A COOPERATIVE GAME-THEORY MODEL FOR BANDWIDTH ALLOCATION IN COMMUNITY MESH NETWORKS

by

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Author's Declaration For Electronic Submission of a Dissertation

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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ABSTRACT

Multi-hop wireless network are promising techniques in the field of wireless communication. The dynamic topology of the network and the independent selfish participants of the network make it difficult to be modeled by traditional tools. Game theory is one of the most powerful tools for such problems. However, most current works have certain limitations. There has not been a widely accepted solution for the problem yet.

In this thesis we propose our solutions for the problem of bandwidth sharing in wireless networks. We assume the nodes are rational, selfish, but not malicious, independent agents in the game. In our model, nodes are trying to send their data to the gateway. Some nodes may require others to forward their packets to successfully connect to the gateway. However, nodes are selfish and do not wish to help others. Therefore it is possible that some nodes may refuse the requirement. In that case, the unpleasant nodes may punish the others by slowing down their traffic, in which case both parties will suffer. Therefore it is non-trivial to find out the equilibrium for these nodes after the bargaining process. What is the proper distribution of resources among these nodes? We propose a solution based on the game theory. Our solution fulfills the goal of fairness and social-welfare maximization.

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1 INTRODUCTION

Wireless networks play an indispensable role in today's world. Multi-hop wireless networks, such as *ad hoc* networks, wireless mesh networks, and community mesh networks, have been studied since the 1970's. They became popular and received tremendous research interest recently. However, incentives for cooperation in *ad hoc* networks and fairness in wireless mesh networks are currently big problems. In wireless multi-hop networks, there is currently no widely-accepted technique to compensate users for their forwarding services [17]. Good economic models are desired to support the operation of these networks. We introduce the concept of the Raiffa Solution from game theory and propose a cooperative game model to study the behavior of nodes in multi-hop wireless networks.

1.1 MOTIVATION

Multi-hop wireless networks are promising techniques in computer science. In *ad hoc* wireless networks, the topology is not fixed as in traditional networks. New nodes may join in or leave a network at any time. All nodes may forward others' packets and also require other nodes to forward their own packets. The network works best when all the nodes are not selfish, but cooperate well with each other.

In *ad hoc* networks interesting problems arise from the fact that the participants do not necessarily have an incentive to cooperate with each other. The behavior of nodes is not defined in the protocol. Therefore the nodes are somehow similar to agents in a game situation. They are selfish, but not malicious; they are rational, but easily run into the situation of the "prisoner's dilemma", where each node hopes the other nodes will forward its packets while it does not forward packets from the other nodes. However, if every node acts in this way, the network would be non-existent, since no node would forward any other node's packets.

In wireless mesh networks we can force the nodes to cooperate. However, just like the difference of market economics and planned economics, the bandwidth schedule is usually unfair to the some of the nodes [13]. It causes fairness problems in wireless mesh networks, which received a lot of research interest recently.

We want to build a model that helps us understand the role of nodes in such a game, and reveals the equilibrium in such a game. These studies will also help us understand fairness from a new perspective.

1.2 CONTRIBUTIONS

In this thesis we propose our solutions for the bandwidth sharing problem. We assume the nodes are rational, selfish, but not malicious, independent agents in the game. Our model works when every node is trying to compete with other nodes for more bandwidth. The incentive for the nodes to forward others' packets is the fact that if they do not cooperate the other nodes may punish them by competing more stringently and they will get less bandwidth in this case. We modeled the idea of "cooperation while threatening" in this thesis.

We adopt the cooperative-game model to solve the problem. We first study the two-node game where we accept the solution proposed by Raiffa [16, 18, 21]. Then we claim that there are only two basic ways nodes can participate in the network, either completely competing or completely cooperating. We use the routing tree to represent the network topology. By treating a subtree as the same as a node in the game, we reduce the game to a two-player game, recursively. An algorithm to determine the appropriate bandwidth allocation among the nodes in the system is then proposed.

The solution works well for networks without concurrent transmissions. For larger networks that are not in a single collision domain, we adopt the method from Jakubczak et al [12] and propose a more-realistic solution for wireless mesh networks.

Our contributions include:

1. We formalize the two-node game and solve the game.
2. We generalize the model to solve multi-node games.
3. We simulate our approach using the *ns-2* simulator.
4. We find out that our solution leads to temporal fairness when the nodes are cooperating, subject to certain conditions which are non-trivial.

1.3 Thesis Structure

The rest of the thesis consists of the following chapters. In Chapter 2 we present a survey on the background and related work as well as introduce some basic knowledge of game theory which is most related to our work. In Chapter 3 we present our model in the order we studied the problems. In Chapter 4 we take interference range into account and discuss the simulation results. We present our conclusion and future work in Chapter 5.

2 Background and Related Work

In this chapter we review the basics of wireless networks and game theory. We show there are incentive and fairness problems in multi-hop wireless networks. We show why game theory is desired in wireless networks. We also introduce the related work in applying game theory in wireless networks.

2.1 Wireless Networks

Wireless networks consist of nodes that are not connected with wires or fibers, but communicate through radio signals. Wireless communications can be modeled by transmission range and interference range. The transmission range and interference range are usually from several meters to several kilometers. The transmission range is smaller than the interference range. If the receiver is within transmission range of the sender, the receiver can successfully receive the signal from the sender and decode the message. If the receiver is out of interference range of the sender, the receive can neither receive nor sense the signal from the sender. If the receiver is within interference range, but out of transmission range of the sender, it cannot receive the signal. However, it can sense the signal and the interference may cause it fail to receive from another sender.

Some of the nodes in the wireless network may be connected by wire to the Internet. We call such nodes gateways. Usually several nodes connect to one gateway to access the Internet, and form the many-to-one traffic.

2.1.1 MULTI-HOP WIRELESS NETWORKS

If two nodes are not within each other's transmission range, they cannot communicate directly. However, if there exists another node which is within the transmission range of both nodes and agrees to forward data for them, they can communicate with each other via the intermediate node. Sometimes the network flow may traverse multiple such intermediate nodes. We call such networks multi-hop wireless networks [1, 3].

In a multi-hop wireless network, nodes have to join the network to benefit from the network, while having the obligation to forward other nodes' packets. The problem is, because of energy and bandwidth limitations, nodes would not wish to forward these packets. Thus the problem arises: how to decide whether or not to forward data, and how to decide the proper portion of the received data that will be forwarded? A lot of interesting discussions and research arises from these problems.

2.1.2 *Ad-hoc* WIRELESS NETWORKS

Ad-hoc networks [22] are one example of multi-hop wireless networks. *Ad-hoc* networks are wireless networks without fixed infrastructure or centralized administration. Such networks are instantaneously formed when interested nodes come within each other's transmission range. *Ad-hoc* networks can be very useful in situations where there is no need for an infrastructure or where its creation would be too costly. Sometimes nodes in *ad-hoc* networks are powered by batteries and only participate in the network for a short time. The advantages of *ad-hoc* networks include: it is very fast to deploy the network; it is robust to changes; it is flexible; it allows nodes in the network with either high or low mobility, *etc.*

A lot of research has studied how to motivate nodes to cooperate with each other to make the network operate well. We will survey these works in the last section of this chapter.

2.1.3 Wireless Mesh Networks

Another example of multi-hop wireless networks is called Wireless Mesh Networks [3, 8]. These networks are composed of regular mesh nodes that act as both data sources/sinks and as routers, and gateway nodes that bridge traffic between the mesh and the wired network (usually the Internet) [4]. The traffic in a Wireless Mesh network is usually from one of the node to the gateway, or the reverse.

Generally there exists a single administrative authority in wireless mesh networks. Nodes are designed to work appropriately.

2.1.4 Community Mesh Networks

Neighbors connecting their home networks together with radios form a Community Mesh Network. When enough neighbors cooperate and forward each others packets, they do not need to individually install a gateway but instead can share Internet access via gateways that are distributed in their neighborhood. Packets dynamically find a route, hopping from one neighbor's node to another to reach the Internet through one of these gateways.

In our model, we assume low mobility, no power constraints, and no single administrative domain. Therefore, our model works best in the situation of community mesh networks.

2.2 Game Theory

Game theory is the mathematical study of the interaction among independent, self-interested agents. It has been applied to a wide range of fields including economics, political science, biology, psychology, linguistics, and computer science. This section introduces some basic knowledge of game theory, which will be referred to in the reminder of the thesis. Most of the contents of the sections come from the book of Von Neumann et al [16, 18].

2.2.1 Basic elements of a game

The basic elements of a game consist of the participants of the game, the action space of these participants, the consequences of these actions, and the preference (utility) of these participants.

2.2.1.1 Self-interested agents

The participants of a game are self-interested independent agents. "Self-interested" does not necessarily mean that agents want to cause harm to each other. Instead, it means that each agent has its own description of which states of the world it prefers, which can include good things happening to other agents and that it acts to make these states realized.

In multi-hop networks we discuss in this thesis, we assume the nodes are self-interested agents only caring for themselves.

2.2.1.2 Utility

Each agent may have different preferences for the same outcome of a game. Utility is the numerical value that represents the preference of the agents. For a fixed player P, and two outcomes M and N, the utility function $U()$ satisfies: $U(M) < U(N)$ if and only if P prefers N to M.

The *expected-utility hypothesis* is widely accepted in the field of game theory. The hypothesis asserts that when faced with uncertainty about which outcomes it will receive, the agent prefers outcomes that maximize its expected utility. If the utility function satisfies this hypothesis, then we say the utility function is linear.

The absolute value of the utility function evaluated at different outcomes is unimportant. Instead, every positive affine transformation of a utility function yields another utility function for the same agent. In other words, if $U(A)$ is a linear utility function for a given agent A then $U'(A) = aU(A) + b$ is also a linear utility function for the same agent, if a and b are constants and a is positive. Therefore, we can always perform a linear transformation on a utility function without changing the preference represented by that utility function. In many situations people linearly transform the utility functions such that the utility ranges from 0 to 1.

2.2.1.3 Action space

The action space is the set of actions an agent can take. In many situations each agent has exactly two actions that it can choose from. The smaller the action space is, the more such games have been studied. Games with an infinite action space are generally hard to analyze.

2.2.1.4 Outcome of a game

Once each agent chooses an action from the action space, there will be some outcome of the game. Sometimes there are several steps in each of which the agents have to take actions. In the view of game theory, the outcome can be expressed as an array of utilities of all the agents, which reflects the preferences of the agents to the outcome.

2.2.2 Two-player normal-form game

Two-player normal-form games are the most-studied games. In this case the game can be represented by a payoff matrix. In the matrix all four possible combinations of the agents' actions are shown, and the utility of the two agents in each outcome is given. Here we introduce some examples of two-player normal-form games.

2.2.2.1 Prisoner's dilemma

The most famous game in game theory is the prisoner's dilemma. The story is: suspect 1 and suspect 2 are arrested by the police. The police have insufficient evidence for a conviction, and, having separated both prisoners, visit each of them to offer the same deal: if one testifies for the prosecution against the other and the other remains silent, the betrayer goes free and the silent accomplice receives the full 10-year sentence. If both stay silent, both prisoners are sentenced to only six months in jail for a minor charge. If each betrays the other, each receives a two-year sentence. Each prisoner must make the choice of whether to betray the other or to remain silent. However, neither prisoner knows for sure what choice the other prisoner will make. This dilemma poses the question: how should the prisoners act?

The game may be represented by the payoff matrix shown in Table (2.1), where we assume the utility of each agent is simply zero minus the number of years in prison.

2.2.2.2 Battle of sexes

Imagine a couple, husband and wife. The husband prefers to go to the football game. The wife would like to go to the opera. Both would prefer to go to the same place rather than different ones. Where should they go?

The payoff matrix is shown in Table (2.2).

2.2.3 Pareto Efficiency

Pareto efficiency is usually a desirable requirement for any solution of a game. A strategy profile is said to be Pareto efficient if for any other strategy profile, there will be at least

		Suspect 2	
		Stays silent	Betrays
Suspect 1	Stays silent	$(-\frac{1}{2},-\frac{1}{2})$	(-10,0)
	Betrays	(0,-10)	(-2,-2)

Table 2.1: Prisoner's Dilemma

one agent with lower utility. The principle is that if some of the agents can get higher utility without harming other agents, they should. It is natural to expect Pareto efficiency in any solution.

2.2.4 Strategy

The strategy of an agent may be any action from the actions space, or a combination of them. We use the notation $(p_1A_1, p_2A_2, \ldots, p_nA_n)$, where $\sum_{k=1}^{n} p_n = 1$ to denote a strategy of an agent. The strategy means the agent would play action A_i with probability p_i, where $i = 1, 2, \ldots, n$. If one of these p_i's is 1, then it is called a pure strategy; otherwise, it is called a mixed strategy.

A strategy profile is an array of strategies of each agent in the game. For example, let the strategy for player $i \in 1, 2, \ldots, k$ be s_i, then $s = \{s_1, ..., s_k\}$ is a strategy profile. The solution of a game can be represented by a strategy profile.

		Wife	
		Football	Opera
Husband	Football	(2,1)	(0,0)
	Opera	(0,0)	(1,2)

Table 2.2: Battle of Sexes

The utility of a strategy profile is a vector of the expected utilities of all the agents when every one acts according to the strategy. *I.e.*, $u(s) = (u_1(s), u_2(s), \ldots, u_k(s))$ where $u_i(s)$ is the expected utility of agent i when agent j plays s_j, for all $j = 1, 2, \ldots, k$.

2.2.4.1 MAX-MIN STRATEGY

To ensure some certain level of safety, the straightforward strategy of a game is the "max-min" strategy, in which case the agent chooses its action from the action space such that it maximizes its worse-case payoff. The rational of this strategy is obvious: choosing any other action may lead to some outcome where the agent gets a lower utility.

2.2.4.2 DOMINATED STRATEGY

To define dominated strategy, we use the following notation. Given a strategy profile $s = \{s_1, ..., s_n\}$, we define $s_{-i} = \{s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n\}$, and $\{s_i, s_{-i}\} = s$.

