Effects of LDPC Code on the BER Performance of MPSK System with Imperfect Receiver Components over Rician Channels

Goran T. Djordjevic, Ivan B. Djordjevic, and Predrag N. Ivanis

ABSTRACT—In this letter, we study the influence of receiver imperfections on bit error rate (BER) degradations in detecting low-density parity-check coded multilevel phase-shift keying signals transmitted over a Rician fading channel. Based on the analytical system model which we previously developed using Monte Carlo simulations, we determine the BER degradations caused by the simultaneous influences of stochastic phase error, quadrature error, in-phase-quadrature mismatch, and the fading severity.

Keywords—Bit error rate (BER), low-density parity-check (LDPC) codes, receiver imperfections, Rician fading.

I. Introduction

To achieve high spectral efficiency in contemporary wireless communication systems, multilevel phase-shift keying (MPSK) is used. As the number of phase levels increases, the system performance is more sensitive to receiver imperfections. The bit error rate (BER) is generally influenced by several receiver imperfections, including phase error, quadrature error, and in-phase-quadrature (I-Q) mismatch [1]-[4]. Since the reference carrier signal in the receiver is reconstructed from a noise-corrupted signal transmitted over a fading channel, there is a difference between the extracted reference signal phase and the received signal phase [1]-[3]. Quadrature error results from the phase shifts other than 90° between the I and Q receiver branches [1], [2], [4]. The I-Q gain mismatches are generated by imperfect mixers and filters at the receiver branches [1], [2].

In [1], the influence of static phase error, quadrature error, and I-Q mismatch on the conditional symbol error probability (SEP) in detecting uncoded MPSK signals transmitted over a fading channel was studied. In [2], a new expression was presented for the average SEP of the uncoded MPSK signals with an I-Q imbalance over additive white Gaussian noise (AWGN), Rician, and Rayleigh fading channels. Taking the quadrature error into account, Park and Cho [4] presented new expressions for the average SEP of MPSK, as well as the individual SEP of each MPSK signal in the AWGN channel.

In this letter, the BER degradations caused by receiver imperfections when low-density parity-check (LDPC) coded MPSK signals are transmitted over the Rician fading channel are determined by computer simulations. Unlike [1]-[3], where the static or uniformly distributed phase error was observed, we consider the phase error as a stochastic process with the Tikhonov probability density function (PDF) [5], which is a more realistic model. We also take into account all other receiver imperfections from [1]-[4]. While other studies treated the problem of the influence of receiver imperfections on BER performance, where uncoded MPSK signal transmission was observed [1]-[4], we use large-girth LDPC codes [6], [7] for signal encoding.

II. System Model

Before being transmitted over a fading channel, a signal is

Manuscript received Apr. 22, 2009; revised June 3, 2009; accepted June 15, 2009.

This work was supported in part by the Ministry of Science of Serbia under Grants TR-11030 and TR-11036, and in part by the National Science Foundation (NSF), USA, under Grant IHCS-0725405.

Goran T. Djordjevic (phone: +381 18 529 304, email: goran@elfak.ni.ac.rs) is with the Faculty of Electronic Engineering, University of Nis, Nis, Serbia.

Ivan B. Djordjevic (e-mail: ivan@ece.arizona.edu) is with the Department of Electrical and Computer Engineering, University of Arizona, Arizona, USA

Predrag N. Ivanis (e-mail: predrag.ivanis@etf.bg.ac.rs) is with the Faculty of Electrical Engineering, University of Belgrade, Belgrade, Serbia.

doi:10.4218/etrij.09.0209.0163

encoded, block-interleaved, and modulated at the transmitter. If we assume Rician propagation, the fading envelope $r(t)$ is given by

$$r(t) = \sqrt{(x_F(t) + \sigma_F \sqrt{2K})^2 + y_F{}^2(t)}, \tag{1}$$

where K denotes the Rician factor, and $x_F(t)$ and $y_F(t)$ are the zero-mean Gaussian random variables with variance σ_F^2.

