Improved Design Criterion for Space-Frequency Trellis Codes over MIMO-OFDM Systems

Shouyin Liu and Jong-Wha Chong

In this paper, we discuss the design problem and the robustness of space-frequency trellis codes (SFTCs) for multiple input multiple output, orthogonal frequency division multiplexing (MIMO-OFDM) systems. We find that the channel constructed by the consecutive subcarriers of an OFDM block is a correlated fading channel with the regular correlation function of the number and time delay of the multipaths. By introducing the first-order auto-regressive model, we decompose the correlated fading channel into two independent components: a slow fading channel and a fast fading channel. Therefore, the design problem of SFTCs is converted into the joint design in both slow fading and fast fading channels. We present an improved design criterion for SFTCs. We also show that the SFTCs designed according to our criterion are robust against the multipath time delays. Simulation results are provided to confirm our theoretic analysis.

Keywords: Space-time coding, space-frequency trellis code, MIMO-OFDM system.

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I. Introduction

Space-time coding [1] as a promising approach to improve high data rate wireless communications by deploying multiple antennas at a transmitter or/and at a receiver in order to establish multiple input multiple output (MIMO) channels has attracted considerable attention and research interest in recent years. This technique represents an integration of diversity, modulation and channel coding. In [1], Tarokh et al derived the design criteria for space-time trellis codes (STTCs) in slow and fast flat fading channels and presented some handcraft STTCs. Based on the Tarokh's criteria, several optimized STTCs with better coding gain have since been reported in [2] and [3] for slow flat fading channels and in [5] and [7] to [9] for fast flat fading channels. Moreover, taking account of the special cases of large diversity, low signal-to-noise ratio (SNR), or distance spectrum [20], several improved design criteria have been derived: trace criteria in [4] and [6] and distance spectrum criteria in [8], [9] and [10].

In the presence of frequency selectivity, since the orthogonal frequency division multiplexing (OFDM) technique can significantly reduce the receiver complexity, a combined system of STTCs with OFDM was first proposed in [13], called a MIMO-OFDM system. Due to incorporating both the advantages of space-time coding and OFDM, MIMO-OFDM seems to be an attractive solution for future broadband wireless communication systems. Since the coding is performed across both transmit antennas and subcarriers of OFDM, i.e., space-frequency coding, MIMO-OFDM can exploit both spatial and frequency diversity (multipath diversity) without requiring the channel state information (CSI) at the transmitter. It was pointed out in [14] and [15] that the maximum achievable diversity is the product of the number of transmit antennas, receive antennas and the propagation paths between a pair of transmit and receive antennas. So far, a large number of papers [14]-[17] have been dedicated to obtaining the frequency diversity and to reduce the decoding complexity for MIMO-OFDM systems. However, several remaining fundamental issues are not well confirmed: for example, 1) the design criteria for space-frequency trellis codes (SFTCs); 2) the robustness of space-frequency codes in various multipath propagation environments; and 3) an efficient approach to obtain frequency diversity with low decoding complexity.

Recently, the robustness of SFTCs was addressed in [10] to [12]. In [10], the ideal interleaver and deinterleaver are employed at the transmitter and at the receiver, respectively, to convert the correlated fading channel constructed by consecutive subcarriers of an OFDM block into an ideal fast fading channel, so that the code design problem for SFTCs is simply converted into the conventional STTC's design problem over fast fading channels. In [11], from the general *pairwise error probability* (PEP) analysis, the good SFTCs with better robustness against the multipath time delays were studied using an exhaustive computer search. Using the closer-to-reality channel model, the impact of the propagation environment on the performance of SFTCs was investigated in [12]. However, the general design criteria for space-frequency codes are not yet available.

In this paper, we first analyze the characteristics of the fading channel constructed by all the subcarriers of an OFDM block in the frequency domain. We show that all the subcarriers of the OFDM block establish a frequency-correlated fading channel having a regular auto-correlation function related to the number of multipaths. The frequency-correlated fading channel is equivalent to the time-correlated fading channel resulting from the Doppler frequency in the conventional space-time coding systems. By introducing the first-order auto-regressive channel model, we derive the design criterion for SFTCs. Those SFTCs subjected to our design criterion possess a large coding gain and robustness for various multipath propagation environments.

The rest of the paper is organized as follows. We review the system model and preliminary results of the STTC's design criteria for slow and fast fading channels in section II. In section III, we investigate the frequency-selective fading channel. The design criterion for SFTCs is proposed in section IV. In section V, we provide several simulation results to confirm our theory. Finally, we give our conclusion in section VI.

II. System Model and Preliminary Results

1. System Model

Let us consider a baseband communication system with n_T transmit antennas and n_R receive antennas. The information data is encoded by a space-time encoder, which generates n_T parallel data sequences. At each symbol period t, the n_T parallel outputs $c_t^1, c_t^2, \cdots, c_t^{n_T}$ are simultaneously transmitted by n_T transmit antennas. We assume that the frame length is L and that the elements of the signal constellation are contracted so that the average energy of the constellation is 1. At symbol period t, the received signal at antenna j, $j = 1, 2, \ldots, n_R$, is given by [1]

$$r_t^j = \sqrt{E_s} \sum_{i=1}^{n_T} h_{i,j}(t) c_t^i + n_t^i, \tag{1}$$

where E_s denotes the average energy per symbol at the transmit antenna and n_t^i is independent Gaussian white noise with zero-mean and a variance of $N_0/2$ per dimension at receive antenna j at time t. The coefficient $h_{i,j}(t)$ is the fading attenuation for the path between the transmit antenna i and receive antenna j pair at time t. Throughout this paper, we assume that there is no spatial correlation between antennas at the transmitter and antennas at the receiver, and the coefficient $h_{i,j}(t)$ is modeled as a complex Gaussian random variable with zero-mean and a variance of 1/2 per dimension.

