

# Wideband Capon Beamforming for a Planar Phased Radar Array with Antenna Switching

Moon-Sik Lee

**ABSTRACT**—A wideband beamforming algorithm for estimating the azimuth angle, elevation angle, velocity, and range using a planar phased radar array with antenna switching is proposed. It uses the time-variant steering vector model. Simulation results illustrating the performance of the proposed beamformer are presented.

**Keywords**—Planar array, phased radar array, antenna switching, wideband beamforming, Capon beamforming.

## I. Introduction

Recently, there has been great interest in the development of advanced radar systems with phased arrays [1], [2]. In traditional multichannel radar array systems, the number of receivers should be equal to the number of receiving antennas, so the hardware expense and power consumption are very high [3]. A radar array system based on antenna switching is a promising substitute due to its lower cost and simpler front-end circuitry. Recently, several beamforming schemes for such switching-type linear phased radar array systems have been proposed, such as MUSIC signal processing to estimate the angle and velocity [4], an ESPRIT method to estimate the angle [5], and a Capon beamformer to find the angle, velocity, and range [6]. However, these algorithms using linear antenna arrays do not give a 2D direction finding of the azimuth and elevation angles.

In this letter, we consider a planar phased radar array with antenna switching and propose a new wideband beamforming

algorithm to simultaneously estimate the azimuth angle, elevation angle, velocity, and range using the considered radar array system.

## II. System Model

Consider a planar phased radar array with a rectangular configuration of  $N_1 \times N_2$  receiving antennas as shown in Fig. 1. Each antenna element is denoted by  $(n_1, n_2)$ , where  $1 \leq n_1 \leq N_1$  and  $1 \leq n_2 \leq N_2$ . Assume that the transmitted linear frequency modulated (FM) signal is of the form  $x_0(t) = \exp[j2\pi(f_0 t + f_1 t^2 / 2)]$ ,  $0 \leq t < T$ , where  $T$  is the pulse period,  $f_0$  is the initial frequency, and  $f_1$  is the chirp rate given by  $f_1 = F/T$ , where  $F$  is the bandwidth. The pulses are transmitted starting at the time instants  $[(u-1)N_2 + n_2 - 1]T$ ,  $u=1, 2, \dots, U$  and  $n_2=1, 2, \dots, N_2$ , where  $U$  is the number of cycles.

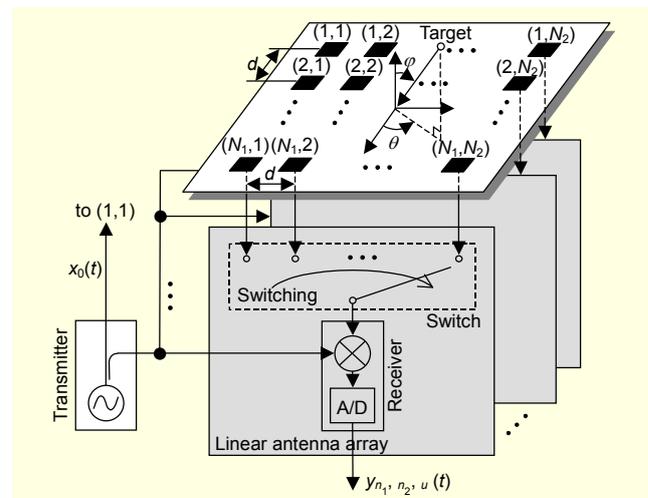


Fig. 1. System block diagram of the planar phased radar array with antenna switching.