The receiver input signal is given by

$$\begin{aligned} s(t) &= r(t)\cos(\omega_0 t + \theta_i(t) + \gamma(t)) \\ &\quad + x(t)\cos(\omega_0 t) - y(t)\sin(\omega_0 t), \end{aligned} \tag{2}$$

where $r(t)$ is the fading envelope, $\theta_i(t)$ is the information bearing phase $\theta_i(t)=2\pi(i-1)/M$, $i=1, 2,\cdots, M$, where M is the number of phase levels. The stochastic phase due to transmission over the fading channel is denoted by $\gamma(t)$, and the in-phase and quadrature white Gaussian noise components with zero mean value and variance σ^2 are respectively denoted by $x(t)$ and $y(t)$. After multiplication by reference signals with the estimated phase denoted by $\hat{\gamma}(t)$, the outputs of the low-pass filters at the in-phase and quadrature branches are

$$\begin{aligned} z_{\mathrm{I}}(t) &= a_{\mathrm{I}} \cdot r(t) \cdot \cos(\theta_i(t) + \varphi(t)) + x_g(t), \\ z_{\mathrm{Q}}(t) &= a_{\mathrm{Q}} \cdot r(t) \cdot \sin(\theta_i(t) + \varphi(t) + \psi) + x_d(t), \end{aligned} \tag{3}$$

where a_{I} and a_{Q} are the signal amplifications in the in-phase (I) and quadrature (Q) branches, respectively. The quadrature error due to phase shift between the I and Q branches is denoted by ψ. The phase error $\varphi(t) = \gamma(t) - \hat{\gamma}(t)$ is a stochastic process with the Tikhonov PDF [4]

$$p_\varphi(\varphi) = (1/2\pi) \times \exp(\alpha \cos\varphi) / I_0(\alpha), \tag{4}$$

where $I_0(.)$ is the modified Bessel function of the first kind and zero order, and α is the signal-to-noise ratio (SNR) in the phase-locked loop (PLL) circuit, which can be specified in terms of the standard deviation of the phase error as $\alpha = 1/\sigma_\varphi{}^2$ [5]. The I-Q mismatch is specified by the parameter $a_r=a_{\mathrm{I}}/a_{\mathrm{Q}}$. In (3), $x_g(t)$ and $x_d(t)$ are the zero mean Gaussian processes with variance σ^2. The correlation coefficient between $x_g(t)$ and $x_d(t)$ is $\rho=\sin\psi$ [1], [2].

The samples of the signals $z_{\mathrm{I}}(t)$ and $z_{\mathrm{Q}}(t)$ in the time instant t_k are fed to the *a posteriori* probability (APP) demapper, deinterleaver, and LDPC decoder. The LDPC codes under study in this letter belong to the class of quasi-cyclic (QC) LDPC codes [6], [7]. Detailed descriptions of the encoding and decoding processes, as well as symbol and bit log-likelihood ratios for this type codes are given in [6], [7].

Example 1. By selecting p=773 and S={0, 2, 7, 3, 30, 38, 65, 97, 138, 158, 209, 298} (parameter p and set S are defined in [6], [7]), an LDPC code LDPC (9276, 6959) of rate R=0.75, column weight 3, and girth 8 is obtained.

Example 2. By selecting p=593 and S={0, 2, 7, 3, 30, 38, 65, 97, 138, 158, 239, 436}, an LDPC code LDPC (7116, 5337) of rate R=0.75, column weight 3, and girth 8 is obtained.

Example 3. By selecting p=360 and using different indices for the first three block-rows as follows S_1={5, 275, 310, 13, 19, 284, 3, 2, 151, 176, 46, 218}, S_2={344, 5, 275, 310, 13, 19, 284, 3, 2, 151, 176, 46} and S_3={151, 334, 275, 13, 284, 2, 176, 218, 5, 310, 19, 3}, an LDPC code LDPC (4320, 3242) of rate R=0.75, column weight 3, and girth 8 is obtained.

We design the LDPC codes with codeword lengths of 9276, 7116, and 4320 to compromise between the code performance and decoding algorithm complexity. For decoding, 25 iterations are sufficient because the improvement with more iterations is negligible.