2. Pairwise Error Probability

Assume that a codeword, $\mathbf{c} = c_1^1 c_1^2 \cdots c_1^{n_T} c_2^1 \cdots c_2^{n_T} \cdots c_L^1 \cdots c_L^{n_T}$, was transmitted. A maximum likelihood decoder might select erroneously in favor of another codeword, $\mathbf{e} = e_1^1 e_1^2 \cdots e_1^{n_T} e_2^1 \cdots e_2^{n_T} \cdots e_L^1 \cdots e_L^{n_T}$. Assuming that ideal channel state information is available at the receiver, the conditional PEP and its upper bound are given by [1]

$$\begin{aligned} P(\mathbf{c}, \mathbf{e} | \mathbf{h}) &= Q\left(\sqrt{\frac{E_s}{2N_0} d_h^2(\mathbf{c}, \mathbf{e})}\right) \\ &\le \frac{1}{2} \exp\left(-d_h^2(\mathbf{c}, \mathbf{e}) \frac{E_s}{4N_0}\right), \end{aligned} \tag{2}$$

where $\mathbf{h}$ is the channel matrix and $d_h^2(\mathbf{c}, \mathbf{e})$ is the *modified Euclidean distance* (MED) between codeword $\mathbf{c}$ and $\mathbf{e}$. The MED is given by

$$d_h^2(\mathbf{c}, \mathbf{e}) = \sum_{t=1}^{L} \sum_{j=1}^{n_R} \left| \sum_{i=1}^{n_T} h_{i,j}(t)(c_t^i - e_t^i) \right|^2. \tag{3}$$

3. Design Criterion for Space-Time Trellis Codes in Rayleigh Slow Fading Channels

In the case of Rayleigh slow fading channels (quasi-static fading channels), the fading coefficients $h_{ij}(t)$ are constant in a frame but change independently from one frame to another such that

$$h_{i,j}(1) = h_{i,j}(2) = \cdots = h_{i,j}(L) = h_{i,j}. \tag{4}$$

Following the derivation in [1], the MED can be rewritten as

$$\begin{aligned} d_h^2(\mathbf{c},\mathbf{e}) &= \sum_{t=1}^{L}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T} h_{i,j}(c_t^i - e_t^i)\right|^2 \\ &= \sum_{j=1}^{n_R}\sum_{i=1}^{r}\lambda_i\left|\beta_{i,j}\right|^2, \end{aligned} \tag{5}$$

where λ_i ($i=1,2,\ldots,r$) and r are the non-zero eigenvalues and the rank of the *codeword distance matrix* $\mathbf{A}(\mathbf{c},\mathbf{e})=\mathbf{B}(\mathbf{c},\mathbf{e})\mathbf{B}^H(\mathbf{c},\mathbf{e})$, respectively, in which $\mathbf{B}(\mathbf{c},\mathbf{e})$ is the *codeword difference matrix* defined as $\mathbf{B}(\mathbf{c},\mathbf{e})=\mathbf{B}(\mathbf{c},\mathbf{e})=(\mathbf{b}_1,\mathbf{b}_2,\ldots,\mathbf{b}_t,\ldots,\mathbf{b}_L)$, and $\mathbf{b}_t = [c_t^1 - e_t^1,\ldots,c_t^{n_T} - e_t^{n_T}]^T$, where $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and the conjugate transpose. $\beta_{i,j}$ are independent complex Gaussian random variables with zero-mean and a variance of 1/2 per dimension. Substituting (5) into (2) and averaging over all $|\beta_{i,j}|$ term by term, then the PEP can be obtained [1] as

$$P(\mathbf{c},\mathbf{e}) \le \left(\prod_{i=1}^{r}\left(1+\lambda_i\frac{E_s}{4N_0}\right)\right)^{-n_R}. \tag{6}$$

At a high SNR, (6) can be further simplified [1] as

$$P(\mathbf{c},\mathbf{e}) \le \left(\frac{E_s}{4N_0}\right)^{-rn_R}\left(\prod_{i=1}^{r}\lambda_i\right)^{-n_R}. \tag{7}$$

As defined in [1], rn_R and $(\prod_{i=1}^{r}\lambda_i)^{1/r}$ are called diversity and coding gain, respectively, which are used to describe the performance measure of space-time coding. The code design is to maximize the minimum rank r over all pairs of distinct codewords while maximizing the coding gain. This is the so-called rank-determinant criterion [1].

4. Design Criterion for Space-Time Trellis Codes in Rayleigh Fast Fading Channels

In this case of Rayleigh fast fading channels, the fading coefficient $h_{ij}(t)$ varies independently from one symbol period to another. Following the derivation in [1], the $d_h^2(\mathbf{c},\mathbf{e})$ for a fast fading channel can be expressed as [1]

$$d_h^2(\mathbf{c},\mathbf{e}) = \sum_{t\in\rho(\mathbf{c},\mathbf{e})}\sum_{j=1}^{n_R}\left|\beta_{1,j}(t)\right|^2\left\|\mathbf{c}_t-\mathbf{e}_t\right\|^2, \tag{8}$$