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An echo signal from a target received by the antenna element denoted by  $(n_1, n_2)$  at the  $u$ -th cycle,  $x_{n_1, n_2, u}(t)$ , is mixed with the complex conjugate transmitted signal  $x_0^*(t)$ . Then, we obtain a frequency down converted signal. The received signal in vector notation,  $\mathbf{y}(t) = (y_{n_1, n_2, u}(t))_{N_1 N_2 U \times 1}$ , is given by

$$\mathbf{y}(t) = \mathbf{a}(\theta, \varphi, \nu, \rho, t)s(t) + \mathbf{m}(t), \quad (1)$$

where  $s(t)$  and  $\mathbf{a}(\theta, \varphi, \nu, \rho, t)$  denote the desired signal and its corresponding time-variant steering vector,  $\mathbf{a}(\theta, \varphi, \nu, \rho, t) = (a_{n_1, n_2, u}(\theta, \varphi, \nu, \rho, t))_{N_1 N_2 U \times 1}$ , respectively;  $\theta$  is the azimuth angle,  $\varphi$  is the elevation angle,  $\nu$  is the velocity,  $\rho$  is the range, and  $\mathbf{m}(t)$  refers to the ‘‘other-signals-plus-noise’’ vector. In radar array systems transmitting linear FM waveforms, the range resolution  $\Delta\rho$  significantly depends on the sweep bandwidth  $F$  as  $\Delta\rho = c/(2F)$ , where  $c$  is the velocity of propagation.

In (1), the signal  $s(t)$  and the steering vector  $a_{n_1, n_2, u}(\theta, \varphi, \nu, \rho, t)$  are of the form

$$s(t) = g_{1,1,1}(t) \exp \left[ j2\pi \left( -f_0 t_{1,1,1} + f_1 \frac{t_{1,1,1}^2}{2} \right) \right] \times \exp[j2\pi(-f_1 t_{1,1,1} t)], \quad (2)$$

$$a_{n_1, n_2, u}(\theta, \varphi, \nu, \rho, t) = \exp \left\{ j2\pi \left[ f_0 (t_{1,1,1} - t_{n_1, n_2, u}) + \frac{f_1}{2} (2t - (t_{1,1,1} + t_{n_1, n_2, u}))(t_{1,1,1} - t_{n_1, n_2, u}) \right] \right\}, \quad (3)$$

where  $g_{1,1,1}(t)$  is the attenuated amplitude,  $t_{1,1,1} = 2(\rho + \nu t)/c$ ,  $t_{1,1,1} - t_{n_1, n_2, u} = -2\nu T[(u-1)N_2 + n_2 - 1]/c - \tau_{n_1, n_2}$ ,  $t_{1,1,1} + t_{n_1, n_2, u} = 2\{2\rho + \nu[(u-1)N_2 + n_2 - 1]T + 2\nu t\}/c + \tau_{n_1, n_2}$ , and  $\tau_{n_1, n_2} = \{[d(n_1 - 1)\cos\theta + d(n_2 - 1)\sin\theta]\sin\varphi\}/c$ , where  $\tau_{n_1, n_2}$  is the time-delay of the wave propagation from the reference antenna located at  $(1, 1)$  to the  $(n_1, n_2)$ th antenna, and  $d$  is the inter-element spacing of the antennas.

### III. Wideband Capon Beamforming

The main difference between linear phased arrays and planar phased arrays for the algorithm development is in the used steering vector model. The time-variant steering vector  $a_{n_1, n_2, u}(\theta, \varphi, \nu, \rho, t)$  in (3) derived for the considered planar phased array with antenna switching is a function of time  $t$  and carries information about the azimuth angle  $\theta$ , the elevation angle  $\varphi$  through  $\tau_{n_1, n_2}$ , the velocity  $\nu$  through  $t_{1,1,1} - t_{n_1, n_2, u}$  and  $t_{1,1,1} + t_{n_1, n_2, u}$ , and the range  $\rho$  through  $t_{1,1,1} + t_{n_1, n_2, u}$ . Therefore, the proposed beamforming algorithm can give estimations about the elevation angle as well as the azimuth angle,

$$\hat{s}(t) = \mathbf{h}^H(\theta, \varphi, \nu, \rho, t)\mathbf{y}(t), \quad (4)$$

where  $(\cdot)^H$  is the Hermitian transpose, and  $\mathbf{h}(\theta, \varphi, \nu, \rho, t)$  is

the beamforming weight given by

$$\mathbf{h}(\theta, \varphi, \nu, \rho, t) = \frac{\hat{\mathbf{R}}_y^{-1} \mathbf{a}(\theta, \varphi, \nu, \rho, t)}{\mathbf{a}^H(\theta, \varphi, \nu, \rho, t) \hat{\mathbf{R}}_y^{-1} \mathbf{a}(\theta, \varphi, \nu, \rho, t)}, \quad (5)$$

where  $\hat{\mathbf{R}}_y$  is the estimate of the covariance matrix defined by  $\hat{\mathbf{R}}_y = \frac{1}{W} \sum_t \mathbf{y}(t)\mathbf{y}^H(t)$ , where  $W$  is the total number of observations.