Definition 1 (Dominated strategy) *For some agent i, if there exist two strategies* $s_i, s_i* \in S_i$ *such that* $u_i(s_i, s_{-i}) \leq u_i(s_i*, s_{-i})$ *for all strategies of the other agents* s_{-i} *and for at least one strategy* s_{-i}, $u_i(s_i, s_{-i}) < u_i(s_i*, s_{-i})$ *, then we say the strategy* s_i *is dominated by* s_i**. If* $u_i(s_i, s_{-i}) < u_i(s_i*, s_{-i})$ *for all strategies of the other agents* s_{-i}*, then we say the strategy* s_i *is strictly dominated by* s_i**.*

The dominated strategy should not be used.

2.2.5 Nash Equilibrium

The Nash Equilibrium is a strategy profile such that no player has anything to gain by changing only his or her own strategy unilaterally. It is the likely outcome of a game if agents are non-cooperative; *i.e.*, they do not communicate with each other and choose the actions by themselves. The formal definition is constructed as follows:

Definition 2 (Best response to a strategy profile) *A best response of Player* i *to the strategy profile* s_{-i} *is a mixed strategy* $s_i* \in S_i$ *such that* $u_i(s_i*, s_{-i}) \geq u_i(s_i, s_{-i})$ *for all strategies* $s_i \in S_i$*.*

The best response may not be unique.

Definition 3 (Nash Equilibrium) *A strategy profile* $s = (s_1, \ldots, s_n)$ *is a Nash equilibrium if, for all agents* i, s_i *is a best response to* s_{-i}*.*

In other words the Nash Equilibrium is a strategy profile where no agent can benefit by playing any other strategy if the other agents do not change their strategy. Therefore, the Nash Equilibrium is a stable outcome of the game. However, since the best response may not be unique, the Nash Equilibrium may not be unique either.

Sometimes we can find the Nash Equilibrium of a game by deleting the dominated strategies from the Payoff matrix. For example, recall the game of Prisoner's Dilemma illustrated in Table (2.1). "Stay Silent" is dominated by "Betray" for both A and B. If we delete the outcomes related to "Stay Silent" of both A and B, there will be only one outcome left, which is the Nash Equilibrium of the game: both agents betray.

2.2.6 Cooperative Game

In a cooperative game the agents can communicate with each other and take actions after they have an agreement. The communication makes some cooperative strategy feasible. For example, assume there are two agents, 1 and 2. The action space of 1 is A_1, A_2, and the action space of 2 is B_1, B_2. Then a cooperative strategy may be $\{p_1(A_1, B_1), p_2(A_2, B_2)\}$, which means with probability p_1, agent 1 plays A_1 and agent 2 plays B_1; with probability p_2, agent 1 plays A_2 and agent 2 plays B_2.

The above cooperative strategy is impossible to carried out unless the agents can communicate with each other and agree to cooperate. We will show examples where the cooperative strategy performs much better than either the Nash Equilibrium or the max-min strategy in a cooperative game.

In the game of "Prisoner's dilemma", as illustrated in Table 2.1, one easily identifies that both the Nash Equilibrium and the max-min strategy lead to the outcome $\{$Betrays, Betrays$\}$. The utility of both agents will be -2. On the other hand, the cooperative strategy is $\{$1(Stay silent, Stay silent)$\}$, where both agents get a utility of -0.5. However, it is impossible to reach to the cooperative optimal unless the agents can communicate and there exists some way to enforce the cooperative strategies.

In the game "Battle of sexes", there are three Nash Equilibria $\{$Football, Football$\}$ (where the husband gets utility 2 and the wife gets utility 1), $\{$Opera, Opera$\}$ (where the husband gets utility 1 and the wife gets utility 2) and $\{$(0.75 Football, 0.25 Opera), (0.25 Football, 0.75 Opera)$\}$ (where both get expected utility 0.75). However, none of these are good solutions for the game. Instead, in the case of a cooperative game where agents can communicate, we have the optimal solution $\{0.5(Football, Football),$ $0.5(Opera, Opera)\}$, where both agents get expected utility 1.5.

The cooperative solution is difficult to implement unless the agents can communicate with each other and the game is repeated for many times. However, it is usually reasonable to assume the availability of communication and repetition, especially in a wireless network. In general, cooperative solutions may be much better than non-cooperative ones.

2.2.7 Raiffa Solution

In the thesis we adopt the Raiffa solution to find the outcome in a cooperative game. We introduce it in this section.

For two player games we can use a graph to help us understand the idea of Raiffa solution. For any strategy profile $s = (s_A, s_B)$, we will have an outcome of the game and the utility is $(U(A), U(B))$. Since it is a cooperative game with mixed strategies, if we plot all possible outcome utility points $(U(A), U(B))$ in a graph, we have a convex set as shown in Figure 2.1

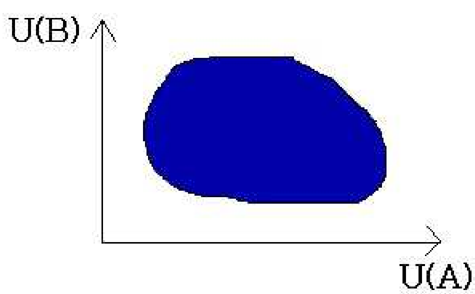


Figure 2.1: All possible outcome of the game

However, not all of these outcomes are possible in a game. The agents will not adopt dominated strategies; using max-min reasoning they can also guarantee themselves some minimal level of utility. We call the utility in the max-min strategy of an agent the security level. Only the outcomes that equal or exceed the security-level point are possible, as illustrated in Figure 2.2

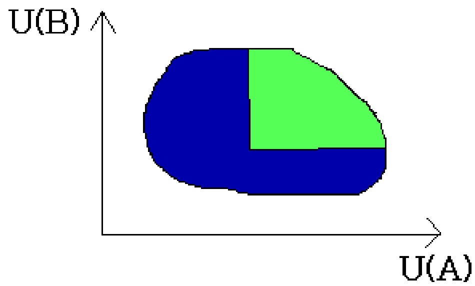


Figure 2.2: Security Level: only the grey area is feasible

By Pareto efficiency, the outcomes on the boundary form the optimal sets, as depicted in Figure 2.3; the points not on the boundary are always dominated by some other point on the boundary. Therefore, a cooperative solution should be on the black line in Figure 2.4.

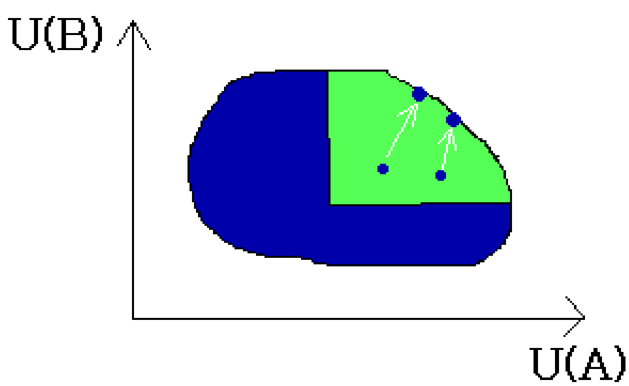


Figure 2.3: Any outcome not on the boundary is not preferable

Adding the last constraint will lead to the Raiffa solution. Raiffa suggests the reasonable solution should be on the 45-degree line starting from the security-level point. In

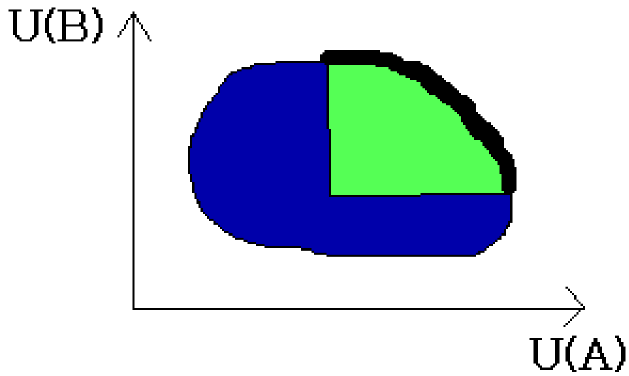


Figure 2.4: Pareto efficiency

other words, the difference between the security level of the two agents should be maintained in the solution. As illustrated in Figure 2.5, any point not on the 45-degree line, like the white points, will cause unfairness. The consequence of unfairness is that agent A may threaten B that it will terminate the cooperation, in which case the new equilibrium points will be the security level, and therefore agent B loses more utility than A does.

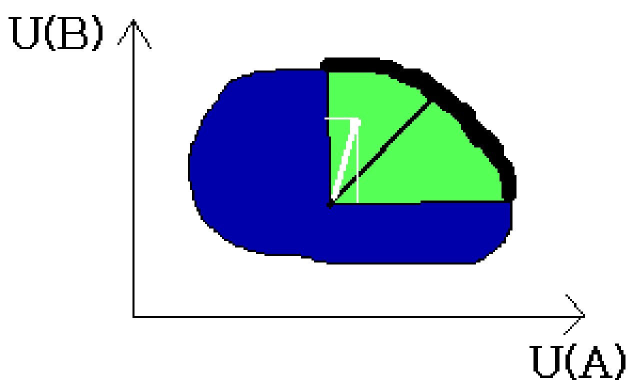


Figure 2.5: 45-degree fair line

The above reasoning suggests that the Raiffa solution is the ideal solution concept for cooperative games. We adopt the solution idea in our studies.

2.3 GAME THEORY IN WIRELESS NETWORKS

There is a lot of research studying wireless networks from the game-theory view recently (*e.g.*, see [2, 5]), especially for *ad hoc* wireless networks. It is not surprising that many people try to explain the behavior of nodes in *ad hoc* networks as a game. The topology structure of *ad-hoc* networks is extremely dynamic; nodes may join or leave the network arbitrarily according to their own interest. Even if a node stays in the network, it may decide on its own to forward the packets of other nodes or not. Therefore, *ad hoc* networks form a typical situation where a game is played between rational agents.

Most such research assumes that the nodes are selfish, but not malicious, independent agents in a game. Most game-theory-based approaches falls in two categories: reputation mechanism and virtual-currency mechanism.

The reputation mechanism assumes that each node should forward all the packets and nodes can monitor the behavior of its neighborhood. Any misbehavior will be reported, and any node that does not forward packets will not be able to get its own packets forwarded in the future [26]. However, this mechanism does not tell us how many packets should be forwarded; it assumes all packets should be forwarded. Our focus is on what the reasonable expectation is of packets to be forwarded.

The virtual-currency mechanism assumes there exists some form of virtual currency in the networks such that nodes can earn money by forwarding packets or make other contributions, and need to pay money to get their own packets sent [11]. This method, along with the well-studied VCG [24] payment mechanism, seems to be a good solution for the problem. However, it may be very complex.

Both mechanism are promising ways. However, it may not be easy to implement reputation or banking services in the network. In this thesis, the factor we are concerned with is the proper proportion of bandwidth that is allocated for these nodes. Therefore, we are studying the problem from a different aspect from these works.

2.3.1 Energy Constraints

The bottleneck for most nodes in *ad hoc* networks may be energy constraints. It costs energy for nodes to act as routers for others. For mobile nodes, energy may be a very limited resource and usually should be reserved for the node itself.

A lot of research starts from the energy constraint (*e.g.*, [25]). However, power is not a major constraint for nodes in community wireless networks. In this thesis we will not take energy constraints into account. Rather, we focus on throughput.

2.3.2 Nash Equilibrium

A lot of research models the network as a non-cooperative game (*e.g.*, [19]). Each node tries to maximize its own utility. It is widely accepted that the Nash Equilibrium is usually the outcome of such a game. However, we do not prefer the Nash Equilbirium because of the following reasons:

1. Sometimes the Nash Equilibrium is far from optimal, as in the famous game "Prisonner's Dilemma".

2. We prefer to model the network as a cooperative game, in which case agents can improve each other's utility by wise cooperation which cannot be taken in the Nash Equilibrium of a non-cooperative game.

3. It is still an open problem to find the equilibrium point in an efficient way when there are a lot of agents in the game. It is now known that finding a Nash Equilibrium with even 2-players belongs to PPAD which is thought to be harder than P.

Moreover, finding a Nash Equilibrium that max social welfare is NP hard. In fact, most research in this category differ in their ways to find the Nash equilibrium, or in their mechanisms whose outcome will converge to the Nash equilibrium with high probability.

2.3.3 Maximize the Aggregate Utility

Some research assigns the resources among agents such that the sum of the utility of all nodes is maximized [6, 9]. However, it is unclear whether this assignment will be advocated by the nodes or not. Not surprisingly, in many cases maximizing social welfare means some nodes have to sacrifice. Since each node is an independent agent, we do not think it will accept the aggregate-utility maximizing assignment if it conflicts with the node's own interest. If there exists any node which can improve its utility if it plays some other strategy, the cooperation has to be unstable. Instead, in both the Nash equilibrium and our solution, nodes are not supposed to sacrifice for aggregate utility.

2.4 Fairness in Wireless Mesh Networks

In wireless mesh networks, operators enforce cooperation through predefined protocols or programs. For example, gateway control [13] controls the resource distribution in wireless mesh networks such that every node get a fair (as pursued in the work) throughput.

Ad hoc networks and community mesh networks are more suitable to be modeled as a game, since the decision of forwarding packets is made individually based on the individual interest of the nodes. On the other hand, in wireless mesh networks where nodes do not have the right to make a decision the incentive problem appears in the form of fairness. After the network operator has made a policy decision, people would ask: is this a fair resource assignment? [20]

Current wireless mesh networks based on the IEEE 802.11 MAC and standard network-layer protocols cannot provide fairness to each node in the network. In particular, it has been demonstrated that nodes close to the gateway can starve or even shut off those that are more hops away without rate-control mechanisms [13]. However, it is not self-evident that different nodes having different bandwidth is unfair. We have to note the fact that different nodes in the network are indeed not symmetric and some may contribute much more for the network. A very deliberate design decision has to be made to advocate for any "fair" schedule.