III. Simulation Results and Conclusions

The LDPC encoder codewords of length n are written row-wise into the mxn block interleaver. The m bits at time instance i are taken column-wise from the block interleaver and Gray-mapped to determine the information bearing phase $\theta_i(t)$ out of $M=2^m$ possible phases. A Rician fading channel is simulated using (1), where samples corresponding to $x_F(t)$ and $y_F(t)$ are generated by the improved Jakes model [8]. The phase error $\varphi(t)$ with the Tikhonov PDF is generated by the acceptance/rejection method [9]. The Gaussian noise components $x_g(t)$ and $x_d(t)$ are generated using the Box-Muller method [9]. The variables a_{I}, a_{Q}, and ψ from (3) are used as the system parameters, as well as σ_φ, which is needed for calculating (4). The BER values are estimated by applying Monte Carlo simulations based on 3,000 information bit errors.

The influence of imperfect reference signal recovery on the BER performance of LDPC-coded QPSK signal is presented in Fig. 1. The proposed code LDPC (9276, 6959) has large

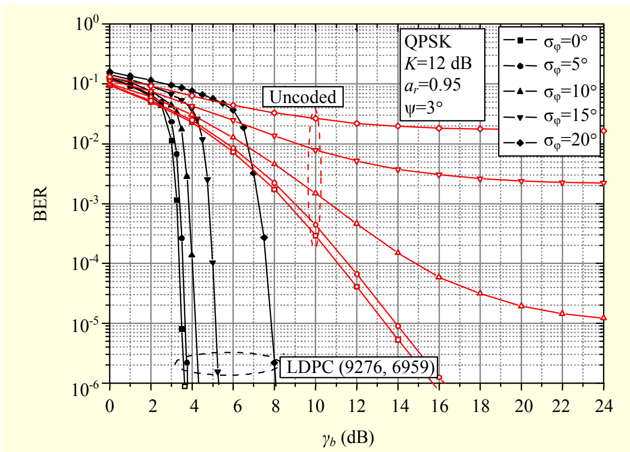

Fig. 1. Influence of imperfect reference signal extraction on BER in detecting uncoded and LDPC-coded QPSK signals.

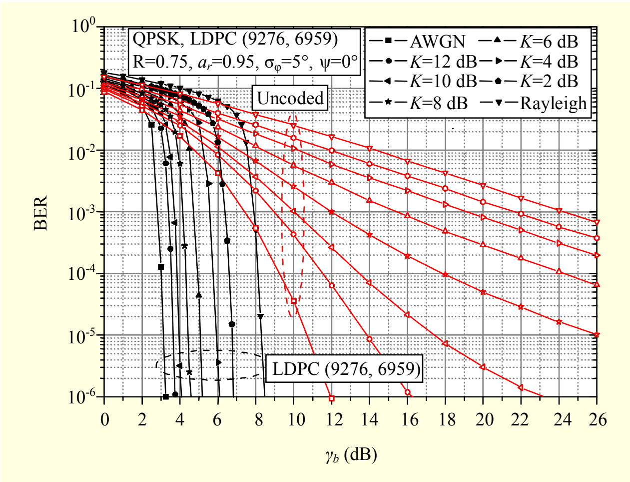

Fig. 2. Influence of fading severity on BER in detecting uncoded and LDPC-coded QPSK signals.

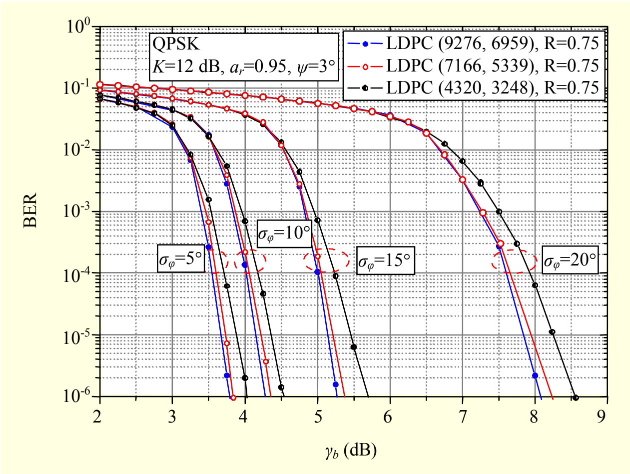

Fig. 3. Influence of LDPC code length on QPSK receiver performance.

