac{x^{2\alpha} + q(\alpha+1-\beta) \times x^{\alpha+\beta} + (w(1-\alpha) + p(\alpha+1)) \times x^\alpha + w(1-\beta)q \times x^\beta + wp}{(x^\alpha + q \times x^\beta + p)^2}$$

If $\alpha, \beta \leq 1$ this derivative is always positive and so the social cost of fraud is minimized for $x = \sqrt[\alpha]{U_\kappa}$. But those parameter restrictions are very reasonable. A fraudster who is not risk seeking over successful fraud payoffs would have $\alpha \leq 1$, while $\beta \leq 1$ means that the fraudster is risk seeking over punishment. As we noted in Section 3.2, this is what we would expect for the fraudster's utility function, as empirical work suggests people tend to be risk averse over gains and risk seeking over losses.[18][12]

This analysis showed that a socially optimal regulator will set regulations that induce low levels of fraud. We derived a socially optimal realized fraud of $\sqrt[\alpha]{U_\kappa}$, coupled with very low detection rates.

What type of regulations would be set by a regulator incentivized to maximize the number of successful prosecutions? We characterize this type of regulator using $v_F(x) = 1$ and $v_S(x) = 0$ — a regulator who tries to maximize the number of successfully apprehended fraudsters. This regulator would solve

$$\max_{x \geq \sqrt[\alpha]{U_\kappa}} \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}.$$

We again look at the first partial derivative:

$$\frac{\partial}{\partial x} \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p} = \frac{(\alpha - \beta + U_\kappa \beta)q \times x^\beta + (U_\kappa + p)\alpha}{(x^\alpha + q \times x^\beta + p)^2} \times x^{\alpha-1}.$$

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[1]For simplicity, we write this using $w = \frac{p+(1-c)\times U_\kappa}{c}$. Note that w will be reused later in this thesis.

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This time we see that it is always positive if $\beta < \frac{\alpha}{1-U_\kappa}$. If this held, then the prosecutions-driven regulator would seek to maximize convictions at the expense of failing to deter large frauds. This is the opposite of our socially optimal regulation.

It is very plausible that β would satisfy this inequality. The inequality holds unless the fraudster is very risk averse over successful frauds and the regulator has a large budget (and thus a large U_κ). Excluding the case of well funded regulators and extremely risk averse criminals, we see that regulators are likely to induce an extremely high level of fraud.

The same conclusion holds for a regulator who cares only about the dollar value of frauds captured; or in fact, any regulator who cares only about an increasing function of the number of frauds captured and the dollar value of frauds captured.

Suppose $v_S(x) = g(x)$ for some increasing and positive g, and that $v_F(x) = 0$ again holds. A regulator characterized by these functions would solve

$$\max_{x \geq u_S^{-1}(U_\kappa)} g(x) \times \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}.$$

But the first derivative of this maximand is just

$$\frac{\partial}{\partial x} g(x) \times \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}$$

$$= g'(x) \times \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p} + g(x) \times \frac{\partial}{\partial x}\left[\frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}\right].$$

But this is again always positive for $\alpha > \beta$. We know the first product is non-negative because g is increasing and the detection rate cannot be negative. The second product must be positive because g is positive and $\frac{\partial}{\partial x}\left[1 - \frac{x^\alpha - U_\kappa}{x^\alpha + q \times x^\beta + p}\right]$ is also positive, as we have already shown.

Thus, the same results apply to regulators whose incentives are based only on the dollar value and number of frauds detected. Under our model, these types of regulators set ineffective regulations.

4.3   General Results on Misaligned Incentives

We can generalize the results from the previous section. The following two lemmas make general observations about the regulations set by regulators who care about either the

number of frauds detected or about the dollar value of frauds detected. We show that under reasonable assumptions about fraudster utility, these types of regulators set regulations that lead to very large frauds.

Lemma 4.3.1 Any regulator who only considers the number of successful prosecutions will set regulations that result in very high levels of frauds if any of the following statements about the fraudster's utility functions hold:

- u_F is bounded and u_S is unbounded; or

- $u_F < 0$, $u_S > 0$ and $\frac{u'_S(x)}{u_S(x)} > \frac{u'_F(x)}{u_F(x)}$.