We study the problem in this way: assume the nodes are free agents as in other multi-hop wireless networks, then find the cooperative outcome of the game, and compare it with existing fairness conceptions.

Before we start our analysis, we show some well-known definitions of fairness below. Some forms of fairness are with respect to cost, and some others are with respect to outcome.

2.4.1 Absolute Fairness

Some people referring to absolute fairness require fairness with respect to the outcome. Under absolute fairness with respect to outcome, the rates are equally distributed between all the streams. All the nodes in the network get the same throughput.

2.4.2 Max-min Fairness

Assume each node get a fair share of throughput defined by absolute fairness. Sometimes the network topology is such that a few nodes can improve their throughput without any other node's throughput decreasing. Therefore, it is not necessary to insist that all nodes should get the same throughput.

The definition of max-min fairness [20] assumes a single bottleneck. All nodes that are limited by the bottleneck get equal share of the bottleneck link. Other nodes can get higher throughput.

The max-min fairness concept is consistent with the idea of Pareto efficiency. People observe that in addition to absolute fairness, some nodes in the network can get higher throughput without reducing the performance of the others. Therefore, people introduce max-min fairness to maximize the overall throughput, while providing basic fairness guarantees.

2.4.3 Proportional Fairness

An allocation x is said to be proportionally fair if for any other feasible allocation x', the aggregate of the proportional change is 0 or negative, *i.e.*,

$$\sum_{i \in I} \frac{x'_i - x_i}{x_i} \leq 0 \tag{2.1}$$

Kelly [14] showed that if the utility function is logarithmic to throughput, and the fairness goal is to maximize the sum of utility of all the nodes, then we reach proportional fairness.

2.4.4 Temporal Fairness

The link capacity of different links in the network may be quite different. It is not necessarily fair to have a 55Mbps link have the same throughput as another link which is only 1Mbps. Temporal fairness [10] considers time, instead of throughput as the resource to be fairly distributed. In temporal fairness, each stream takes the same amount of spectrum time to arrive at the gateway, subject to the max-min limitations.

In this thesis, we assume the nodes in the wireless network can make independent decisions as in *ad hoc* networks and community wireless networks. Then we find the Raiffa equilibrium of the game, and advocate the outcome to be the fair share. We show that if the utility function is the throughput, the security level of each node is 0, and the fairness goal is to realize the Raiffa equilibrium, then we reach the temporal fairness.

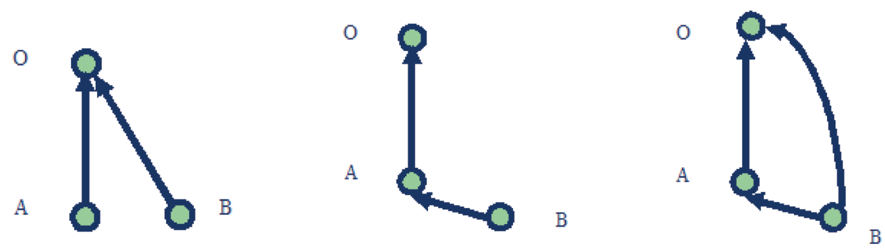
3 Games in Many-to-One Routing

In this chapter, we focus on those problems where there are many nodes but only one gateway in the network. As observed by Cheng et al. [7], the network topology will form a confluent tree. We also make the assumption that no concurrent transmitting is allowed in such networks. This assumption is true if the network is small and all links are within each other's interference range. On the other hand, it may be far from real life when applied to larger networks. However, making these assumptions makes things easier initially, so we can focus on the game-theory view; this restriction will be removed in Chapter 4.

This chapter is organized as follows: first, we study some representative specific examples of the simplest scenario with two nodes; we then apply our solution to general two-node games; finally, with induction and recursion, we determine the solution for the general case.

3.1 Two-Node Game: Numerical Scenario

The simplest game happens between two nodes. Let us assume there are two nodes, denoted by A and B, both trying to connect to the gateway O. Depending on the available links, there are three potential scenarios, as illustrated in Fig.3.1. In Fig. 3.1(a), both A and B can access O directly, while they cannot communicate with each other. In this case, the interests of A and B are incompatible. They compete with each other for bandwidth. In Fig. 3.1(b), A can access O directly, but B can only access O indirectly if A agrees to forward B's packets. In this case B has to cooperate with A to access O. In Fig. 3.1(c), both A and B can access O directly, while B can also send packets to A and ask A to forward them. Thus, they may be either competing or cooperating with the other party.



(a) (b) (c)

Figure 3.1: Three possible scenarios between two nodes

There are some assumptions and settings throughout the chapter. We assume the nodes always have data to transmit to the gateway. This is true if the network is busy and no node has enough bandwidth. This is also a reasonable simplification at the beginning of the analysis. The objective of these nodes is to get as much throughput as possible.

In this chapter, we define the utility U_X of a node X to be equal to its throughput, T_X:

$$U_X = U_X(T_X) = T_X \tag{3.1}$$

Since the throughput of any node in any network must be finite, the range of our utility for any node is $[0, \infty)$.

It might be reasonable to have some more elaborate utility function defined here. However, it is nontrivial to choose the proper utility functions. Advocating for a certain utility function for nodes in wireless network is out of this scope of the thesis; however, we believe the utility function can be replaced with most other functions (as long as they are continuous and monotonically increasing) and the solution procedure will still be effective.

We assume the nodes are selfish. They only care for the interest of themselves. They do not wish to contribute to the whole network unless it is beneficial for themselves. They can make independent decisions as agents do in a typical game. As such, each agent will attempt to maximize its own utility, and not care about the aggregate utility of the network.

In this section, we will assume all packets are of the same size; the link capacities are known specific numerical values. We study how the bargain will proceed and how equilibrium will be found in the three cases in the following discussions.

3.1.1 Competition Game

Consider the situation illustrated in Fig. 3.1(a). Assume A can communicate with O at 10Mb/s and B can communicate with O at 1Mb/s. What is the likely outcome of the competition?

Let t_A be the fraction of time A can access O in one second, and t_B be the fraction of time B can access O in one second.

The utility of node A is given by:

$$U_A = T_A = t_A \times 10 \tag{3.2}$$

The utility of node B is given by:

$$U_B = T_B = t_B \times 1 \tag{3.3}$$

By the selfish assumption, both A and B wish to maximize their own utility, therefore their goal is to get as much time as possible. On the other hand, since the network is busy, we have:

$$t_A + t_B = 1 \tag{3.4}$$

That is, the interest of A and B are incompatible and there is no way that they can cooperate to get any better outcome. There is no better strategy other than to compete with each other for bandwidth. In this case, the 802.11MAC protocol will ensure that each packet from both parties has the same chance to get transmitted. We assume the packets are of the same size; therefore:

$$t_A \times 10 = t_B \times 1 \tag{3.5}$$

From Eq. (3.4) and Eq. (3.5) the solution is $t_A = \frac{1}{11}$ seconds and $t_B = \frac{10}{11}$ seconds. Both parties get a throughput of $\frac{10}{11}$ Mbps.

The same results are reported by Gambiroza et al. [10]. Intuitively, in the equilibrium both nodes should try to send as much as possible, which results in the above outcome. We now study this as a normal-form game and advocate this result from the game-theory view.

The basic elements of the game are:

- The participants of the game: A and B
- Action space of A: A can keep silent, try to send all the time, or try to send some of the time and keep silent in the rest of the time.
- Action space of B: B can keep silent, try to send all the time, or try to send some of the time and keep silent in the rest of the time.

It is impossible to write down the payoff matrix for games with infinite actions. We have to parameterize the action space for each agent. Let the action space of X (either A

or B) be $\{$Active, Silent$\}$. Then the strategy X plays in the game can be expressed as (pActive, $(1-p)$Silent), where $p \in [0,1]$.
This is a mixed strategy formed by the two pure strategies "Silent" and "Active", and p is the probability of X being active. Then we have the following game with the payoff matrix shown in Table (3.1):

- The participants of the game: A and B
- Action space of A: Silent, Active
- Action space of B: Silent, Active
- Strategy of A: (p_AActive, $(1-p_A)$Silent); $p_A \in [0,1]$
- Strategy of B: (p_BActive, $(1-p_B)$Silent); $p_B \in [0,1]$

A \ B	Silent	Active
Silent	(0,0)	(0,1)
Active	(10,0)	$(\frac{10}{11}, \frac{10}{11})$

Table 3.1: Payoff Matrix for the First Game

No matter what strategy B uses, A can always get a higher utility by playing “Active”; therefore “Silent” is strictly dominated by “Active” for A. For B,“Silent” is also strictly dominated by “Active”. Thus, the Nash Equilibrium of the game is (Active, Active), which results in the situation where both agents try to send all the time and finally both get a throughput of $\frac{10}{11}$ Mbps.

We will show there is no strategy such that *both* nodes get higher utility; therefore there is no desire for any cooperation. Assume that in the final outcome of the game the proportion of (Active, Active) is p_1, the proportion of (Active, Silent) is p_2, the proportion of (Silent, Active) is p_3, the proportion of (Silent, Silent) is p_4. Because at any time, the nodes must be in one and only one of the four situations, we must have:

$$p_1 + p_2 + p_3 + p_4 = 1; p_i \in [0, 1], i = 1, 2, 3, 4 \tag{3.6}$$

In order to make cooperation possible we should also have:

$$p_1 \begin{pmatrix} \frac{10}{11} \\ \frac{10}{11} \end{pmatrix} + p_2 \begin{pmatrix} 10 \\ 0 \end{pmatrix} + p_3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + p_4 \begin{pmatrix} 0 \\ 0 \end{pmatrix} > \begin{pmatrix} \frac{10}{11} \\ \frac{10}{11} \end{pmatrix} \tag{3.7}$$

Which leads to:

$$\frac{10}{11}p_1 + 10p_2 > \frac{10}{11} \tag{3.8}$$

$$\frac{10}{11}p_1 + p_3 > \frac{10}{11} \tag{3.9}$$

However, Eq. (3.8) $+$ Eq. (3.9) $\times 10$ yields:

$$10(p_1 + p_2 + p_3) > 10 \tag{3.10}$$

or

$$p_1 + p_2 + p_3 > 1 \tag{3.11}$$

which contradicts with Eq. (3.6).

Therefore, there is no outcome of the game where *both* nodes get higher utility than they do in the Nash Equilibrium. Therefore, the Nash Equilibrium in this game is Pareto optimal outcome. In other words, the outcome of the game must be the Nash Equilibrium.

3.1.2 COOPERATION GAME

We now consider the second possibility, shown in Fig. 3.1(b). We assume A can communicate with O at 10Mbps. B can access A at 10Mbps, but cannot access O directly.

There are a lot of possible outcomes of the game. B can keep silent all the time; then A gets a bandwidth of 10 Mbps and B gets a bandwidth of 0 Mbps. If B keeps active, and A does not forward any of B's data, just trying to send to O, then A gets 5Mbps, and B gets 0Mbps.

Obviously, neither outcome is favorable to B. On the other hand, the throughput A gets varies when B adopts different strategies. It is also possible that B sends data and A forwards some of it. There may be such a conversion between A and B:

- B: Hi friend; can you forward these packets for me?
- A: I do not wish to. If I do, I will have less time to transmit my own data.
- B: If you do not forward my data, I will punish you by keeping active to slow down your traffic.
- A: Ok; let's make a deal. I will forward some of your data; please don't bother me the rest of the time.

- B: Sounds like a good deal. I will send to you at 3Mbps.
- A: No way; I will forward at most 1Mbps.
- B: ...

Note that this time it is not a zero-sum game. A and B may agree to cooperate. The difficulty is how to determine the proper allocation of bandwidth between the two nodes. We analyze the cooperation in the game-theory view.

The basic elements of the game are:

- The participants of the game: A and B
- Action space of A: Forward B's packets (Cooperative), Send its own packets (Non-cooperative)
- Action space of B: Keep active or keep silent

The exact definition of these actions needs to be emphasized. A plays "cooperative" if it forwards all of B's packets even if this means that A does not have time to transmit its own packets. A plays "Non-cooperative" if it keeps trying to send its own packets, and never forwards B's packets. B plays "Keep Silent" if it does not attempt to send any packets. B plays "Keep Active" if B keeps trying sending all the time. However, since A is also trying to send (either forwarding B's or sending A's own packets), B can succeed in sending only half the time. Therefore when playing "Keep active", B sends packets half the time.

A's utility is maximized if the strategy profile (Non-cooperative, Keep silent) is carried out. In this case A gets 10Mbps, and B gets 0Mbps. B's utility is maximized if the strategy profile (Cooperative, Keep active) is carried out. In this case, B tries to send all the time but B can only succeed half the time, because the other half of the time is taken by A forwarding packets from B. Therefore in this case A gets 0Mbps and B gets 5Mbps.

As in the previous game, the actions they actually play can be considered as mixed strategies of these actions. Then we have the payoff matrix shown in Table 3.2:

		B	
		Silent	Active
A	Cooperative	(10,0)	(0,5)
	Non-cooperative	(10,0)	(5,0)

Table 3.2: Payoff Matrix for the Second Game

For A, the strategy "Cooperative" is dominated by "Non-cooperative". For B, "Silent" is dominated by "Active". Then it is straightforward to see that the pure-strategy Nash Equilibrium of this game is (Non-cooperative, Active), which is apparently not what we wish to happen. However, unlike the first game, there are cooperative outcomes that are better than the Nash Equilibrium for both nodes. For example, suppose the outcome consists of 20% of (Cooperative, Active) and 80% of (Non-Cooperative, Silent); the utility vector shows:

$$0.2\begin{pmatrix}0\\5\end{pmatrix}+0.8\begin{pmatrix}10\\0\end{pmatrix}+0\begin{pmatrix}5\\0\end{pmatrix}+0\begin{pmatrix}10\\0\end{pmatrix}=\begin{pmatrix}8\\1\end{pmatrix}>\begin{pmatrix}5\\0\end{pmatrix} \tag{3.12}$$

Therefore we consider the cooperative version of the game, and find the Raiffa solutions with the following steps.