coding gain in the Rician fading channel and is also robust to the deleterious effects of the stochastic phase noise. For BER=10^{-5}, when reference signal recovery is perfect, the coding gain is 10 dB. When $\sigma_\varphi=10^0$, the coding gain is 24.3 dB. For uncoded QPSK, when $\sigma_\varphi=15^0$, a BER of 10^{-5} cannot be reached at all because the BER floor slightly greater than 10^{-3} appears; however, with application of LDPC (9276, 6959), a BER of 10^{-6} is achieved for γ_b=5.3 dB, where γ_b denotes the average SNR per information bit.

Figure 2 shows the BER performance of LDPC-coded QPSK signal detection for different values of fading severity. Although the slopes of BER dependences for uncoded signals are considerably different for different fading severities (different values of K), the slopes of BER dependences for LDPC coded signals are nearly the same. This code performs very well for a wide range of the Rician parameter K.

Figure 3 compares the performance of three LDPC codes of the same code rate and different values of codeword lengths. The LDPC (9276, 6959) with a codeword length twice that of LDPC (4320, 3242) outperforms it by about 0.3 dB to 0.5 dB at a BER of 10^{-6}. From a comparison of BER dependences for codes LDPC (7166, 5339) and LDPC (9276, 6959), we conclude that further increase in codeword length for the same girth and code rate leads to little improvement in coding gains. Simulation results show that these LDPC codes do not exhibit a BER floor as low as BER=10^{-6}.

The mean-time to lock loss could be a significant problem when PLL is used in a fading environment, especially at low SNRs. It is also important to perform closed-loop nonlinear-model simulations including the carrier recovery PLL in order to determine the LDPC performance degradation at low SNRs due to repeated loss of lock. However, this is beyond the scope of this study and will be addressed in future.

References

[1] S. Park and S.H. Cho, "SEP Performance of Coherent MPSK over Fading Channels in the Presence of Phase/Quadrature Error and I-Q Gain Mismatch," *IEEE Trans. Commun.*, vol. 53, no. 7, July 2005, pp. 1088-1091.

[2] J. Park, S. Park, and K.R. Cho, "Performance Analysis of Symbol Error Probability for MPSK with I-Q Unbalance over a Rician Fading Channel," *IEICE Trans. Commun.*, vol. E88-B, no. 4, Apr. 2005, pp. 1702-1704.

[3] C.M. Lo and W.H. Lam, "Average BER of BPSK and QPSK Systems with Noisy Phase Reference over Nakagami-m Fading Channels," *IEICE Trans. Commun.*, vol. E84-B, 2001, pp. 1687-1689.

[4] S. Park and S.H. Cho, "Computing the Average Symbol Error Probability of the MPSK System Having Quadrature Error," *ETRI J.*, vol. 28, no. 6, Dec. 2006, pp. 793-795.

[5] N.C. Sagias and G.K. Karagiannidis, "Effects of Carrier Phase Error on EGC Receivers in Correlated Nakagami-m Fading," *IEEE Commun. Lett.*, vol. 9, no. 7, 2005, pp. 580-582.

[6] M.P.C. Fossorier, "Quasi-cyclic Low-Density Parity-Check Codes from Circulant Permutation Matrices," *IEEE Trans. Inf. Theory*, vol. 50, no. 8, Aug. 2004, 1788-1793.

[7] I.B. Djordjevic et al., "Large Girth Low-Density Parity-Check Codes for Long-Haul High-Speed Optical Communications," *Proc. OFC/NFOEC*, Feb. 24-28, 2008, Paper no. JWA53.

[8] Y.R. Zheng and C. Xiao, "Simulation Models with Correct Statistical Properties for Rayleigh Fading Channels," *IEEE Trans. Commun.*, vol. 51, no. 6, 2003, pp. 920-928.

[9] M.C. Jeruchim, P. Balaban, and K. Shanmugan, *Simulation of Communication Systems – Modeling, Methodology, and Techniques*, Kluwer Academic, NY, 2000, pp. 381-384.