where $\rho(\mathbf{c},\mathbf{e})$ denotes the set of time instances for $t=1,2,\ldots,L$ such that $\mathbf{c}_t \ne \mathbf{e}_t$ and $\left\|\mathbf{c}_t-\mathbf{e}_t\right\|^2 = \sum_{i=1}^{n_T}\left|c_t^i - e_t^i\right|^2$ and $\beta_{1,j}(t)$ are independent complex Gaussian random variables for different $j=1,2,\ldots,n_R$ and $t=1,2,\ldots,L$ with zero-mean and the variance of 1/2 per dimension. Let δ denote the number of $\rho(\mathbf{c},\mathbf{e})$, called *symbol-wise Hamming distance*. Then, there are a total of δn_R independent random variables in (8). Substituting (8) into (2) and averaging (2) over all the δn_R independent variables, $|\beta_{1,j}|$, the PEP for fast fading channels can be obtained [1] as

$$P(\mathbf{c},\mathbf{e}) \le \left(\prod_{t\in\rho(\mathbf{c},\mathbf{e})}\left(1+\left\|\mathbf{c}_t-\mathbf{e}_t\right\|^2\frac{E_s}{4N_0}\right)\right)^{-n_R}. \tag{9}$$

At a high SNR, (9) can be further simplified [1] as

$$P(\mathbf{c},\mathbf{e}) \le \left(\frac{E_s}{4N_0}\right)^{-\delta n_R}\prod_{t\in\rho(\mathbf{c},\mathbf{e})}\left\|\mathbf{c}_t-\mathbf{e}_t\right\|^{-2n_R}. \tag{10}$$

As defined in [1], δn_R is called diversity and $\prod_{t\in\rho(\mathbf{c},\mathbf{e})}\left\|\mathbf{c}_t-\mathbf{e}_t\right\|^{2/\delta}$ is called coding gain. The code design for fast fading channels is to maximize the minimum *symbol-wise Hamming distance*, δ, over all pairs of distinct codewords while providing the maximum coding gain. This is the so-called distance-product criterion [1].

5. Space-Frequency Coding

In order to deal with the frequency-selectivity problem, the combined MIMO-OFDM system was first reported in [13], in which the output codeword from the space-time encoder was allocated into subcarriers of OFDM blocks and was transmitted by its corresponding transmit antennas. We use c_k^i to denote the symbol transmitted at subcarrier k, $k=0,1,\ldots,K-1$, and at transmit antenna i, $i=1,2,\ldots,n_T$, where K is the number of total subcarriers of one OFDM block. We use $H_{ij}(k)$ to denote the fading coefficient (frequency response) for the k-th subcarrier from transmit antenna i to receive antenna j. At the receiver, the received signal at receive antenna j, $j=1,2,\ldots,n_R$, on the subcarrier k is expressed as

$$r_k^j = \sqrt{E_s}\sum_{i=1}^{n_T}H_{i,j}(k)c_k^i + n_k^j, \tag{11}$$

where n_k^j is independent complex Gaussian white noise with zero-mean and a variance of $N_0/2$ per dimension. Since the codeword is mapped onto subcarriers of the OFDM blocks, the space-time coding is renamed as space-frequency coding. Comparing (11) with (1), the only difference is that the time index t in (1) is replaced by subcarrier index k in (11).

III. Frequency-Selective Fading Channels and Correlation Properties

A frequency-selective fading channel can be described by the tapped-delay-line model in the time domain. The time-variant impulse response at time t to an impulse applied at time t-τ between transmit antenna i and receive antenna j can be expressed [18] as

$$h_{i,j}(t,\tau) = \sum_{l=0}^{P-1} q_{i,j}(t,l)\delta(\tau - lT_s), \tag{12}$$

where P is the number of non-zero channel taps, $q_{i,j}(t, l)$ is the complex coefficient of the l-th tap, and T_s is the sampling interval of the OFDM system. We usually model $q_{i,j}(t, l)$ as an independent Gaussian random variable with zero-mean and the normalized power, $E[|q_{i,j}(t, l)|^2]=1/P$. Since we deal with the quasi-static fading channels, i.e., the fading coefficients are constant during an OFDM block and vary independently from one OFDM block to another, the time index can be dropped for the sake of brevity. With a proper cyclic prefix, a perfect sampling time, and tolerable leakage, the frequency response, i.e., fading coefficient, for the k-th subcarrier between transmit antenna i and receive antenna j is expressed as

$$H_{i,j}(k) = \sum_{l=0}^{P-1} h_{i,j}(l)\,\mathrm{e}^{-j2\pi kl/K}. \tag{13}$$

Let us investigate the statistical characteristics of $H_{ij}(k)$ by evaluating its mean, variance, and auto-correlation. Using the known conditions, $E[h_{i,j}(l)]=0$, $E[|h_{i,j}(l)|^2]=1/P$, and $E[h_{i,j}(l)h^*_{i',j'}(l')] = 0$, $i \neq i'$, or $j \neq j'$, or $l \neq l'$, the results are summarized as follows:

$$\begin{aligned}
&E\left[H_{i,j}(k)\right] = 0 \\
&E\left[\left|H_{i,j}(k)\right|^2\right] = 1 \\
&R_{HH}(k,k+n) = E\left[H_{i,j}(k)H^*_{i,j}(k+n)\right]. \\
&\qquad = \frac{1}{P}\frac{1-\mathrm{e}^{j2\pi Pn/K}}{1-\mathrm{e}^{j2\pi n/K}} \\
&i \in \{1,2,\ldots,n_T\},\ \ j \in \{1,2,\ldots,n_R\} \\
&k,n \in \{0,1,\ldots,K-1\}
\end{aligned} \tag{14}$$

Observing (14), we can find the following:

1) Each fading coefficient $H_{ij}(k)$ corresponding to the k-th subcarrier is a complex random variable with zero-mean and a variance of 1 (1/2 per dimension). However, these subcarriers are correlated, and the auto-correlation function $R_{HH}(\cdot)$ only depends on the difference of the subcarrier index k. Therefore, the sequence of fading coefficients along the subcarrier index k can be viewed as a wide-sense stationary narrowband complex Gaussian process. In other words, the fading channel that is established by all the consecutive subcarriers within one OFDM block can be viewed as a frequency-correlated fading channel, which is identical to the conventional time-correlated fading channel described by the well-known Jakes model [18]. In the sequel, the design problem of space-frequency codes over MIMO-OFDM systems is identical to the problem of STTC design over time-correlated fading channels.
2) For a clear observation of the correlation characteristic versus the number of propagation paths, as an example, the correlation of the first subcarrier with all the others is plotted in Fig.1, where K=64 and P=2 to 7. As demonstrated, the larger the number of multipaths or time delay, the smaller the correlation between adjacent subcarriers.
3) One interesting aspect of the correlation function worth mentioning is that the correlation function at each case has P–1 zero-points corresponding to P–1 subcarriers. This means that for each subcarrier there are P–1 subcarriers that are uncorrelated with each other, and they are distributed uniformly among the K subcarriers with interval K/P. This characteristic will be detailed and discussed in another publication for obtaining full diversity over MIMO-OFDM systems.

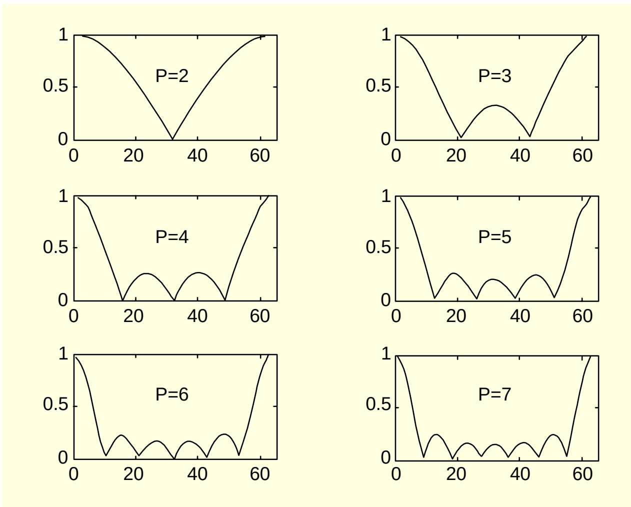

Fig. 1. Correlation of the first subcarrier with the others.

In the next section, we introduce the first-order auto-regressive model to express the frequency-correlated fading channels and derive the design criterion for SFTCs.

IV. Design Criterion for Space Frequency Trellis Codes

From the above analysis, the fading channel constructed by the consecutive subcarriers in an OFDM block can be modeled as a wide-sense stationary narrowband complex Gaussian process. We introduce the first-order auto-regressive model to express the random process (this method was also used by Peel et al. to analyze the performance of the differential unitary space-time modulation in [19]). Then, $H_{i,j}(k)$ is described as

$$H_{i,j}(k)=\sigma_{i,j}(k)H_{i,j}(0)+\eta_{i,j}(k)W_{i,j}(k), \tag{15}$$

where $H_{i,j}(0)$ is the fading coefficient of the 0-th subcarrier as a reference, which is a complex Gaussian random variable changing independently from one OFDM block to another with zero-mean and a variance of 1/2 per dimension. Variable $W_{i,j}(k)$ is an independent complex Gaussian random variable also with zero-mean and a variance of 1/2 per dimension, but it changes independently from subcarrier to subcarrier and is independent of $H_{i,j}(0)$. Variables $\sigma_{i,j}(k)$ and $\eta_{i,j}(k)$ are real numbers and $0\le\sigma_{i,j}(k)\le 1$, $0\le\eta_{i,j}(k)\le 1$. Since the energy is conserved, $[\sigma_{i,j}(k)]^2+[\eta_{i,j}(k)]^2 = 1$. In fact, the meaning of (15) is that one correlated fading channel can be decomposed into two independent components: one corresponds to the slow fading channel, the other to the fast fading channel. Clearly, $\sigma_{i,j}(k)$ and $\eta_{i,j}(k)$ exhibit a degree of correlation; a large $\sigma_{i,j}(k)$ (small $\eta_{i,j}(k)$) corresponds to a fading channel with high correlation (nearing the slow fading channel) and a large $\eta_{i,j}(k)$ (small $\sigma_{i,j}(k)$) to a channel with low correlation (nearing the fast fading channel). When $\sigma_{i,j}(k)=1$ $\eta_{i,j}(k)=0$, $k=1,\ldots,K$, the fading channel is a slow fading channel (quasi-static fading). Conversely, the fading channel is a fast fading one.

Substituting (15) into (14) and putting $\sigma_{i,j}(0)=1$, the relation of $\sigma_{i,j}(k)$ and $\eta_{i,j}(k)$ and correlation factor $R_{HH}(k)$ can be given as

$$\sigma_{i,j}(k)=|R_{HH}(k)| \tag{16}$$

$$\eta_{i,j}(k)=\sqrt{1-(\sigma_{i,j}(k))^2}=\sqrt{1-|R_{HH}(k)|^2}. \tag{17}$$

As shown in Fig. 1, in a narrow range of k (during a *simple-error-event* [20], [21]), $\sigma_{i,j}(k)$ is a descending function as subcarrier index k increases, and $\eta_{i,j}(k)$ is a rising function.