Assume the cooperative strategy profile is
$\{(p_1(Cooperative, Active), p_2(Cooperative, Silent), p_3(Non{-}cooperative, Active), p_4(Non{-}cooperative, Silent)\}; p_1+p_2+p_3+p_4=1, p_i \in [0,1], i=1,2,3,4$
The utility vector for A and B are given by Eq. (3.13):

$$\begin{pmatrix}U_A\\U_B\end{pmatrix}=p_1\begin{pmatrix}0\\5\end{pmatrix}+p_2\begin{pmatrix}10\\0\end{pmatrix}+p_3\begin{pmatrix}5\\0\end{pmatrix}+p_4\begin{pmatrix}10\\0\end{pmatrix} \tag{3.13}$$

Therefore,

$$U_A = 10p_2 + 5p_3 + 10p_4 \tag{3.14}$$

$$U_B = 5p_1 \tag{3.15}$$

The utility of each agent is normalized such that the best utility an agent may get is 1 and the worst is 0. Let U_X be the utility of X (either A or B). Let MAX_{U_X} be the maximal utility a node X can get and MIN_{U_X} be the minimal utility it can get. These are fixed values once the topology and link capacities of the network are given. The normalized utility of node X, NU_X, is a function of U_X (therefore a function of p_1, p_2, p_3, p_4), defined as:

$$NU_X = \frac{U_X - MIN_{U_X}}{MAX_{U_X} - MIN_{U_X}} \tag{3.16}$$

This is the only linear mapping from the interval of the utility of X, $[MIN_{U_X}, MAX_{U_X}]$, to $[0, 1]$. Any non-linear mapping implicitly changes the underlying utility functions and therefore is not preferred. After normalization, the Payoff Matrix is shown in Table 3.3.

		B: Silent	B: Active
A	Cooperative	(1,0)	(0,1)
A	Non-cooperative	(1,0)	(0.5,0)

Table 3.3: Normalized Payoff Matrix for the Second Game

Therefore, the normalized utility of A and B, as a function of p_1, p_2, p_3, p_4, are given by:

$$NU_A = p_2 + 0.5p_3 + p_4 \tag{3.17}$$

$$NU_B = p_1 \tag{3.18}$$

Second, we observe that A and B have different powers in the bargain. A is much stronger since it can ensure itself a normalized utility of 0.5 leaving B with 0 by playing "non-cooperative". The difference between nodes is characterized by the security level. As introduced in Chapter 2, the security level is the highest utility a node can guarantee itself. Any other strategy may lead to lower utility.

The security level, SL_X, of a node X is the highest utility it can guarantee itself. From the normalized-utility matrix shown in Table 3.3 we have:

$$SL_A = 0.5 \quad \text{when } A \text{ takes "Non-cooperative"} \tag{3.19}$$

$$SL_B = 0 \quad \text{when } B \text{ takes either action} \tag{3.20}$$

The normalized utility and security level reveal the asymmetry between the nodes.

Raiffa suggests that the difference between the security levels should be maintained in the solution profile. If the relative advantage is kept, any agent that unilaterally deviates from the solution profile will lose at least the same as the other agent does since the cooperation will be terminated and both will get the utility of the security level.

The Raiffa solution is given by the following optimization problem:

Maximize NU_A under the constraints:

- $NU_A - NU_B = SL_A - SL_B$
- $p_1 + p_2 + p_3 + p_4 = 1$
- $p_1, p_2, p_3, p_4 \in [0, 1]$

Expressing all terms in p_1, p_2, p_3 and p_4 the problem becomes:

Maximize

$$NU_A = p_2 + 0.5p_3 + p_4 \tag{3.21}$$

given:

$$p_2 + 0.5p_3 + p_4 - p_1 = 0.5 \tag{3.22}$$

$$p_1 + p_2 + p_3 + p_4 = 1 \tag{3.23}$$

and $p_1, p_2, p_3, p_4 \in [0, 1]$.

Taking Eq. (3.22) from Eq. (3.21) yields:

$$NU_A = p_1 + 0.5 \tag{3.24}$$

Eq. (3.23)- Eq. (3.22) yields:

$$2p_1 + 0.5p_3 = 0.5 \tag{3.25}$$

Therefore,

$$p_1 = 0.25 - 0.25p_3 \tag{3.26}$$

$$NU_A = 0.75 - 0.25p_3 \leq 0.75 \tag{3.27}$$

Therefore, NU_A is maximized to be 0.75 when $p_3 = 0$, which yields $p_1 = 0.25$ and $p_2 + p_4 = 0.75$.

Therefore, one of the solutions for this game is $\{0.25(Cooperative, Active), 0.75(Cooperative, Silent)\}$; *i.e.*, B should be "active" for 25% of the time, and A should forward all these packets. A has 75% of the time to send its own packets while B keeps silent. In this case A gets 7.5Mbps and B gets 1.25Mbps. Both nodes get better throughput than the Nash Equilibrium, where A gets 5Mbps and B gets 0Mbps.

3.1.3 Cooperation and Competition Game

We now consider the third possibility, shown in Fig. 3.1(c). We assume that A can communicate with O at 10Mbps. B can access A at 10Mbps, and also access O at 1Mbps. First, we identify the participants and their action spaces:

- The participants of the game: A and B
- Action space of A: Cooperative or Non-cooperative
- Action space of B: Keep silent, send to A or send to O

The difference in this case from the scenario in Section 3.1.2 lies in the fact that B can access O directly. However, the link between B and O is weaker, so the more-efficient way is that B sends its packets to A, which will hopefully forward them to O. If B keeps sending to O, thus competing with A, the traffic will be slowed down significantly; both will get $\frac{10}{11}$ Mbps, as analyzed in the first game.

Will the outcome of game be that of the first or the second game, or different from the both? B is likely to get more than it gets in the first game, as long as it has an alternative route which can send data much faster. Moreover, this time B is much stronger than it was in the second game. It can access O even if A does not cooperate; moreover, he can slow down A's traffic from 10Mbps to less than 1 Mbps. This is due to the current 802.11 MAC protocol, which gives the packets from A and B equal chance to be transmitted. One can expect this time the outcome is better for B.

We apply the same reasoning procedure as in Section 3.1.2 to see what the exact solution is. The normal-form game is represented by the matrix in Table 3.4. Normalization of utility will yield the payoff matrix in Table 3.5.

Assume the strategy profile is

$\{p_1(Cooperative, KeepSilent), p_2(Cooperative, SendtoO),$

		B		
		Keep Silent	Sending to O	Sending to A
A	Cooperative	(10,0)	(0,1)	(0,5)
	Non-cooperative	(10,0)	$(\frac{10}{11}, \frac{10}{11})$	(5,0)

Table 3.4: Payoff Matrix for the Third Game

$p_3(Cooperative, SendtoA), p_4(Non-cooperative, KeepSilent),$
$p_5(Non-cooperative, SendtoO), p_6(Non-cooperative, SendtoA)\}$
where $p_i \in [0, 1], and \sum_{i=1}^{6} p_i = 1$, the utility vector is given by:

$$\begin{pmatrix} U_A \\ U_B \end{pmatrix} = p_1 \begin{pmatrix} 10 \\ 0 \end{pmatrix} + p_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + p_3 \begin{pmatrix} 0 \\ 5 \end{pmatrix} + p_4 \begin{pmatrix} 10 \\ 0 \end{pmatrix} + p_5 \begin{pmatrix} \frac{10}{11} \\ \frac{10}{11} \end{pmatrix} + p_6 \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{3.28}$$

Therefore,

$$U_A = 10p_1 + 10p_4 + \frac{10}{11}p_5 + 5p_6 \tag{3.29}$$

A \ B	Keep Silent	Send to O	Send to A
Cooperative	(1,0)	(0,$\frac{1}{5}$)	(0,1)
Non-cooperative	(1,0)	($\frac{1}{11}$, $\frac{2}{11}$)	($\frac{1}{2}$,0)

Table 3.5: Normalized Payoff Matrix for the Third Game

$$U_B = p_2 + 5p_3 + \frac{10}{11}p_5 \tag{3.30}$$

$$NU_A = p_1 + p_4 + \frac{1}{11}p_5 + \frac{1}{2}p_6 \tag{3.31}$$

$$NU_B = \frac{1}{5}p_2 + p_3 + \frac{2}{11}p_5 \tag{3.32}$$

Then the security level of A is given by:

$$SL_A = \frac{1}{11} \quad \text{when } A \text{ is non-cooperative} \tag{3.33}$$

the security level of B is given by:

$$SL_B = \frac{2}{11} \quad \text{when } B \text{ sends to O} \tag{3.34}$$

The Raiffa solution is given by the following optimization problem:

Maximize NU_A under the constraint:

- $NU_A - NU_B = SL_A - SL_B$
- $\sum_{i=1}^{6} p_i = 1$
- $p_i \in [0, 1]$

Expressing all terms in p_i the problem becomes:

Maximize

$$NU_A = p_1 + p_4 + \frac{1}{11}p_5 + \frac{1}{2}p_6 \tag{3.35}$$

given:

$$p_1 + p_4 + \frac{1}{11}p_5 + \frac{1}{2}p_6 - (\frac{1}{5}p_2 + p_3 + \frac{2}{11}p_5) = -\frac{1}{11} \tag{3.36}$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1 \tag{3.37}$$

and $p_i \in [0, 1]$.

Taking Eq. (3.36) into Eq. (3.35) yields:

$$NU_A = \frac{1}{5}p_2 + p_3 + \frac{2}{11}p_5 - \frac{1}{11} \tag{3.38}$$

Eq. (3.35)+ Eq. (3.38) yields:

$$2NU_A = p_1 + p_4 + \frac{3}{11}p_5 + \frac{1}{2}p_6 + \frac{1}{5}p_2 + p_3 - \frac{1}{11} \tag{3.39}$$

Since $p_i \geq 0$, for all i, from Eq. (3.39) and Eq. (3.37) we have:

$$2NU_A \leq p_1 + p_4 + p_5 + p_6 + p_2 + p_3 - \frac{1}{11} = \frac{10}{11} \tag{3.40}$$

$$NU_A \leq \frac{5}{11} \tag{3.41}$$

Where "$=$" is valid if and only if $p_2 = p_5 = p_6 = 0$. If $p_2 = p_5 = p_6 = 0$, from Eq. (3.36) we have

$$p_1 + p_4 - p_3 = -\frac{1}{11} \tag{3.42}$$

from Eq. (3.37) we have

$$p_1 + p_3 + p_4 = 1 \tag{3.43}$$

from Eq. (3.42) and Eq. (3.43) we see $p_3 = \frac{6}{11}$ and $p_1 + p_4 = \frac{5}{11}$. Therefore, NU_A is maximized to be $\frac{5}{11}$ when $p_2 = p_5 = p_6 = 0, p_3 = \frac{6}{11}$,and $p_1 + p_4 = \frac{5}{11}$. Therefore, a solution profile may be

$$\{\tfrac{5}{11}(Cooperative, KeepSilent), \tfrac{6}{11}(Cooperative, SendtoA)\}$$

So B should send data to A for $\frac{6}{11}$ of the time and never try to connect to O directly. A should forward all of the data received and send its own packets for $\frac{5}{11}$ of the time. These strategy lead to the outcome that A gets $\frac{50}{11}$Mbps and B get $\frac{30}{11}$Mbps. As expected, this time the outcome is much more balanced as B's ability to bargain is much stronger than before.

3.1.3.1 EXAMPLE OF COMPETITION

In this section we see that the nodes do not necessarily have to cooperate with each other. Sometimes they compete for the bandwidth.

We now consider the scenario that is similar to that discussed above. However, we now assume that A can communicate with O at 10Mbps. B can access A at 10Mbps, and also access O at 8Mbps. Intuitively, neither B nor A can benefit from cooperation, because it costs more time for B's packets to arrive at O if they are not transmitted directly. A formal proof is shown as below.

The Payoff Matrix is given by:

A \ B	Keep Silent	Send to O	Send to A
Cooperative	(10,0)	(0,8)	(0,5)
Non-cooperative	(10,0)	$(\frac{40}{9}, \frac{40}{9})$	(5,0)

Table 3.6: Payoff Matrix for the Third Game

Note that for B, the strategies "Keep Silent" and "send to A" are both strictly dominated by "Send to O". The only reasonable strategy for A when B plays "Send to O" is "Non-cooperative". Therefore, the Nash equilibrium of the game will be (Non-cooperative, Sending to O), which is the same as the scenario in the first game, where nodes compete with each other to connect to O.

We will show it is impossible for any cooperative strategy to surpass the Nash equilibrium of the game. Assume the strategy profile is
$\{p_1(Cooperative, KeepSilent), p_2(Cooperative, SendtoO),$
$p_3(Cooperative, SendtoA), p_4(Non-cooperative, KeepSilent),$
$p_5(Non-cooperative, SendtoO), p_6(Non-cooperative, SendtoA)\}$
where $p_i \in [0,1], and \sum_{i=1}^{6} p_i = 1$. If both nodes get higher utility than they do in the Nash Equilibrium, we have:

$$p_1\begin{pmatrix}10\\0\end{pmatrix}+p_2\begin{pmatrix}0\\8\end{pmatrix}+p_3\begin{pmatrix}0\\5\end{pmatrix}+p_4\begin{pmatrix}10\\0\end{pmatrix}+p_5\begin{pmatrix}\frac{40}{9}\\\frac{40}{9}\end{pmatrix}+p_6\begin{pmatrix}5\\0\end{pmatrix}>\begin{pmatrix}\frac{40}{9}\\\frac{40}{9}\end{pmatrix} \tag{3.44}$$

Which leads to:

$$10p_1 + 10p_4 + \frac{40}{9}p_5 + 5p_6 > \frac{40}{9} \tag{3.45}$$

$$8p_2 + 5p_3 + \frac{40}{9}p_5 > \frac{40}{9} \tag{3.46}$$

Eq. (3.45) leads to:

$$10p_1 + 10p_4 + 5p_6 > \frac{40}{9}(1 - p_5) \tag{3.47}$$

Eq. (3.46) leads to:

$$8p_2 + 5p_3 > \frac{40}{9}(1 - p_5) \tag{3.48}$$

Eq. (3.47) $\times 8+$ Eq. (3.48) $\times 10$ yields:

$$80p_1 + 80p_2 + 50p_3 + 80p_4 + 40p_6 > 80(1 - p_5) \tag{3.49}$$

$$80p_1 + 80p_2 + 50p_3 + 80p_4 + 80p_5 + 40p_6 > 80 \tag{3.50}$$

Since $p_3 \geq 0$, $p_6 \geq 0$, we have

$$80p_1+80p_2+80p_3+80p_4+80p_5+80p_6 \geq 80p_1+80p_2+50p_3+80p_4+80p_5+40p_6 > 80 \tag{3.51}$$

Therefore

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 > 1 \tag{3.52}$$

Which contradicts with $\sum_{i=1}^{6} p_i = 1$.