The power function of the wideband Capon beamformer is obtained from the signal estimate (4) as

$$P(\theta, \varphi, \nu, \rho) = \frac{1}{W} \sum_t |\hat{s}(t)|^2. \quad (6)$$

Finally, the estimates of the parameters  $(\theta, \varphi, \nu, \rho)$  are defined as solutions of the following problem:

$$(\hat{\theta}, \hat{\varphi}, \hat{\nu}, \hat{\rho}) = \arg \max_{\theta, \varphi, \nu, \rho} P(\theta, \varphi, \nu, \rho). \quad (7)$$

The optimization in (7) can be produced as a straightforward search on the grid of values of  $(\theta, \varphi, \nu, \rho)$ . The grid is made by regular intervals for each parameter of  $(\theta, \varphi, \nu, \rho)$ . For example, the grid of values of  $\theta$  is given by  $[\theta_{\text{start}}: \Delta\theta: \theta_{\text{end}}]$ , where  $\Delta\theta$  is the interval, and  $\theta_{\text{start}}$  and  $\theta_{\text{end}}$  are the start value and the end value of  $\theta$  for the search, respectively. Therefore, the computational complexity of (7) depends on the number of all considered values of  $(\theta, \varphi, \nu, \rho)$ .

Assuming that the chirp rate  $f_1$  in (3) is small, that is,  $f_1 \approx 0$ , we obtain the time-invariant simplified steering vector model as

$$a_{n_1, n_2, u}(\theta, \varphi, \nu) = \exp\{j2\pi[f_0(t_{1,1,1} - t_{n_1, n_2, u})]\}. \quad (8)$$

This simplified model (8) has an important advantage in that all standard beamforming algorithms are immediately available for application. However, a drawback is that information about the range  $\rho$  is lost, and it is impossible to evaluate the role of the sweeping bandwidth of the chirp rate  $f_1$ . The performance of the proposed wideband Capon beamformer is evaluated through a comparison with narrowband beamforming algorithms which use the time-invariant steering vector model (8). To investigate this performance comparison, we have developed two narrowband beamformers, called the narrowband MUSIC and Capon beamformers, which use the MUSIC method [4] and Capon method [7] for the time-invariant model (8), respectively.

### IV. Simulation Results

In our simulations, we assume a planar phased radar array of  $N_1 \times N_2$  receiving antennas with a half-wavelength inter-element spacing, a linear FM signal with  $f_0 = 24$  GHz, the bandwidth  $F = 150$  MHz, and the pulse period  $T = 100$   $\mu$ s. A moving target with  $\theta = 30^\circ$ ,  $\varphi = 30^\circ$ ,  $\nu = 70$  km/h, and  $\rho = 50$  m is considered.

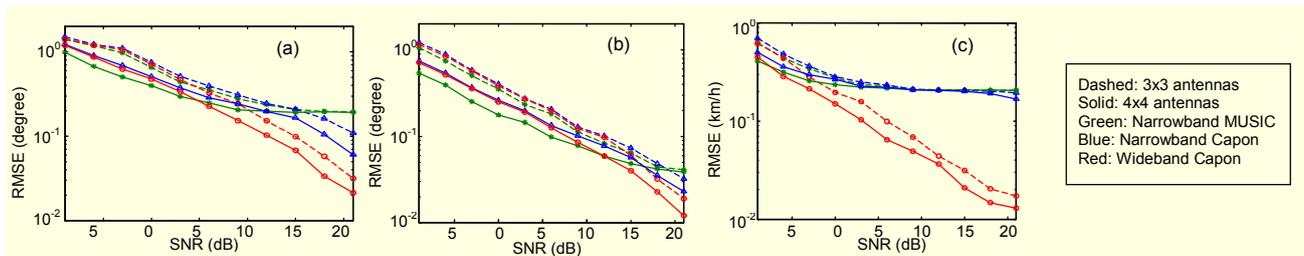


Fig. 2. Comparison of (a) azimuth angle, (b) elevation angle, and (c) velocity estimations as a function of the number of  $N_1 \times N_2$  antennas.