Proof

From (4.1), we have that the regulator works to solve

$$\max_{x \geq u_S^{-1}(U_\kappa) \text{ and } x \geq 0} v_S(x) - \big(v_S(x) - v_F(x)\big) \times \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)}.$$

A regulator focused on maximizing the number of successful prosecutions can be characterized by $v_S(x) = 0$ and $v_F(x) = 1$. Thus, the maximization problem becomes

$$\max_{x \geq u_S^{-1}(U_\kappa) \text{ and } x \geq 0} \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)}.$$

If we have u_F is bounded and u_S is unbounded, then as $x \to \infty$ we get that the maximand approaches 1, which is the maximum value.

Looking at the first order condition,

$$\frac{\partial}{\partial x} \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)} = \frac{u'_S(x)}{u_S(x) - u_F(x)} - (u'_S(x) - u'_F(x)) \frac{u_S(x) - U_\kappa}{(u_S(x) - u_F(x))^2}$$

$$= \frac{-u'_S(x)u_F(x) + u'_F(x)u_S(x) + (u'_S(x) - u'_F(x))U_\kappa)}{(u_S(x) - u_F(x))^2}$$

$$= -\frac{\frac{u'_S(x)}{u_S(x)} - \frac{u'_F(x)}{u_F(x)}}{u_F(x)u_S(x)(u_S(x) - u_F(x))^2} + \frac{(u'_S(x) - u'_F(x))U_\kappa}{(u_S(x) - u_F(x))^2}$$

If our second condition holds, this partial derivative is always positive and thus the regulator's utility is maximized for the highest possible level of fraud. ∎

This lemma shows that a regulator who cares only about the number of frauds caught will set extremely poor regulations in several scenarios; namely, in the cases where punishments grow more quickly than rewards or where rewards are unbounded and punishments are not. Both these scenarios are very reasonable. Punishments are naturally bounded — under the American justice system the worst punishment that can be awarded for white collar crime is life imprisonment. On the other hand, rewards are effectively unbounded (although this may not translate into an unbounded utility function).

Clearly, the statements of Lemma 4.3.1 also hold for a regulator who seeks to maximize the dollar value of fraud discovered: as shown in Section 4.2, this type of regulator has even greater incentives towards higher amounts of fraud. However, we can also prove strong results for such a regulator. We provide these results in the following lemma.

Lemma 4.3.2 A regulator who only considers the dollar value of successful prosecutions will set regulations that result in very high levels of fraud if the fraudster is such that any of the following hold:

- u_F is bounded; or

- $u_F'' \leq 0$.

Proof

From (4.1), we have that the regulator works to solve

$$\max_{x \geq u_S^{-1}(U_\kappa) \text{ and } x \geq 0} v_S(x) - \big(v_S(x) - v_F(x)\big) \times \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)}.$$

A regulator that only considers the dollar value of successful cases has $v_S(x) = 0$ and $v_F(x) = x$, and thus the maximization problem becomes

$$\max_{x \geq u_S^{-1}(U_\kappa) \text{ and } x \geq 0} x \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)}.$$

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First suppose $u_F \geq -a$ for some $a \in \mathbb{R}$. Then we have

$$x \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)} \geq x \left(1 - \frac{U_\kappa - a}{u_S(x) - a} \right) \geq b \times x$$

for some constant $b > 0$. But this means by increasing x, we can increase the regulator's utility without bound. So the regulator will induce very high levels of fraud.

Looking at the first order condition,

$$\frac{\partial}{\partial x} x \frac{u_S(x) - U_\kappa}{u_S(x) - u_F(x)} = \frac{u_S(x) - U_\kappa + x u'_S(x)}{u_S(x) - u_F(x)} - x(u'_S(x) - u'_F(x)) \frac{u_S(x) - U_\kappa}{(u_S(x) - u_F(x))^2}$$

$$= \frac{(u_S(x) - u_F(x) - x u'_S(x) + x u'_F(x))(u_S(x) - U_\kappa) + (u_S(x) - u_F(x)) x u'_S(x)}{(u_S(x) - u_F(x))^2}$$

$$> (u_S(x) - u_F(x) + x u'_F(x)) \frac{(u_S(x) - U_\kappa)}{(u_S(x) - u_F(x))^2}$$

If $u''_F \leq 0$, this is never negative and thus the derivative is always positive. Again, it is reasonable to assume that this second derivative will be negative and that fraudsters will be risk seeking over failed frauds. If this holds, the regulator will maximize its utility by inducing very large amounts of fraud: a very undesirable outcome. ∎

These two lemmas suggest that aligning regulatory incentives to the dollar value or amount of fraud captured works spectacularly badly for some types of fraudster utility function. Such incentives place too little emphasis on minimizing damage, which means regulators focus on catching fraudsters and not on inducing fraudsters to commit less fraud.