Therefore, it is impossible for any cooperative strategy to surpass the Nash equilibrium of the game.

3.2 General solutions

After studying four basic scenarios in two-nodes games, we present our general solutions for two-nodes games in this section.

The generalized two-nodes game is as shown in Fig. 3.1(c) : There are two nodes, A and B competing for a single access node O. The bandwidth is as follows: from A to O, x Mbps, from B to O, z Mbps, and $x > z$. Instead of sending data directly, B also has the option to send its packets to A with the hope that A will forward its packets. It is y

Mbps from B to A. Suppose A and B are rational, selfish, but agreed to try to cooperate to maximize both's utility; what is the best outcome which both would likely to agree to?

The payoff matrix for the game is shown in Table. 3.7:

		B		
		Keep Silent	Send to O	Send to A
A	Cooperative	(x,0)	(0,z)	(0,$\frac{1}{\frac{1}{x}+\frac{1}{y}}$)
	Non-cooperative	(x,0)	($\frac{1}{\frac{1}{x}+\frac{1}{z}}$, $\frac{1}{\frac{1}{x}+\frac{1}{z}}$)	($\frac{1}{\frac{1}{x}+\frac{1}{y}}$,0)

Table 3.7: Payoff Matrix for the General Two-node game

Depending on the value of x,y and z, there may be different scenarios, as the examples we have shown in Section 3.1 illustrates. One of the following two will be true:

- $\frac{1}{z} > \frac{1}{x} + \frac{1}{y}$
- $\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y}$

We study the game in each case.

3.2.1 CASE 1: $\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y}$

Lemma 1 *If $\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y}$, "Send to O" is a dominant strategy for B; (Non-cooperative, Send to O) is the Nash equilibrium of the game.*

Proof:

If A plays "Cooperative", then

$$U_B(SendtoO) = z > 0 = U_B(KeepSilent) \tag{3.53}$$

$$U_B(SendtoO) = z \geq \frac{1}{\frac{1}{x} + \frac{1}{y}} = U_B(SendtoA) \tag{3.54}$$

If A plays "Non-cooperative", then

$$U_B(SendtoO) = \frac{1}{\frac{1}{x} + \frac{1}{z}} > 0 = U_B(KeepSilent) \tag{3.55}$$

$$U_B(SendtoO) = \frac{1}{\frac{1}{x} + \frac{1}{z}} > 0 = U_B(SendtoA) \tag{3.56}$$

Therefore, no matter which strategy A plays, we always have $U_B(SendingtoO) \geq U_B(SendingtoA)$ and $U_B(SendingtoO) \geq U_B(KeepSilent)$; therefore, all other strategies for B are dominated by "Sending to O". Similarly, we can show "Cooperative" is dominated by "Non-cooperative" for A. Therefore, (Non-cooperative, Send to O) is the Nash equilibrium of the game.

Lemma 2 *If $\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y}$, no cooperative strategy improves the utility of both nodes simultaneously compared with (Non-cooperative, Send to O).*

Proof: Assume the cooperative strategy
$\{p_1(Cooperative, KeepSilent), p_2(Cooperative, SendtoO),$
$p_3(Cooperative, SendtoA), p_4(Non-cooperative, KeepSilent),$

$p_5(Non-cooperative, SendtoO), p_6(Non-cooperative, SendtoA)\}$
where $p_i \in [0,1], and \sum_{i=1}^{6} p_i = 1$, improves the utility of both nodes simultaneously, we have:

$$p_1 \begin{pmatrix} x \\ 0 \end{pmatrix} + p_2 \begin{pmatrix} 0 \\ z \end{pmatrix} + p_3 \begin{pmatrix} 0 \\ \frac{1}{\frac{1}{x}+\frac{1}{y}} \end{pmatrix} + p_4 \begin{pmatrix} x \\ 0 \end{pmatrix} + \tag{3.57}$$

$$p_5 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{z}} \\ \frac{1}{\frac{1}{x}+\frac{1}{z}} \end{pmatrix} + p_6 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{y}} \\ 0 \end{pmatrix} > \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{z}} \\ \frac{1}{\frac{1}{x}+\frac{1}{z}} \end{pmatrix} \tag{3.58}$$

Which leads to:

$$p_1 x + p_4 x + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} + p_6 \frac{1}{\frac{1}{x}+\frac{1}{y}} > \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.59}$$

$$p_2 z + p_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} > \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.60}$$

However, $y \geq 0$, therefore $x > \frac{1}{\frac{1}{x}+\frac{1}{y}}$, so Eq. (3.59) leads to:

$$p_1 x + p_4 x + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} + p_6 x > \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.61}$$

$$p_1 x + p_4 x + p_6 x > (1-p_5) \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.62}$$

$\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y} \Rightarrow z > \frac{1}{\frac{1}{x}+\frac{1}{y}}$, so Eq. (3.60) leads to:

$$p_2 z + p_3 z + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} \geq \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.63}$$

$$p_2 z + p_3 z \geq (1-p_5) \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.64}$$

Eq. (3.62) $/x+$ Eq. (3.64) $/z$ yields:

$$p_1 + p_2 + p_3 + p_4 + p_6 > (1 - p_5)\frac{1}{\frac{1}{x} + \frac{1}{z}}(\frac{1}{x} + \frac{1}{z}) = 1 - p_5 \tag{3.65}$$

$$p_1 + p_2 + p_3 + p_4 + +p_5 + p_6 > 1 \tag{3.66}$$

which contradicts with the fact $\sum_{i=1}^{6} p_i = 1$. Proof completed.

Lemma (1) and Lemma (2) show that there is no strategy such that both nodes get higher utility than they do in the Nash equilibrium. Moreover, the Nash equilibrium consists of dominant strategies; therefore we have the following theorem:

Theorem 1 *If $\frac{1}{z} \leq \frac{1}{x} + \frac{1}{y}$, the outcome of the game must be (Non-cooperative, Send to O); both nodes get utility of $\frac{1}{\frac{1}{x}+\frac{1}{z}}$.*

The case $y = 0$, where B cannot access A can be regarded as a special case in this category.

3.2.2 CASE 2: $\frac{1}{z} > \frac{1}{x} + \frac{1}{y}$

If $\frac{1}{z} > \frac{1}{x} + \frac{1}{y}$, cooperation becomes possible. Before we try to find the cooperative solution of the game, we have the following lemma to simplify our work.

In the following discussion, we assume the cooperative strategy is
$\{p_1(Cooperative, KeepSilent), p_2(Cooperative, SendtoO),$
$p_3(Cooperative, SendtoA), p_4(Non - cooperative, KeepSilent),$
$p_5(Non - cooperative, SendtoO), p_6(Non - cooperative, SendtoA)\}$
where $p_i \in [0, 1], and \sum_{i=1}^{6} p_i = 1$.

Lemma 3 *If $\frac{1}{z} > \frac{1}{x} + \frac{1}{y}$, for any given strategy characterized by $(p_1, p_2, p_3, p_4, p_5, p_6)$, there exists some $q_1, q_3 \in [0, 1]$ such that $q_1 + q_3 = 1$, and both nodes get at least the same utility if they play the strategy characterized by $(q_1, 0, q_3, 0, 0, 0)$.*

The lemma shows that any Pareto-efficient strategy should have $p_2 = p_4 = p_5 = p_6 = 0$. This observation will greatly simplify our work.

Proof:

$$\begin{aligned}
\frac{1}{z} > \frac{1}{x} + \frac{1}{y} \quad &\Rightarrow \quad \frac{x}{z} > 1 + \frac{x}{y} && (3.67)\\
&\Rightarrow \quad 1 + \frac{x}{z} > 2 + \frac{x}{y} && (3.68)\\
&\Rightarrow \quad 1 > \frac{2 + \frac{x}{y}}{1 + \frac{x}{z}} && (3.69)\\
&\Rightarrow \quad 1 > \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}} + \frac{1}{1 + \frac{x}{z}} && (3.70)\\
&\Rightarrow \quad 1 - \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}} > \frac{1}{1 + \frac{x}{z}} && (3.71)
\end{aligned}$$

Therefore, we can choose some $w \in R$, such that

$$1 - \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}} > w > \frac{1}{1 + \frac{x}{z}} \tag{3.72}$$

Let $q_1 = p_1 + p_4 + p_6 + wp_5$ and $q_3 = p_2 + p_3 + (1-w)p_5$; obviously, $q_1 + q_3 = \sum_{i=1}^{6} p_i = 1$, and $q_1, q_3 \in [0, 1]$. We will show both nodes get at least the same utility if they plays the strategy characterized by $(q_1, 0, q_3, 0, 0, 0)$.

$$\begin{aligned}
\text{E}q.\ (3.72) \quad &\Rightarrow \quad w > \frac{1}{1 + \frac{x}{z}} && (3.73)\\
&\Rightarrow \quad xw > \frac{x}{1 + \frac{x}{z}} = \frac{1}{\frac{1}{x} + \frac{1}{z}} && (3.74)
\end{aligned}$$

We also have:

$$x \geq x \tag{3.75}$$

and $\frac{1}{x} \leq \frac{1}{x} + \frac{1}{y}$ so

$$x \geq \frac{1}{\frac{1}{x} + \frac{1}{y}} \tag{3.76}$$

Eq. (3.75) $\times(p_1 + p_4)+$ Eq. (3.76) $\times p_6+$Eq. (3.74) $\times p_5$ yields:

$$p_1x + p_4x + p_6x + p_5xw \geq p_1x + p_4x + p_6\frac{1}{\frac{1}{x} + \frac{1}{y}} + p_5\frac{1}{\frac{1}{x} + \frac{1}{z}} \tag{3.77}$$

or

$$q_1x \geq p_1x + p_4x + p_6\frac{1}{\frac{1}{x} + \frac{1}{y}} + p_5\frac{1}{\frac{1}{x} + \frac{1}{z}} \tag{3.78}$$

On the other hand,

$$\begin{aligned} \mathrm{E}q.\ (3.72) &\Rightarrow\ 1 - \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}} > w & (3.79)\\ &\Rightarrow\ 1 - w > \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}} & (3.80)\\ &\Rightarrow\ (1 - w)\frac{1}{\frac{1}{x} + \frac{1}{y}} > \frac{1 + \frac{x}{y}}{1 + \frac{x}{z}}\frac{1}{\frac{1}{x} + \frac{1}{y}} & (3.81)\\ &\Rightarrow\ (1 - w)\frac{1}{\frac{1}{x} + \frac{1}{y}} > \frac{1}{\frac{1}{x} + \frac{1}{z}} & (3.82) \end{aligned}$$

We also have:

$$\frac{1}{\frac{1}{x} + \frac{1}{y}} \geq \frac{1}{\frac{1}{x} + \frac{1}{y}} \tag{3.83}$$

and $\frac{1}{z} > \frac{1}{x} + \frac{1}{y}$ so

$$\frac{1}{\frac{1}{x} + \frac{1}{y}} > z \tag{3.84}$$

Eq. (3.84) $\times p_2+$ Eq. (3.83) $\times p_3+$Eq. (3.82) $\times p_5$ yields:

$$p_2 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5(1-w) \frac{1}{\frac{1}{x}+\frac{1}{y}} \geq p_2 z + p_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.85}$$

or

$$q_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} \geq p_2 z + p_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} \tag{3.86}$$

The utility vector $(U_A, U_B)$of the strategy $(p_1, p_2, p_3, p_4, p_5, p_6)$ is given by:

$$\begin{pmatrix} U_A \\ U_B \end{pmatrix} = p_1 \begin{pmatrix} x \\ 0 \end{pmatrix} + p_2 \begin{pmatrix} 0 \\ z \end{pmatrix} + p_3 \begin{pmatrix} 0 \\ \frac{1}{\frac{1}{x}+\frac{1}{y}} \end{pmatrix} \tag{3.87}$$

$$+p_4 \begin{pmatrix} x \\ 0 \end{pmatrix} + p_5 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{z}} \\ \frac{1}{\frac{1}{x}+\frac{1}{z}} \end{pmatrix} + p_6 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{y}} \\ 0 \end{pmatrix} \tag{3.88}$$

I.e.,

$$\begin{pmatrix} U_A \\ U_B \end{pmatrix} = \begin{pmatrix} p_1 x + p_4 x + p_6 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} \\ p_2 z + p_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} + p_5 \frac{1}{\frac{1}{x}+\frac{1}{z}} \end{pmatrix} \tag{3.89}$$

The utility vector (U'_A, U'_B) of the strategy $(q_1, 0, q_3, 0, 0, 0)$ is given by:

$$\begin{pmatrix} U'_A \\ U'_B \end{pmatrix} = q_1 \begin{pmatrix} x \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ z \end{pmatrix} + q_3 \begin{pmatrix} 0 \\ \frac{1}{\frac{1}{x}+\frac{1}{y}} \end{pmatrix} \tag{3.90}$$

$$+0 \begin{pmatrix} x \\ 0 \end{pmatrix} + 0 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{z}} \\ \frac{1}{\frac{1}{x}+\frac{1}{z}} \end{pmatrix} + 0 \begin{pmatrix} \frac{1}{\frac{1}{x}+\frac{1}{y}} \\ 0 \end{pmatrix} \tag{3.91}$$

I.e.,

$$\begin{pmatrix} U_A' \\ U_B' \end{pmatrix} = \begin{pmatrix} q_1 x \\ q_3 \frac{1}{\frac{1}{x}+\frac{1}{y}} \end{pmatrix} \tag{3.92}$$

From Eq. (3.89), Eq. (3.92), Eq. (3.78) and Eq. (3.86), we see $U_A' \geq U_A$ and $U_B' \geq U_B$. Proof completed.