We assume that the attenuated amplitude  $g_{1,1,1}(t)$  in (2) is replaced by  $g_{1,1,1} + \eta(t)$ , modeling the time-variant echo-signal attenuation effects by random  $\eta(t)$ . We use  $g_{1,1,1} = 2$ ,  $N_1 = 4$  antennas,  $N_2 = 4$  antennas,  $U = 3$  cycles, and  $W = 100$  observations.

In Fig. 2, we compare the performance of the narrowband MUSIC, narrowband Capon, and wideband Capon beamformers in terms of estimation accuracy. The RMSEs of the azimuth angle, elevation angle, and velocity estimations are computed using 200 independent Monte Carlo simulation runs. The narrowband beamformers use the time-invariant steering vector model (8) in which information about the range is lost; therefore, they do not give the range estimation.

As seen in Fig. 2, the wideband Capon beamformer yields a clear improvement in the accuracy of the target parameters with an increase in the SNR and the number of  $N_1 \times N_2$  receiving antennas. The RMSEs for the azimuth angle and elevation angle of the narrowband Capon beamformer are close to those of the wideband Capon beamformer, whereas the performance of the narrowband MUSIC beamformer is not so good for higher SNRs. The accuracy of velocity estimation by the narrowband MUSIC and Capon beamformer is clearly worse than that by the wideband Capon beamformer. This comparison shows that the use of the time-variant steering vector model with the wideband

Capon beamformer significantly improves estimation accuracy, as compared with the narrowband beamformers which exploit the time-invariant steering vector model.

Figure 3 shows beam patterns for the wideband Capon beamformer for the azimuth angle, elevation angle, velocity, and range. The ten curves correspond to various samples of random observations. As seen in Fig. 3, the wideband Capon beamformer yields good performance in terms of estimation accuracy and resolution. The 3 dB resolutions for the azimuth angle, elevation angle, velocity, and range are about  $0.95^\circ$ ,  $0.54^\circ$ ,  $0.36$  km/h, and  $0.91$  m, respectively.

## V. Conclusion

In this letter, we proposed a wideband beamforming algorithm to estimate the target parameters using a planar phased radar array with antenna switching. We demonstrated that the proposed wideband beamformer offers significant performance gain over narrowband beamformers.

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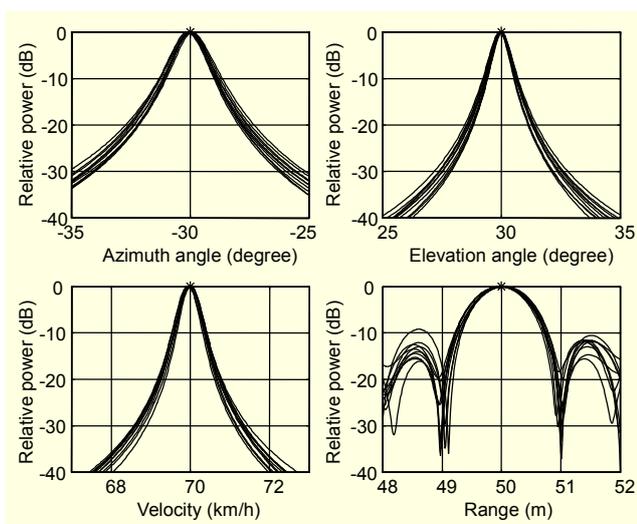


Fig. 3. Beam patterns for the wideband Capon beamformer. True target locations are indicated by '\*' signs on the figures.