Substituting (15) into (3), the MED can be rewritten as

$$\begin{aligned}
d_h^2(\mathbf{c},\mathbf{e}) &= \sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T}H_{i,j}(k)(c_k^i-e_k^i)\right|^2\\
&=\sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T}(\sigma_{i,j}(k)H_{i,j}(0)+\eta_{i,j}(k)W_{i,j}(k))(c_k^i-e_k^i)\right|^2\\
&=\sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T}H_{i,j}(0)\sigma_{i,j}(k)(c_k^i-e_k^i)\right|^2 \qquad \textit{part } 1\\
&+\sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left|\sum_{i=1}^{n_T}W_{i,j}(k)\eta_{i,j}(k)(c_k^i-e_k^i)\right|^2 \qquad \textit{part } 2\\
&+\sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left\{2\,\mathrm{Re}\left[\sum_{i=1}^{n_T}H_{i,j}(0)\sigma_{i.j}(k)(c_k^i-e_k^i)\right]\right.\\
&\qquad\left.\times\mathrm{Re}\left[\sum_{i'=1}^{n_T}W_{i',j}(k)\eta_{i',j}(k)(c_k^{i'}-e_k^{i'})\right]\right\} \qquad \textit{part } 3\\
&+\sum_{k=0}^{K-1}\sum_{j=1}^{n_R}\left\{2\,\mathrm{Im}\left[\sum_{i=1}^{n_T}H_{i,j}(0)\sigma_{i.j}(k)(c_k^i-e_k^i)\right]\right.\\
&\qquad\left.\times\mathrm{Im}\left[\sum_{i=1}^{n_T}W_{i',j}(k)\eta_{i',j}(k)(c_k^{i'}-e_k^{i'})\right]\right\} \qquad \textit{part } 4.
\end{aligned} \tag{18}$$

where the real and imaginary parts of $W_{i,j}(k)$ are both independent random variables with zero-mean and a variance of 1/2, and $H_{i,j}(0)$ and $W_{i,j}(k)$ are independent of each other. After processing, the sum of *part* 3 and *part* 4 is expressed as

$$\begin{aligned}
s &= part3+part4\\
&=\sum_{k\in\rho(\mathbf{c},\mathbf{e})}\sum_{j=1}^{n_R}\sum_{i=1}^{n_T}\sum_{i'=1}^{n_T}2\sigma_{i,j}(k)\eta_{i',j}(k)\\
&\times\Big\{X_{i,i',j}^1(k)\big[\mathrm{Re}(c_k^i-e_k^i)\,\mathrm{Re}(c_k^{i'}-e_k^{i'})+\mathrm{Im}(c_k^i-e_k^i)\,\mathrm{Im}(c_k^{i'}-e_k^{i'})\big]\\
&+X_{i,i',j}^2(k)\big[\mathrm{Im}(c_k^i-e_k^i)\,\mathrm{Re}(c_k^{i'}-e_k^{i'})-\mathrm{Re}(c_k^i-e_k^i)\,\mathrm{Im}(c_k^{i'}-e_k^{i'})\big]\\
&+X_{i,i',j}^3(k)\big[\mathrm{Re}(c_k^i-e_k^i)\,\mathrm{Im}(c_k^{i'}-e_k^{i'})-\mathrm{Im}(c_k^i-e_k^i)\,\mathrm{Re}(c_k^{i'}-e_k^{i'})\big]\\
&+X_{i,i',j}^4(4)\big[\mathrm{Re}(c_k^i-e_k^i)\,\mathrm{Re}(c_k^{i'}-e_k^{i'})+\mathrm{Im}(c_k^i-e_k^i)\,\mathrm{Im}(c_k^{i'}-e_k^{i'})\big]\Big\},
\end{aligned} \tag{19}$$

where

$X_{i,i',j}^1(k)=\mathrm{Re}(H_{i,j}(0))\,\mathrm{Re}(W_{i',j}(k))$,

$X_{i,i',j}^2(k)=\mathrm{Re}(H_{i,j}(0))\,\mathrm{Im}(W_{i',j}(k))$,

$X_{i,i',j}^3(k)=\mathrm{Im}(H_{i,j}(0))\,\mathrm{Re}(W_{i',j}(k))$, and

$X_{i,i',j}^4(k)=\mathrm{Im}(H_{i,j}(0))\,\mathrm{Im}(W_{i',j}(k))$,

which are independent of each other with zero-mean and have a variance of 1/4. We use $\rho(\mathbf{c},\mathbf{e})$ to denote denotes the set of subcarriers $k=1,\ldots,K$, such that $\mathbf{c}_k\neq\mathbf{e}_k$. Let δ denote the number of $\rho(\mathbf{c},\mathbf{e})$. Then, s maximally contains $\delta\times n_T\times n_T\times n_R\times 4$ independent Gaussian random variables, $X^n_{i,i',j}(k)$, $n=1,...,4$, each with zero-mean and a variance of 1/4. Substituting (19) into (2), and averaging (2) over all the $\delta\times n_T\times n_T\times n_R\times 4$ Gaussian random variables, the contribution of *part* 3 and *part* 4 to PEP, denoted as $P_s(\mathbf{c},\mathbf{e})$, can be obtained. Obviously, $P_s(\mathbf{c},\mathbf{e})$ is related to the SNR, the number of $\delta\times n_T\times n_T\times n_R\times 4$, and the SFTCs' construction; however, it is not a large number. To simplify the discussion, we omit the effect of the construction of SFTCs on $P_s(\mathbf{c},\mathbf{e})$. As an example, we give an estimation value of $P_s(\mathbf{c},\mathbf{e})$ for the 16-state, two transmit antennas and one receive antenna SFTCs. Due to the high correlation between the adjacent subcarriers in MIMO-OFDM systems, $\eta_{ij}(k)$ are often very small numbers. It is reasonable to assume that $2\sigma_{i,j}(k)\eta_{i,j}(k)E_s/4N_0$ approaches 1 at a moderate SNR. Then, $P_s(\mathbf{c},\mathbf{e})$ is to average (2) over 48 random variables, $X^n_{i,i',j}(k)$. Consequently, $P_s(\mathbf{c},\mathbf{e})$ has the order of 4×10^2.