We now try to find the Raiffa solution for the game. Normalized utility yields the payoff matrix in Table (3.8)

		B		
		Keep Silent	Send to O	Send to A
A	Cooperative	(1,0)	(0,$z(\frac{1}{x}+\frac{1}{y})$)	(0,1)
	Non-cooperative	(1,0)	$(\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{z}}, \frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}+\frac{1}{z}})$	($\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{y}}$,0)

Table 3.8: Normalized Payoff Matrix for the General Two-node game

By Lemma (3) we know any strategy that is Pareto efficient must be characterized by $(q_1, 0, q_3, 0, 0, 0)$. The Raiffa solution is Pareto efficient. So we can assume the desired strategy profile is p(Cooperative, Silent),q(Cooperative,Send to A), where $p, q \in [0, 1]$

and $p + q = 1$. Then we can write down the normalized utility and security level as functions of p and q:

$$NU_A = p \tag{3.93}$$

$$NU_B = q \tag{3.94}$$

$$SL_A = \frac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{z}} \tag{3.95}$$

$$SL_B = \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} + \frac{1}{z}} \tag{3.96}$$

We require $NU_A - NU_B = SL_A - SL_B$, therefore,

$$p - q = -\frac{\frac{1}{y}}{\frac{1}{x} + \frac{1}{z}} \tag{3.97}$$

We also know $p + q = 1$, therefore

$$p = (1 - \frac{\frac{1}{y}}{\frac{1}{x} + \frac{1}{z}})/2 \tag{3.98}$$

$$q = (1 + \frac{\frac{1}{y}}{\frac{1}{x} + \frac{1}{z}})/2 \tag{3.99}$$

Finally we get the solution of the game: $(1 - \frac{\frac{1}{y}}{\frac{1}{x}+\frac{1}{z}})/2$(Cooperative, Silent),$(1 + \frac{\frac{1}{y}}{\frac{1}{x}+\frac{1}{z}})/2$(Cooperative,Send to A). A gets a throughput of $x(1 - \frac{\frac{1}{y}}{\frac{1}{x}+\frac{1}{z}})/2$, and B gets a throughput of $(1 + \frac{\frac{1}{y}}{\frac{1}{x}+\frac{1}{z}})/2(\frac{1}{x} + \frac{1}{y})$.

Concluding the discussions in this section, we see for two nodes there are only two cases, either completely competing or completely cooperating. We address the problem by given the analytical formula for the strategy profile for each situation.

3.3 MULTIPLE-NODE GAMES

The fact that only the most efficient path should be used does not only apply to two-node networks if we assume all nodes are within the same collision domain. The proof is as follows. Suppose an inefficient route is used with non-zero probability in the strategy. We rearrange the data sending along the inefficient way to be sent by the more efficient route. This will cost less time since it is more efficient. Note that all the nodes still get the same bandwidth but we have some extra time that may distributed to any route to improve the benefits of at least one other node. In other words, there exist other strategy profiles such that at least one node's utility increases without any other node's utility decreasing. Therefore, by Pareto efficiency, it would be a better allocation.

3.3.1 ROUTING TREE

The wireless network can be regarded as a graph. Each participant of the network is a node in the graph. The edges are the used network links between the nodes. As observed above, only the most efficient path is ever used. Then we will get a confluent tree which represents the topology of the network. This is usually referred to as the routing tree of the network formed by the shortest path. The gateway will be the root.

It is possible to recursively play two-player games to get the solution for a multiple-player game if they form a tree topology. A similar observation is discussed by Cheng et al. [7]. There is some research of combinatorial agency which also resembles our ideas [4].

3.3.2 GROUPS

To extend our solution to multi-node networks we first introduce the concept of group and group coordinator. In this section we assume all the information is public.

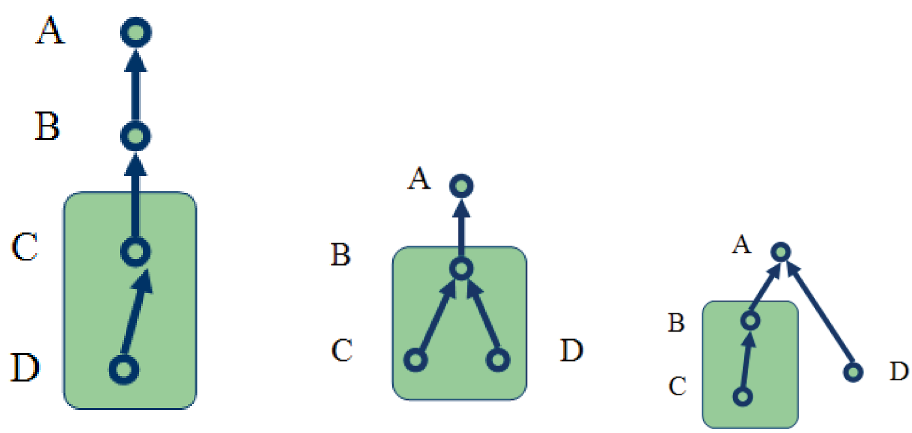
Definition 4 (Group) *A group is a set of nodes which forms a subtree in the routing tree.*

A group may consist of a node and its parent, or two or more competing nodes and their parent. It may also contains smaller groups. A group appears to be a single node to the outside and acts (either compete or cooperate) as a single node. After it gets the bandwidth from the outside, it will share it within the group according to the agreement of the group memebers.

Definition 5 (Group Coordinator) *The root of the subtree formed by the nodes of a group is called the group coordinator.*

The group coordinator acts as the representative of the group communicating with the outside world. It has the responsibility to forward packets from other nodes in the group.

Some examples of groups are shown below:



(a) Ex. 1 (b) Ex. 2 (c) Ex. 3

Figure 3.2: Example of groups

In Fig. 3.2(a), Node C and D form a group, which cooperates with B. C is the group coordinator, which will be in charge of distributing the resource between C and D.

In Fig. 3.2(b), Node B,C,D form a group, which cooperates with A. Within the group, B cooperates with C and D while C competes with D for resources.

In Fig. 3.2(c), Node B and C form a group, which competes with D for resources.

A group can be as large as the whole network with the gateway as the root, or as small as a single node. We have to determine how groups play games with each other. There are two types of games: the competition game may happen among several groups; the cooperation game may happen between a node and a group.

3.3.3 Competition Game between groups

As illustrated in Fig. 3.3, several groups are competing to connect to the parent. Each small node may represent a group in the routing tree.

Definition 6 (Competition Factor) *The competition factor for a group is the relative proportion of time assigned to this group with respect to the spectrum time received by the whole group. Each group is assigned a different competition factor, which sum up to 1.*

Assume the group coordinator gets some spectrum time from its parent. Then it divides the time into several slots according to the competition factors assigned to its child groups. In each slot the coordinator makes a deal with a certain child group; *i.e.*, forwards some packets from the group and sends its own packets for the rest of the time. Each group tries to get more time (a bigger competition factor) to cooperate with the group coordinator.

For a competition game, we have the following information:

- Groups involved in the game: $G1, G2, G3, ..., Gn$
- Their parent, P

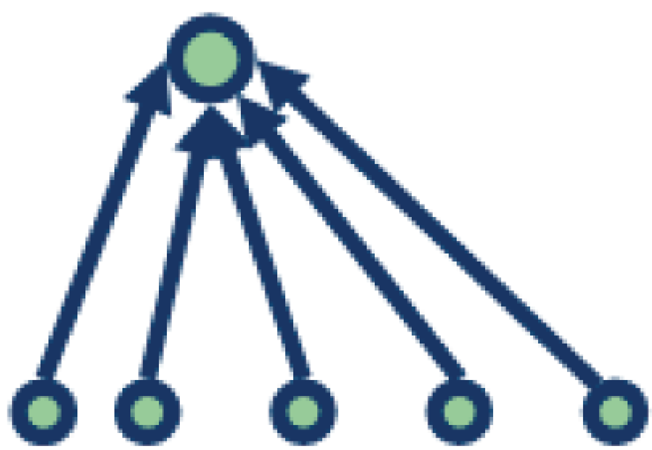


Figure 3.3: Competition between groups

- For each group $Gk, k = 1, 2, 3, ..., n$, we know the number of nodes in the group, n_k, as well as the nodes $Gk_1, Gk_2, ..., Gk_{n_k}$
- For each node Gk_m, we know the bandwidth from them to P, denoted by Gk_m^P

Let T_{Gk} be the *competition factor* for group Gk, $k = 1, 2, ...n$. To be fair to assign the proper competition factor, we have to ensure that each group get enough proportion of time such that no agent can benefit from deviating from the assignment. Any node's deviation from the assignment may lead to pure competition. In this case, all the nodes just try to fight for bandwidth and try to connect to the subtree node directly. Denote the time node Gk_m gets by T_{Gk_m}; the following conditions will be true:

- The sum of all times should be 1. *i.e.*,

$$\sum_{k=1}^{n} \sum_{m=1}^{n_k} T_{Gk_m} = 1 \tag{3.100}$$

- By 802.11 scheduling, for any k,m,i,j,

$$T_{Gk_m} Gk_m^P = T_{Gi_j} Gi_j^P \tag{3.101}$$

The solution for the above conditions are:

$$T_{Gk_m} = \frac{\frac{1}{Gk_m^P}}{\sum_{k=1}^{n} \sum_{m=1}^{n_k} \frac{1}{Gk_m^P}} \tag{3.102}$$

Group $k(k = 1, 2, n)$ as a whole will get the following proportion of time, which we define as the competition factor:

$$T_{Gk} = \sum_{m=1}^{n_k} T_{Gk_m} = \frac{\sum_{m=1}^{n_k} \frac{1}{Gk_m^P}}{\sum_{k=1}^{n} \sum_{m=1}^{n_k} \frac{1}{Gk_m^P}} \tag{3.103}$$

3.3.4 Cooperation Game between nodes and subtrees

A cooperation game always happens between a node and a group, which may be as small as a single node. As illustrated in Fig. 3.4, node O help forwarding the packets from the group G to P. As observed in the previous sections, in the optimal case node O should always forward all the packets from G. The only uncertainty is the proportion of time the link between G and O is active.

Definition 7 (Cooperation Factor) *The cooperation factor for a group is the relative proportion of time it is active when it is cooperating with its parent.*

We wish to assign the group G a *cooperation factor*. If node O, the parent of group G, has only one child, then the *cooperation factor* will be the relative time for group G to be active. Otherwise, in case O has more than one child, the proportion of time for group G being active should be the product of its *cooperation factor* and *competition factor*, since group G should first compete with other groups for access to O, and then distribute the time between O and G after that bargain.

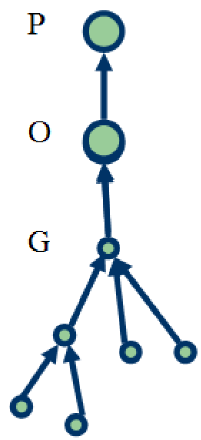


Figure 3.4: Cooperation between a group and a node

For a cooperation game, we have the following information:

- Participants of the game: Group G and Node O
- For group G, we know the number of nodes in the group, n, as well as the nodes $G_0, G_1, G_2, ..., G_n$, where G_0 is the group coordinator
- For each node $G_k, k = 0, 1, 2, ...n$, we know the bandwidth from them to P and to O, denoted by G_k^P and G_k^O
- Bandwidth from O to P is OP

Denote the cooperation factor, the time G is sending in 1 second, by C. The throughput of group G would be:

$$U_G = G_0^O \times C \tag{3.104}$$

The throughput of node O would be:

$$U_O = (1 - C - CG_0^O/OP) \times OP \tag{3.105}$$

The minimal throughput of both parties may be zero. The maximum possible throughput for group G is:

$$MAX_U(G) = \frac{1}{\frac{1}{G_0^O} + \frac{1}{OP}} \tag{3.106}$$

The maximum possible throughput for node O is OP.

The normalized utility for G and O are:

$$NU_G = G_0^O C(\frac{1}{G_0^O} + \frac{1}{OP}) \tag{3.107}$$

$$NU_O = 1 - C - CG_0^O/OP \tag{3.108}$$

The security level of G would be the sum of all its members' throughput in case there is no cooperation; *i.e.*,

$$SL_G = \frac{|\{k : G_k^P \neq 0\}|}{\sum_{G_k^P \neq 0} \frac{1}{G_k^P} + \frac{1}{OP}} \tag{3.109}$$

Where $|X|$ denotes the number of elements in the finite set X.

The security level of O is a little more difficult to determine. Some nodes will try to access P directly, and some may just bother O by interfering with O's ability to transmit. The security level is:

$$SL_O = \min \frac{1}{\sum_{G_k^P neq0} \frac{1}{G_k^P} + \frac{1}{OP}}, \frac{OP}{|\{k : G_k^O \neq 0\}|} \tag{3.110}$$

The Raiffa solution suggests finding C to maximize $NU(G)$ under the constraint:

- $NU_G - NU_O = SL_G - SL_O$
- $0 \leq C, NU_G, NU_O \leq 1$

Solving for C we have:

$$C = \frac{1 + SL_G - SL_O}{2(1 + \frac{G_0^O}{OP})} \tag{3.111}$$

3.4 Solution Algorithms

Being able to find the equilibrium of both kinds of games that may appear in a network, the solution algorithm can recursively solve the game. The basic idea of the algorithm is that the gateway distributes bandwidth among the several largest groups; then each group coordinator recursively distributes the resource among its subgroups. Subgroups compete with each other, and cooperate with the group coordinator such that the group coordinator forwards data for the subgroups.