Chapter 5

Structured Regulations

The previous chapter showed that with perfect information, regulators who are overly focused on catching fraudsters will design ineffective regulations. This chapter imposes additional restrictions on the structure of regulations. Doing so will allow us to apply our model to the case where the precise fraudster's utility function is not known.

In this chapter, we will assume that regulators can only set regulations that follow certain parameterized forms. This additional structure gets around two key weaknesses of the analysis in Chapter 4:

- The assumption that the regulator possesses precise knowledge of the fraudster utility function; and

- The assumption that regulations can take any form.

Again, we compare the regulations set by a regulator with skewed incentives to the regulations that are socially optimal; again we find that regulations set by a regulator who focuses on maximizing the number of successful prosecutions or the dollar amount of frauds apprehended are far from socially optimal. However, this chapter derives this result using constraints of scope and additional generality.

5.1 Uncertain Levels of Risk Aversion

This section considers how regulators might set regulations when faced with fraudsters with unknown constant relative risk aversion. We will again show that regulators who seek to maximize frauds detected have perverse incentives and set regulations with undesirable consequences.

Our results are similar to the ones we derive in Section 4.2. However, there are three key differences in our method:

- We assume an unknown level of fraudster utility;

- We do not consider penalty functions; and

- We constrain the set of possible regulations.

This exclusion of penalty functions is questionable, but needed to ensure tractable results. It is also applicable to in-progress Ponzi schemes, as in effect, where an increased chance of failure due to liquidity concerns is the penalty for failing to defraud a sufficient amount.

As we have stated, we consider a fraudster with power utility over successful frauds, and no penalty function: $u_S(x) = x^\alpha$ for some $\alpha > 0$ and $u_F(x) = 0$. We will assume that α is unknown to the regulator, but known to the fraudster.

We further assume that the regulator faces a budget constraint of $\kappa \geq \int_0^\infty \lambda e^{-\lambda y} \psi(y) dy$, but also faces a constraint of scope. We assume that the regulator sets regulations of the form

$$\psi^{\eta_0, \eta_1}(y) = 1 - \eta_0 \times e^{-\eta_1 \times y} \tag{5.1}$$

for $0 < \eta_0 \leq 1$ and $0 < \eta_1$.

This type of constraint gives the regulator power over the level of scrutiny to apply to large frauds versus small frauds. By increasing η_1, the regulator can focus more on larger frauds, by decreasing η_0, the regulator can shift the focus to smaller frauds.

Because the regulations are of a known form, we can write the fraudster's utility maximization problem in terms of the regulations. From (3.2) and (5.1), the fraudster's problem is

$$\max_{x \geq 0} x^\alpha \times \eta_0 \times e^{-\eta_1 \times x}.$$

Because there is no penalty of failure, it always makes sense for the fraudster to commit a fraud. So we can solve for the optimal value of this fraud using the first order condition:

$$0 = \frac{\partial}{\partial x} x^{\alpha} \times \eta_0 \times e^{-\eta_1 \times x} = (\alpha - \eta_1 x) \times x^{\alpha-1} \eta_0 e^{-\eta_1 \times x}.$$

Thus we get that $x = \frac{\alpha}{\eta_1}$ is the fraudster's optimal level of fraud.[1]

Consider a regulator trying to set an optimal regulation, while being uncertain about the fraudster's precise utility function. This regulator will solve equation (3.6),

$$\max_{\psi \in A} E\Big[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\Big].$$

We can substitute the parameterization for the regulation and our solutions for the level of fraud in the above expression:

$$\max_{\psi \in A} E\Big[v_S\big(\frac{\alpha}{\eta_1}\big) \times \eta_0 \times e^{-\alpha} + v_F\big(\frac{\alpha}{\eta_1}\big) \times (1 - \eta_0 \times e^{-\alpha})\Big].$$