Note that according to the consistent estimation theory, when $\delta\times n_T\times n_T\times n_R\times 4$ approaches a very large value (infinite), s approaches zero, i.e., $P_s(\mathbf{c},\mathbf{e})$ approaches 1. For convenience, we define the following expression: $P_s(\mathbf{c},\mathbf{e})=F(SNR, \delta, n_T, n_R)$.

Comparing *part* 1 with (5), we find that *part* 1 corresponds to a slow Rayleigh fading channel, and only the *codeword difference matrix* $\mathbf{B}(\mathbf{c},\mathbf{e})$ is impacted by $\sigma_{i,j}(k)$. For simplicity of discussion, it is a reasonable assumption that all the channels between each pair of transmit and receive antennas have the same fading property, such as $\sigma_{i,j}(k)=\sigma(k)$ and $\eta_{i,j}(k)=\eta_i(k)$, $i=1,\ldots,n_T$, $j=1,\ldots,n_R$. Then, $\mathbf{B}(\mathbf{c},\mathbf{e})$ and $\mathbf{A}(\mathbf{c},\mathbf{e})$ are modified as

$$\mathbf{B}_s(\mathbf{c},\mathbf{e})=(\sigma(1)\mathbf{b}_0,\sigma(1)\mathbf{b}_1,...,\sigma(K-1)\mathbf{b}_{K-1})$$
$$\mathbf{A}_s(\mathbf{c},\mathbf{e})=\mathbf{B}_s(\mathbf{c},\mathbf{e})\mathbf{B}_s^H(\mathbf{c},\mathbf{e}).$$

Similarly, compared with (8), *part* 2 clearly corresponds to the fast Rayleigh fading channel. Following the derivation of section II, the PEP of SFTCs over frequency-correlated Rayleigh fading channels can be obtained as

$$P(\mathbf{c},\mathbf{e})\le 2F(SNR,\delta,n_T,n_R)\left(\prod_{i=1}^{r}\left(1+\lambda_i'\frac{E_s}{4N_0}\right)\right)^{-n_R}\times\left(\prod_{k\in\rho(\mathrm{c},\mathrm{e})}\left(1+\eta^2(k)\|\mathrm{c}_k-\mathrm{e}_k\|^2\frac{E_s}{4N_0}\right)\right)^{-n_R},\tag{20}$$

where r and λ_i', $i=1,...,r$, are the rank and nonzero eigenvalues, respectively, of the modified *codeword distance matrix* $\mathbf{A}_s(\mathbf{c},\mathbf{e})$. As mentioned above, the fading channels constructed by all the consecutive subcarriers mainly exhibit the slow fading characteristic. In addition, only the first several *simple-error-events* may occur since the maximum likelihood estimator is used at the receiver [20], [21]. This means that we consider the correlation of subcarriers only in a narrow range for several consecutive subcarriers. Therefore, the number of $\sigma(k)$ is close to one, but $\eta(k)$ is near zero. Consequently, to ensure $\eta^2(k)E_s/4N_0 >> 1$seems to be impossible; therefore (20) cannot be simplified in the same way as in the types of (7) and (10). In what follows, the simplification of (20) for different cases of SNR is investigated.

Because $\sigma(k)$ is close to one and we are only interested in the several consecutive subcarriers (within a *simple-error-event*), we assume $\sigma(k)\approx\sigma$ as a constant and $\eta(k)\approx\eta$. Thus, (20) is expressed as

$$P(\mathbf{c},\mathbf{e})\le 2F(SNR,\delta,n_T,n_R)\left(\prod_{i=1}^{r}\left(1+\lambda_i\frac{\sigma E_s}{4N_0}\right)\right)^{-n_R}\times\left(\prod_{k\in\rho(\mathrm{c},\mathrm{e})}\left(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2\frac{\eta^2E_s}{4N_0}\right)\right)^{-n_R}\tag{21}$$

Case 1.

Assuming that the SNR is lower, so $\eta^2(k)E_s/4N_0 << 1$, it follows from (21), as discussed in [4] and [9], that

$$\begin{aligned}P(\mathbf{c},\mathbf{e})&\le 2F(SNR,\delta,n_T,n_R)\left(\prod_{i=1}^{r}\left(1+\lambda_i\frac{\sigma E_s}{4N_0}\right)\right)^{-n_R}\\&\quad\times\left(1+\sum_{k\in\rho(\mathrm{c},\mathrm{e})}\|\mathbf{c}_\mathrm{k}-\mathbf{e}_k\|^2\frac{\eta^2E_s}{4N_0}+o\left(\frac{\eta^2E_s}{4N_0}\right)\right)^{-n_R}\\&\approx 2F(SNR,\delta,n_T,n_R)\left(\prod_{i=1}^{r}\left(1+\lambda_i\frac{\sigma E_s}{4N_0}\right)\right)^{-n_R}\\&\quad\times\left(\sum_{k\in\rho(\mathrm{c},\mathrm{e})}\|\mathbf{c}_\mathrm{k}-\mathbf{e}_k\|^2\frac{\eta^2E_s}{4N_0}\right)^{-n_R}\\&\le 2F(SNR,\delta,n_T,n_R)\left[\left(\prod_{i=1}^{r}\lambda_i\right)\left(\sum_{k\in\rho(\mathrm{c},\mathrm{e})}\|\mathbf{c}_\mathrm{k}-\mathbf{e}_k\|^2\right)\right]^{-n_R}\\&\quad\times\left[\sigma^{\frac{r}{r+1}}\eta^{\frac{2}{r+1}}\frac{E_s}{4N_0}\right]^{-(r+1)n_R},\end{aligned}\tag{22}$$