We assume the tree topology is given, and all the link-quality infomation needed is known. Every node maintains two numbers, nodeShare and treeShare, which are the share distributed to the node and the subtree with the node as the root. Then Algorithm 1 calculates the proper share for each node:

The function solve(A), where A is a node in the network, works as stated in Algorithm 2:

The complexity of the algorithm is $O(n^2)$, where n is the number of nodes in the network. In algorithm 1, the function "solve()" will be called exactly n times. In algorithm

Algorithm 1 Solution for multiple nodes game

Set the nodeShare and treeShare of all the nodes to be 1
root $\leftarrow$ the gateway
solve(root)

Algorithm 2 Solve(A)

if A is not a leaf **then**
  $n =$ number of A's first-level children
  $B =$ set of A's first-level children
  $BW_{XY} =$ Bandwidth from X to Y
  Play a competition game among $B_1, B_2, \ldots, B_n$
  for $i = 1 \ldots n$ **do**
    B_i.treeShare $\leftarrow$ B_i's competition factor $\times$ A.treeShare
    if A is not the gateway **then**
      $P =$ A's parent
      Play a cooperation game between B_i and A
      B_i.treeShare $\leftarrow$ B_i's cooperation factor $\times B_i$.treeShare
      A.nodeShare $\leftarrow$ A.nodeShare - B.treeShare $\times (1 + \frac{BW_{AP}}{BW_{B_iA}})$
    end if
    B_i.nodeShare $\leftarrow$ B_i.treeShare
  end for
end if

2, most steps are within constant time. only the for loop may involve at most $O(n)$ steps. Therefore, the complexity of the algorithm is $O(n^2)$

3.5 Results and Discussions

We have implemented the algorithm presented in the previous section. Given any tree topology and link quality information, the program is able to calculate the proper bandwidth sharing according to the Raiffa solution.

We list several examples here to illustrate the results given by the algorithm.

3.5.1 The power of threatening

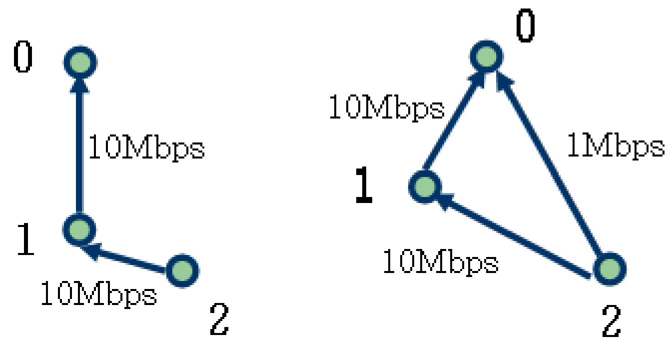
Experiments show that with a potential alternative link, nodes get much better throughput even if the link is never used in the optimal Equilibrium. It is the ability to disrupt communication of the other party that makes the improvement possible.

In Fig. 3.5(a), two nodes A and B are trying to access the gateway G, and the outcome is:

- Throughput of Node 1: 7.5Mbps
- Throughput of Node 2: 1.25Mbps

In Fig. 3.5(b), B has an alternative link to G. This time, the outcome is:

- Throughput of Node 1: 4.4Mbps
- Throughput of Node 2: 2.8Mbps



(a) (b)

Figure 3.5: The power of threatening

3.5.2 CHAINS

Our studies show that even in the view of game theory, distant nodes tend to get starved. This should not be a big surprise to us. Distant nodes have very few, or even no contribution to the network at all. It costs other nodes great effort to forward their traffic. There is no hope they should get equal throughput as other nodes.

Figure 3.6: Distant nodes get starved

In Fig. 3.6, we assume each node can communicate with its neighbor at 10Mbps. The outcome of the game is:

- Throughput of Node 1: 7.64Mbps
- Throughput of Node 2: 0.956Mbps

- Throughput of Node 3: 0.12Mbps
- Throughput of Node 4: 0.02Mbps

If we assume in addition to the 10Mbps link with the neighbors, nodes can communicate with each other at 1Mbps, the outcome is more balanced, but centered nodes still enjoy a higher shares:

- Throughput of Node 1: 4.76Mbps
- Throughput of Node 2: 1.42Mbps
- Throughput of Node 3: 0.43Mbps
- Throughput of Node 4: 0.28Mbps

We are not alone. Leino [15] shows consistent results. In his simulation, the nodes in the center of the network always tend to escape from the network, which results in the next centered node wishing to escape. Our study also suggests that to keep the interest of the center node to stay in the network, it should be assigned very high bandwidth.

3.5.3 SYMMETRY

Let us study another example.

Assuming all links shown in Fig. 3.7 are 10Mbps, the game generates the following outcome:

- Throughput of Node 1: 2.51Mbps
- Throughput of Node 2: 3.35Mbps
- Throughput of Node 3: 2.51Mbps

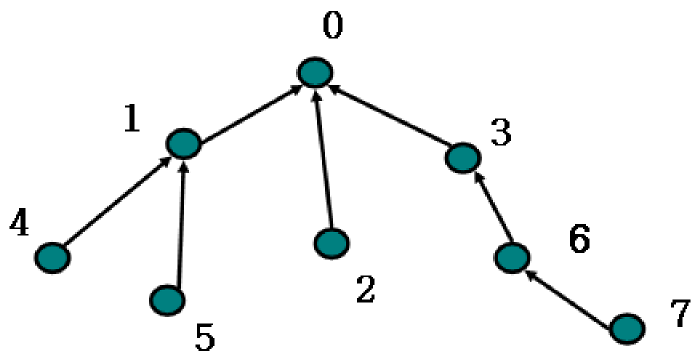


Figure 3.7: Another example

- Throughput of Node 4: 0.21Mbps
- Throughput of Node 5: 0.21Mbps
- Throughput of Node 6: 0.31Mbps
- Throughput of Node 7: 0.05Mbps

Again we see the nodes get less throughput as they are further from the gateway. Moreover, we see that symmetric nodes in the network get the same throughput, which naturally makes sense. Nodes 4 and 5 get the same throughput because they are symmetric in the network. Nodes 1 and 3 get the same throughput because both of they have the same link quality to the gateway and both of them have children with the same link quality.

Now we assume there are additional links between every two nodes in the network with the link quality of 1Mbps; the outcome would be different:

- Throughput of Node 1: 2.23Mbps

- Throughput of Node 2: 0.24Mbps
- Throughput of Node 3: 2.13Mbps
- Throughput of Node 4: 0.71Mbps
- Throughput of Node 5: 0.71Mbps
- Throughput of Node 6: 0.65Mbps
- Throughput of Node 7: 0.41Mbps

The surprise may be the fact that this time node 2 gets the poorest throughput. However, if the cooperation senario is broken, all nodes just try to connect to the gateway by their competition; then all nodes will get a throughput of 0.232 Mbps. Therefore it is still beneficial for node 2 to accept the equilibrium. For the other nodes, it is obviously much better than the non-cooperative case. However, due to the overhead of protocols, the improvements is so small to make the solution impractical. This is a simplified mathematic model and a lot of further work should be done. We also notice that simplifying the multi-node game to recursively two-node game may lose some information and lead to some problems. We will keep studying in future works.

These examples show the fact that nodes that act as a important router in the center of the network get the highest throughput. The fewer hops it is away from the gateway, the higher the throughput it gets; likewise, if the node acts as a router for many other nodes.

3.5.4 LIMITATIONS

Our algorithm assumes the whole network is within a single clique. Therefore, there are no concurrent trasmissions in the network. While this may be true if the network is small and all nodes are within the interference range of each other, it does not apply to larger networks. We will address this problem in the following chapter.

4 Realistic Solutions and Simulations

The algorithm in Chapter 4 only works within a single clique; *i.e.*, we assumed all the nodes are within the same collision domain. Therefore, the algorithm works well only if the network is small. For larger networks we need further tools to study and analyze the problem.

Interference and collision [23] are a nontrivial problems to deal with. For example, the nodes 1,2 and 3 in Fig. 4.1 are within each other's interference range; therefore they cannot transmit at the same time. Moreover, nodes 2, 3 and 4 are also within each other's interference range. However, node 4 can transmit to node 3 while node 1 transmits to the gateway as long as link l_1 and link l_4 are not in each other's interference range.

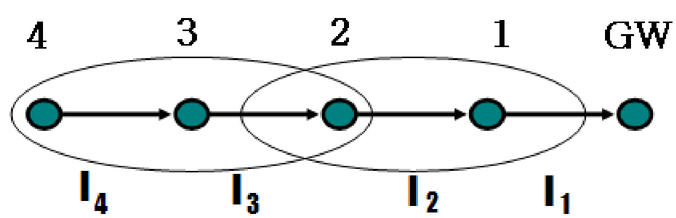


Figure 4.1: Interference range in a chain

In larger networks the problem may be even more complicated. Fortunately, there has already been some research to study this problem.

4.1 RELATED WORK

By adopting the concepts of link-usage matrix and medium-usage matrix, people managed to achieve a certain form of fairness in their simulations. These works are introduced by Jakubczah et al. [12]. We first have a look how these matrices work.

Before we introduce the matrix, we list all our notations below:

- number of streams in the network: n
- number of links in the network: m
- streams in the network: $s_1, s_2, \ldots, s_n$
- throughput of stream s_i: $R_i, i = 1, 2, \ldots, n$
- links in the network: $l_1, l_2, \ldots, l_m$
- link capacity of link l_i: $C_i, i = 1, 2, \ldots, m$
- collision domain for link l_i: $u_i \subset \{l_1, l_2, \ldots, l_m\}, i = 1, 2, \ldots, m$

The first matrix L is called the link-usage matrix. L is a $m \times n$ matrix defined as:

$$L[i,j] = \begin{cases} 1, & \text{if stream } s_j \text{ uses link } l_i; \\ 0, & \text{otherwise;} \end{cases}$$

The link-usage matrix provide us with the information about which links are involved in each stream. To also included the link-capacity information, we define the $m \times n$weighted link-usage matrix L' as:

$$L'[i,j] = \frac{1}{C_i} L[i,j] \tag{4.1}$$

The third matrix M is called the medium-usage matrix, which is a $m \times m$ matrix defined as:

$$M[i,j] = \begin{cases} 1, & \text{if } l_j \in u_i; \\ 0, & \text{otherwise;} \end{cases}$$

The stream-throughput vector R is defined as:

$$R = (R_1, R_2, \ldots, R_n)^T \tag{4.2}$$

The constraint of the throughput of the streams in a network can be represented by the following formula:

$$ML'R \leq 1_m \tag{4.3}$$

Where 1_m is a m-dimensional vector $(1, 1, 1...1)^T$.

Note that M is a $m \times m$ matrix, L' is a $m \times n$ matrix, and R is a $n \times 1$ matrix, therefore both $ML'R$ and 1_m are m-dimensional vector. Thus, the above vector inequality actually contains n numerical inequalities.

We use the senario in Fig. 4.1 as an example to show how to calculate the absolute fairness share with the matrix.

- number of streams in the network: $n = 4$
- number of links in the network: $m = 4$
- streams in the network: s_1, s_2, s_3, s_4, where s_i denotes the stream originated from node i to the gateway.
- throughput of stream s_i: $R_i, i = 1, 2, 3, 4$

- links in the network: l_1, l_2, l_3, l_4
- link capacity of link l_i: $C_i, i = 1, 2, 3, 4$
- collision domain for link l_i: $u_i \subset \{l_1, l_2, l_3, l_4\}, i = 1, 2, 3, 4$

Matrix L is:

$$L = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Matrix L' is:

$$L' = \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix}$$

Matrix M is:

$$M = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix}$$

By the definition of absolute fairness, $R_1 = R_2 = R_3 = R_4 = R$, therefore,

$$R = (R_1, R_1, R_1, R_1)^T \tag{4.4}$$

Finally, the constraint can be expressed as:

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} R_1 \\ R_1 \\ R_1 \\ R_1 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Calculating this yields:

$$\begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} + \frac{1}{C_2} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \\ \frac{1}{C_1} & \frac{1}{C_1} + \frac{1}{C_2} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \\ \frac{1}{C_1} & \frac{1}{C_1} + \frac{1}{C_2} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} & \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} + \frac{1}{C_3} & \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} R_1 \\ R_1 \\ R_1 \\ R_1 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Which is equivalent to the combination of the four inequalities:

$$(\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}) \times R_1 \leq 1 \tag{4.5}$$

$$(\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}) \times R_1 \leq 1 \tag{4.6}$$

$$(\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}) \times R_1 \leq 1 \tag{4.7}$$

$$(\frac{1}{C_2} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}) \times R_1 \leq 1 \tag{4.8}$$

We hope to maximize R_1 under the above constraints, which yields:

$$R_1 = \frac{1}{\frac{4}{C_1} + \frac{3}{C_2} + \frac{2}{C_3} + \frac{1}{C_4}} \tag{4.9}$$

So R_1 is the fair share for the four nodes if absolute fairness is applied.

4.2 OUR SOLUTION

Our work may be considered as an extension of the above work. Instead of absolute fairness, we take game playing into account.

The core idea of the Raiffa solution is that the difference between the normalized security level of agents should be maintained in their final assigned normalized utility. Therefore, when we maximize the normalized utility of any agent, the normalized utility of all agents reach their maximum.

Recall that, in the previous section, with the goal of absolute fairness, we have the following constraint:

$$R = (R_1, R_1, R_1, R_1)^T \tag{4.10}$$

If we replace this constraint with the constraint that the difference between the normalized utilities of agents should be maintained, we get another solution, which meets all the interference constraints as well as implements the Raiffa-solution concept.