As before, we characterize a socially optimal regulator by $v_S(x) = -x$ and $v_F(x) = -c \times x$. This type of regulation would solve

$$\max_{\psi \in A} E\Big[(c-1)\frac{\alpha}{\eta_1} \times \eta_0 \times e^{-\alpha} - c\big(\frac{\alpha}{\eta_1}\big)\Big] \Leftrightarrow \min_{\psi \in A} \frac{1}{\eta_1} \times \Big(\eta_0 + \frac{cE[\alpha]}{(1-c)E[\alpha e^{-\alpha}]}\Big)$$

To simplify notation, we write $w = \frac{cE[\alpha]}{(1-c)E[\alpha e^{-\alpha}]}$, noting that this will always be positive. We will reassign w for later problems. Using this notation, the problem becomes

$$\min_{\psi \in A} \frac{\eta_0 + w}{\eta_1}$$

To solve this problem, we need to consider the budget constraint, $\kappa \geq \int_0^{\infty} \lambda e^{-\lambda y} \psi(y) dy$. We can easily simplify this by solving the integral:

$$\kappa \geq \int_0^{\infty} \lambda e^{-\lambda y}(1 - \eta_0 e^{-\eta_1 y}) dy$$

$$\Leftrightarrow \kappa \geq 1 - \frac{\eta_0 \lambda}{\eta_1 + \lambda}.$$

---

[1] It is clear that the first derivative is positive only for $x < \frac{\alpha}{\eta_1}$.

As the minimand is a decreasing function of η_1, we can take this constraint as binding and write η_0 as a function of η_1.

$$\kappa \geq 1 - \frac{\eta_0 \lambda}{\eta_1 + \lambda} \Rightarrow \eta_0 = \frac{(1-\kappa)(\eta_1 + \lambda)}{\lambda} \text{ for } \eta_1 \leq \frac{\lambda}{(1-\kappa)} - \lambda$$

So we can rewrite the problem of minimizing social costs as

$$\min_{\eta_1 \leq \frac{\lambda}{(1-\kappa)} - \lambda} \frac{\eta_1 + \lambda(1 + w/(1-\kappa))}{\eta_1 \lambda}.$$

This is minimized for $\eta_1 = \frac{\lambda}{(1-\kappa)} - \lambda$ and $\eta_0 = 1$.

But what types of regulations might a regulator with a flawed incentive structure choose? Consider a regulator whose goal is to maximize some increasing function of the following two metrics:

- The dollar value of fraud captured; and

- The number of fraudsters apprehended.

This type of structure appears to be very close to the internal incentive structures used by a prominent regulator. We express the regulator's utility functions as $v_S(x) = 0$ and $v_F(x) = g(x)$ for a nonnegative and increasing g.

Consider the regulator's maximization (3.6):

$$\max_{\psi \in A} E\left[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big) \right].$$

$$\Leftrightarrow \max_{\psi \in A} E\left[g\big(x(\psi)\big) \times \psi\big(x(\psi)\big) \right]$$

$$\Leftrightarrow \max_{\psi \in A} E\left[g\big(\frac{\alpha}{\eta_1}\big) \times \big(1 - \eta_0 \times e^{-\alpha}\big) \right].$$

For any fixed α, the maximand is a decreasing function of both η_0 and η_1. Thus, the expectation must also be a decreasing function of these two parameters and our poorly incentivized regulator would minimize both η_1 and η_0. A quick check shows that such a regulation leads to large numbers of extremely large frauds.

5.2   Unknown Penalty Functions

This section applies the methods shown in the previous section to the case of unknown penalty functions. This section builds on Examples 3.2.1 and 3.4.2. We make assumptions similar to these examples:

- A fraudster who is risk neutral over successful frauds, $u_S(x) = x$, and faces a constant penalty of failure, $u_F(x) = -p < 0$;

- A regulator who has budget constraint of $\kappa \geq \int_0^\infty \lambda e^{-\lambda y} \psi(y) dy$; and

- A constraint of scope such that only regulations of the form

$$\psi^{\eta_0, \eta_1}(y) = 1 - \eta_0 \times e^{-\eta_1 \times y} \tag{5.2}$$

  for $0 \leq \eta_0 \leq 1$ and $0 < \eta_1$ are attainable.

We generalize beyond these examples by assuming that p is a random variable and providing more analysis.