where $o\left(\frac{\eta^2E_s}{4N_0}\right)$ denotes the summation of all the terms that include the higher order quantities of $\frac{\eta^2E_s}{4N_0}$ in (22). We observe that the coding gain is given as $\left[\left(\prod_{i=1}^{r}\lambda_i\right)\left(\sum_{k\in\rho(\mathrm{c},\mathrm{e})}\|\mathbf{c}_k-\mathbf{e}_k\|^2\right)\right]^{1/(r+1)}$ and that the diversity is changed to $(r+1)n_R$; however, the effective SNR is modified as $\left[\sigma^{\frac{r}{r+1}}\eta^{\frac{2}{r+1}}\frac{E_s}{4N_0}\right]$.

Case 2.

At a moderate SNR, which is a practical case, $\eta^2(k)E_s/(4N_0)$ approximates to one, i.e., $\eta^2(k)E_s/(4N_0)\approx 1$. It follows from (21), as discussed in [4] and [9], that

$$\begin{aligned} P(\mathbf{c},\mathbf{e}) &\le 2F(SNR,\delta,n_T,n_R)\left(\prod_{i=1}^{r}\left(1+\lambda_i\frac{\sigma E_s}{4N_0}\right)\right)^{-n_R} \\ &\times\left(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\left(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2\right)\right)^{-n_R} \\ &=2F(SNR,\delta,n_T,n_R)\left[\left(\prod_{i=1}^{r}(\lambda_i)\right)\left(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\left(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2\right)\right)\right]^{-n_R} \\ &\times\left[\frac{\sigma E_s}{4N_0}\right]^{-rn_R}. \end{aligned} \tag{23}$$

We find that in this case, the coding gain is changed as $\left[\left(\prod_{i=1}^{r}\lambda_i\right)\left(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\left(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2\right)\right)\right]^{1/r}$ and the diversity is still rn_R. After the approximation processing, the effect of the correlation on diversity is hidden. But this change of diversity can be easily understood from (18).

Case 3.

Assuming that the SNR is sufficiently high so that $\eta^2(k)E_s/(4N_0)$ is more than one–but not far more from one–it follows from (21) that

$$\begin{aligned} &P(\mathbf{c},\mathbf{e})\le 2F(SNR,\delta,n_T,n_R) \\ &\times\left[\left(\prod_{i=1}^{r}(\lambda_i)\right)\left(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\left(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2\frac{\eta^2E_s}{4N_0}\right)\right)\right]^{-n_R}\times\left[\frac{\sigma E_s}{4N_0}\right]^{-rn_R}. \end{aligned} \tag{24}$$

Due to the limit of space, we mainly discuss Case 2 in the rest of the paper, since it has a high practicability.

1) Although the effect of channel correlation on diversity is not shown explicitly in (23), it results from the approximate processing, and the change of diversity can be observed clearly from (21). The minimum diversity of SFTCs is always preserved as rn_R, but the maximum achievable diversity is δn_R (upper bound). For various propagation channel environments, the diversity varies from rn_R to δn_R, where r and δ are, respectively, the minimum rank and minimum *symbol-wise Hamming distance* over all pairs of distinct codewords.

2) The coding gain of SFTCs in MIMO-OFDM systems, indeed, is dominated simultaneously by both the determinant $\prod_{i=1}^{r}\lambda_i$ and the modified product distance (*MPD*) $\prod_{k\in\rho(\mathbf{c},\mathbf{e})}(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2)$.

3) The importance of (21) is that it reveals the fact that the performance is affected simultaneously by two factors: the unchanged part (slow fading) and the fast changing part (fast fading).

Therefore, we propose the following code design criterion for SFTCs:

Proposition 1. Rank and distance criterion for space-frequency trellis coding

■ Simultaneously maximize the minimum *symbol-wise Hamming distance* δ and the minimum rank r of matrix $\mathbf{A}(\mathbf{c},\mathbf{e})$ over all the pairs of distinct codewords.

■ Instead of maximizing either the minimum $\prod_{i=1}^{r}\lambda_i$ or the minimum $\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\|\mathbf{c}_k-\mathbf{e}_k\|^2$, maximize the minimum product $\left[\prod_{i=1}^{r}\lambda_i\right]\cdot\left[\prod_{k\in\rho(\mathbf{c},\mathbf{e})}(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2)\right]$.

Note that the new criterion does not need any knowledge of the channel environment (multipath and time delay). Obviously, it is a combination of the rank and distance criteria. Hence, we can use the existing tools to design the SFTCs for MIMO-OFDM systems. The feasibility of the criterion will be verified by simulations in section V.

V. Simulation Results

To corroborate the above analysis, we have performed computer simulations through several examples. In all the simulations, we assume that all the propagation channels between pairs of transmit and receive antennas have the same number of paths, and each path has identical normalized average power 1/P. There is no spatial correlation between antennas of a transmitter and antennas of a receiver. Unless specified particularly, the space-frequency coded MIMO-OFDM system consists of K=64 subcarriers and 16 cyclic prefix samples in an OFDM block, referring to the IEEE 802.11a wireless LAN standards. The sampling interval of the OFDM system is T_s=50 ns. We consider the quasi-static fading channels where the fading coefficients $h_{i,j}(t)$ are constant within one OFDM block and vary independently from one OFDM block to another. The signal-to-noise ratio is defined as SNR=10 log(n_TE_s/N_0). The figure of merit is the bit-error rate.