To implement the solution, we have to address two critical problems: How to normalized the utiltiy of each node? How to find out the securtiy level of each node?

The former problem is relatively easier to address. Let X be any node in the network. The linear transformation from $U_X \in [MAX_{U_X}, MIN_{U_X}]$ to $NU_X \in [0, 1]$ is straightforward:

$NU_X = \frac{U_X - MIN_{U_X}}{MAX_{U_X} - MIN_{U_X}}$

Where $MINU_X = 0$ for all X and $MAXU_X$ is the solution of R_X in Eq. (4.3) when

$$R[i] = \begin{cases} R_X, & \text{if stream i is originated from node X} \\ 0, & \text{otherwise;} \end{cases}$$

The latter problem is difficult to address. In a large network, it is not clear what is the minimum bandwidth a node can guarantee itself. However, as long as there are so many nodes in the network, if all the other nodes act against one of them, then it is very likely that the actual bandwidth that node gets will be approximately zero. In other words, all the nodes have very low, if not zero, security level. Moreover, if we assume nodes can perform a Denial of Service (DoS) attack by continually broadcasting, then every node does have zero security level, as any of them can perform a DoS attack and stop every node from transmitting. Therefore, we adopt an approximation in our final solution: we assume all nodes have zero security level. This may not be true, but it should be very close to the reality.

We will use the example from Section 4.1 to show how we calculate the fair share. The network topology is the same as shown in Fig. 4.1. We use the same notations as in Section 4.1. The matrices M,L, and L' are the same as in Section 4.1.

First we normalize the utility of the nodes. To find the normalized utility of node 1, we need MAX_{U_1}, which is the solution of R_1 in Eq. (4.3) when $R = (R_1, 0, 0, 0)^T$, ***i.e.***,

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} R_1 \\ 0 \\ 0 \\ 0 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Solving these inequalities we find the maximal possible value for $R_1 \leq C_1$. Therefore, the normalized utility of node 1 is given by:

$$NU_1 = \frac{U_1 - MIN_{U_1}}{MAX_{U_1} - MIN_{U_1}} = \frac{R_1 - 0}{C_1 - 0} = \frac{R_1}{C_1} \tag{4.11}$$

Similarly, the solution of R_2 in Eq.(4.3) when $R = (0, R_2, 0, 0)^T$ yields the MAX_{U_2}:

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} 0 \\ R_2 \\ 0 \\ 0 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Solving these inequalities we find the maximal possible value for $R_2 \leq \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$. Therefore, the normalized utility of node 2 is given by:

$$NU_2 = \frac{U_2 - MIN_{U_2}}{MAX_{U_2} - MIN_{U_2}} = \frac{R_2 - 0}{\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} - 0} = R_2(\frac{1}{C_1} + \frac{1}{C_2}) \tag{4.12}$$

Similarly, the solution of R_3 in Eq. (4.3) when $R = (0, 0, R_3, 0)^T$ yields the $MAXU_3$:

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ R_3 \\ 0 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Solving these inequations we find the maximal possible value for $R_3 \leq \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$. Therefore, the normalized utility of node 3 is given by:

$$NU_3 = \frac{U_3 - MIN_{U_3}}{MAX_{U_3} - MIN_{U_3}} = \frac{R_3 - 0}{\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}} - 0} = R_3(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}) \tag{4.13}$$

Similarly, the solution of R_4 in Eq. (4.3) when $R = (0, 0, 0, R_4)^T$ yields the $MAXU_4$:

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \\ R_4 \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Solving these inequations we find the maximal possible value for $R_4 \leq \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}}$. Therefore, the normalized utility of node 4 is given by:

$$NU_4 = \frac{U_4 - MIN_{U_4}}{MAX_{U_4} - MIN_{U_4}} = \frac{R_4 - 0}{\frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}} - 0} = R_4\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}\right) \quad (4.14)$$

Since we assume zero security level, the requirement of

$NU_X - NU_Y = SL_X - SL_Y$

where $X, Y \in \{1, 2, 3, 4\}$ leads to

$$NU_1 = NU_2 = NU_3 = NU_4 \quad (4.15)$$

Taking Eq.(4.11),Eq.(4.12),Eq.(4.13) and ,Eq.(4.14) into Eq.(4.15) yields:

$$R_1\frac{1}{C_1} = R_2\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = R_3\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = R_4\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}\right) \quad (4.16)$$

as our fairness constraint. Finally, let $R = (R_1, R_2, R_3, R_4)$ where

$$\begin{cases} R_1 = R_0 C_1 \\ R_2 = R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} \\ R_3 = R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}} \\ R_4 = R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}} \end{cases}$$

and take R into Eq. (4.3) to find R_0:

$$\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} & \frac{1}{C_1} \\ 0 & \frac{1}{C_2} & \frac{1}{C_2} & \frac{1}{C_2} \\ 0 & 0 & \frac{1}{C_3} & \frac{1}{C_3} \\ 0 & 0 & 0 & \frac{1}{C_4} \end{vmatrix} \begin{vmatrix} R_0 C_1 \\ R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} \\ R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}} \\ R_0 \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}} \end{vmatrix} \leq \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}$$

Solving the inequalities we have $R_0 \leq \frac{1}{4}$. Therefore, our fair shares are:

$$\begin{cases} R_1 = \frac{C_1}{4} \\ R_2 = \frac{1}{4(\frac{1}{C_1}+\frac{1}{C_2})} \\ R_3 = \frac{1}{4(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3})} \\ R_4 = \frac{1}{4(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4})} \end{cases}$$

4.3 SIMULATION AND DISCUSSIONS

We implement our solution in the Shoshin ns2 simulation testbed[lily]. A source-rate-control algorithm limits the rate of each stream. We modified the piece of code that calculates the fair share for each node.

We use the ns-2 simulator to do the simulation. The default physical interface has transmission range of 250.0 meters and interference range of 550.0 meters. We set the

MacDataRate to be 1Mbps for each link. The link capacity is 860 kbps. We used a packetSize of 1500 bytes. The simulation runs for 125 seconds.

We implement the simulation of the scenario depicted in Fig. 4.2, where we place the nodes 200 meters apart.

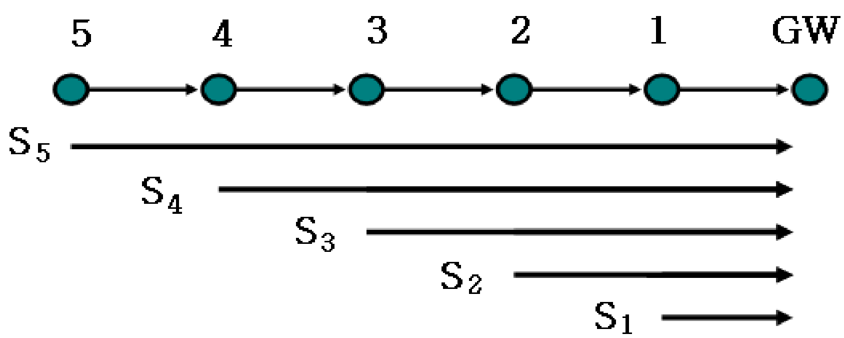


Figure 4.2: Simulation set up

Initially we use TCP traffic without our fairness algorithm, and the throughputs of the four streams are shown in Fig. 4.3. We then turn our algorithm on, and the throughputs of the streams are shown in Fig. 4.4.

We see several things from the simulation:

1. The relative order of the throughputs of the streams are almost the same in both cases

2. The centered nodes get worse throughput in our solution. In Fig. 4.3, stream 1 get more than 200 kbps most of the time, while in Fig. 4.4 we can see that in our solution, stream 1 gets only 150kbps.

3. The distant nodes get better throughput in our solution. In Fig. 4.3, stream 5 get starved some times, and the average throughput is around 20kbps, while in Fig.

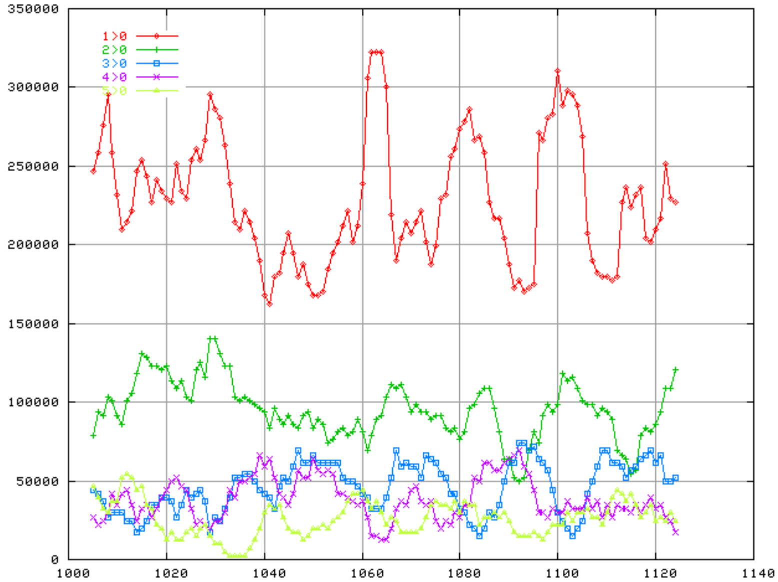


Figure 4.3: TCP without fairness

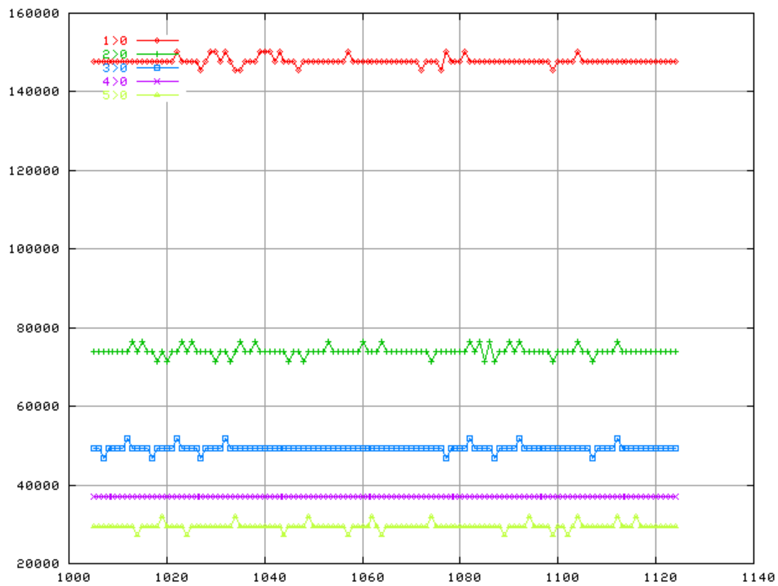


Figure 4.4: TCP with fairness

4.4 we can see that in our solution, stream 5 gets stable 30Kbps. The throughput of node 3 and 4 are also improved a little.

The simulation results make sense. The relative order of the throughputs of the nodes is maintained; therefore the solution of our algorithm reflects the different cost for the nodes to access the gateway. On the other hand, the balanced assignment prevents far away nodes from starving.

4.4 TEMPORAL FAIRNESS AND OUR SOLUTIONS

In the examples in Section 4.1 and 4.2, our fair share assignments are the same as in the definition of temporal fairness. This is non-trivial. We can show our solution does lead to temporal fairness.

In temporal fairness, each stream takes the same amount of spectrum time to arrive at the gateway, subject to the max-min limitations.

Recall in our solution, we assume cooperation and require:

$NU_i - NU_j = SL_i - SL_j$

where

$U_i = T_i$, $NU_i = \frac{U_i}{MAX_{U_i}}$ and $SL_i = 0$

for every two i, j.

Therefore, our requirement leads to:

$\frac{T_i}{MAX_{U_i}} = \frac{T_j}{MAX_{U_j}}$ for every two node i, j.

where MAX_{U_i} is the throughput node i gets when there is only one stream from i to the gateway in the network. Therefore, $\frac{T_i}{MAX_{U_i}}$ equals the spectrum time for the stream from node i to the gateway. Therefore, the requirement becomes “the spectrum time for stream from node i = the spectrum time for the stream from node j” for every two nodes i, j. This is exactly what temporal fairness claims.

5 Conclusions and Future Work

In this thesis we proposed our work in applying game theory in *ad hoc* networks (as well as wireless mesh networks) to determine the proper resource distribution among the nodes. A framework algorithm is proposed, simulations are performed and discussed.

The thesis is organized in the order we studied the problem. We first did a brief survey of *ad hoc* and wireless mesh networks, and evaluated the related research in this field. Then we studied the simplest two-node games. By parameterizing the action space, we reduce the game to a normal-form game with mixed-strategy space. Since the Nash equilibrium is not always Pareto optimal we looked for a solution using cooperative game theory. We adopted the Raiffa solution.

The Raiffa solution is the best solution concept we feel reasonable. However, there may be other solution ideas that are also good in this case. Note that the solution is independent of the framework algorithm, so one can readily choose other solution ideas and still make use of the framework to address the problem.

Then we studied ways to extend our work to multiple nodes. We reduce the network to a tree, which leads to the recursive-solution idea.

After that, we take the interference range into account and a more realistic algorithm is presented. Combined with existing source-rate-control algorithms, we have validated our work by simulation.

Finally, we found out that our solution from the cooperative-game view coincides with temporal fairness, which goes beyond our expectation.

5.1 Future Work

Some assumptions in our algorithm, like the topology knowledge is known to every node, need to be justified. It is possible that local topology knowledge would be enough, but further works needs to be done to make it clear.

The model in the thesis implements the simplest utility function and normalization to convert the throughput to the normalized utility. Our utility function is simply $U(X) = X$; our normalization is just the linear transformation. It may be better to introduce other utility functions and normalization procedure to model the game and get results other than temporal fairness.

We may also consider other solution concept like Nash bargain equilibrium, and see if the outcome is different.

Finally, more simulations with different settings are required to study and validate the model better.

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