From Examples 3.2.1, we know that the fraudster's utility maximization problem is

$$\max_{x \geq 0} x \times \eta_0 \times e^{-\eta_1 \times x} - p \times (1 - \eta_0 \times e^{-\eta_1 \times x}),$$

and that the solution to this problem is

$$x(\psi) = \begin{cases} \frac{1}{\eta_1} - p & \text{for } \frac{\eta_0}{\eta_1} e^{\eta_1 p - 1} > p \\ 0 & \text{otherwise.} \end{cases}$$

In Example 3.4.2, we simplified the budget constraint to $\kappa \geq 1 - \frac{\eta_0 \lambda}{\eta_1 + \lambda}$ and determined that for any known p a socially optimal regulator would set regulations that maximize η_1, and thus set $\eta_1 = \frac{\lambda \kappa}{1 - \kappa}$ and $\eta_0 = 1$ (provided the regulator's budget is sufficiently small). If this result holds for any p, then this result must also hold for the expectation maximization problem (3.6), provided that the regulator's budget κ is sufficiently small that the regulator cannot induce zero fraud with positive probability.

Consider a regulator incentivized to maximize the value or number of frauds successfully prosecuted. We will show that such a regulator will set regulations that are far from the socially optimal form.

Such a regulator would work to solve (3.6),

$$\max_{\psi \in A} E\Big[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\Big].$$

But what types of regulations might a regulator choose? We consider the same regulator as in the previous section, whose goal is the standard maximization problem with $v_S(x) = 0$, a zero loss from a undiscovered fraud, and $v_F(x) = g(x)$, a gain from each apprehended fraud that is positive and nondecreasing in fraud size. We have

$$\max_{\psi \in A} E\Big[v_S\big(x(\psi)\big) \times \big(1 - \psi(x(\psi))\big) + v_F\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\Big]$$

$$\Leftrightarrow \max_{\psi \in A} E\Big[g\big(x(\psi)\big) \times \psi\big(x(\psi)\big)\Big]$$

$$\Leftrightarrow \max_{\psi \in A} E\Big[g\big(\frac{1}{\eta_1} - p\big) \times \big(1 - \eta_0 e^{p\eta_1 - 1}\big)\Big]$$

This problem is similar to the previous section, as we get that lower values of both η_0 and η_1 are preferred by the regulator for any distribution of p. Thus the optimal solution is $\eta_0 = 1 - \kappa$ and $\eta_1 = 0$. This occurs when η_1 is minimized. But as we argued in the previous section, this leads to extremely large frauds and is far from optimal. Again, we have some generality in that this result holds for any bounded distribution of p, given a small enough κ.

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Chapter 6

Conclusion

Our analysis suggests that financial regulations should be designed not just to catch as many criminals as possible, but also to ensure that successful small-time criminals do not have an incentive to become big-time criminals. These regulatory structures have a "harm reduction" effect: by making larger frauds more risky and thus less desirable, they deter rational criminals from committing large frauds.

The performance measures used by the SEC appear to have been flawed and created incentives that contributed to the recent financial crisis. The incoming SEC Enforcement Director Robert Khuzami stated that the SEC should move away from these metrics and focus more on deterrence effect and significance. Our model agrees with this: such regulations might better minimize the damage financial fraud causes.

Note that our model has several major limitations: we focus primarily on the single period setting, whereas most frauds are repeated games; we greatly simplify the process through which regulatory actions are aggregated and transformed into apprehension likelihoods; we assume that white-collar criminals are rational and have fine control over the amount of fraud to commit; and we assume both fraudsters and regulators have unbiased information. However, even with these limitations and simplifying assumptions, we feel the model offers valuable insights. A further weakness of our model is our incomplete treatment of Type I errors - innocent people erroneously prosecuted. These errors have a high social cost, but in the interests of simplicity our model doesn't fully address this.

There are several interesting avenues for future research. It should be possible to derive

closed form solutions for optimal regulations with unspecified fraudster utility. This would have interesting implications for regulatory design.

Empirical validation of the model would be interesting but it would be very challenging to find an appropriate dataset. Because each country's financial regulators are centralized, cross-country data is the obvious choice; however, it would be plagued by cultural factors.

We hope that our research, along with better monitoring and enforcement, will help regulators become more effective at preventing financial fraud and the social costs that result.

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