Example 1. Diversity of SFTCs versus multipaths

This example verifies the fact that the diversity of SFTCs can be improved as the number of propagation paths increases. In this example, the 16-state, two transmit-antennas, quaternary phase shift keying (QPSK) space-time trellis code (STTC) designed in [1], named as TSC after the authors, is directly used as the space-frequency trellis code for the MIMO-OFDM system. The TSC code has the minimum rank, r=2, and the minimum *symbol-wise Hamming distance*, $\delta=3$. From the analyses in sections III and IV, the correlation between subcarriers will be reduced as the number of multipaths increases, i.e., the component exhibiting the fast fading property of the channel constructed by the consecutive subcarriers becomes strong. Hence, the diversity of SFTCs will gradually move from $2n_R$ to $3n_R$. The simulation results are illustrated in Fig. 2. The performance curves of TSC both in the ideal slow fading channels and in fast fading channels are also plotted in Fig. 2. As shown, the performance curve of 2 multipaths (P=2) approximately parallels the slow fading curve, but the curve of 12 multipaths (P=12) approaches the fast fading curve. The simulation results evidently demonstrate the accuracy of our analysis. Obviously, the diversity of space-time trellis codes achieved in slow fading channels, i.e., spatial diversity, can be preserved completely by using the STTCs as SFTCs. When $\delta > r$, in additional to spatial diversity, a portion of frequency diversity (not whole) can be achieved in the MIMO-OFDM systems. But little diversity can be improved when $\delta = r$. The effect of multipaths on the coding gain of SFTCs will be investigated in the following examples.

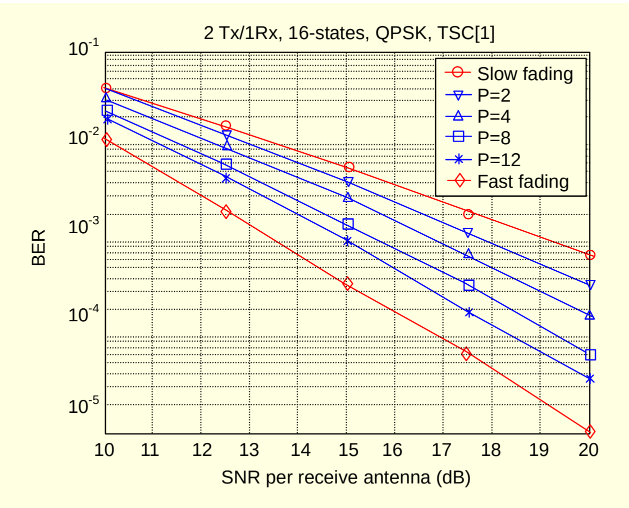

Fig. 2. Performance comparison of the 16-state QPSK TSC [1] code over different multipath channels.

Example 2. Coding Gain of the 16-state SFTCs versus code constructions over MIMO-OFDM systems

In proposition 1, we pointed out that the coding gain of SFTCs is mainly determined by the minimum value of $\left[\prod_{i=1}^{r}\lambda_i\right]\cdot\left[\prod_{k\in\rho(\mathbf{c},\mathbf{e})}(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2)\right]$. This example is used to demonstrate our proposition. The reported 16-state, two transmit antennas, QPSK STTCs are listed in Table 1. These codes are designed according to the rank-determinant criterion [1]-[5], trace criterion [6], and distance-product criterion [7], [8], respectively. It is proved in [20] and [21] that the performance is mainly determined by the first several *simple-error-events* that diverge from the correct path but converge back to it soon after along the shortest error path. The first three *simple-error-events* that have a *symbol-wise Hamming distance* of 3, 4 or 5, respectively, are analyzed. Along all the pairs of distinct codewords with the minimum rank r=2 and minimum *symbol-wise Hamming distance* δ = 3, the corresponding minimum values of determinant $Det(\prod_{i=1}^{r}\lambda_i)$, product distance $PD(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}\|\mathbf{c}_k-\mathbf{e}_k\|^2)$, modified product distance $MPD(\prod_{k\in\rho(\mathbf{c},\mathbf{e})}(1+\|\mathbf{c}_k-\mathbf{e}_k\|^2))$, and the minimum value of the product of both *Det* and *MPD* (*Det***MPD*)–moreover, the number of distinct pairs of codewords corresponding to the minimum value *Det***MPD*, i.e., distance spectrum (*DS*) [20]–are computed and presented in Table 1. One sees that the codes CYV [6] and ZQWL [8] have the maximum minimum *PD*=128, while codes YB [3], TC [4], and VY [5] have the maximum minimum value of *Det***MPD*=4000. The code TSC [1] as a reference has the minimum value of *Det***MPD*=540 and a maximum *DS*=2048. Code BBH [2] also has the maximum *DS*=2048 with medium *Det***MPD*=2940.

Figure 3 displays the simulation results of the eight 16-state STTCs listed in Table 1 in the case where there are eight

Table 1. Two transmit antennas, 16-state, QPSK STTCs designed in the listed reference.

Name	Generation matrix	First three simple-error events				
		Det	PD	MPD	Det*MPD	DS
TSC [1]	$\begin{bmatrix}0&0&1&2&2&0\\1&2&2&0&0&2\end{bmatrix}$	12	16	45	540	2048
BBH [2]	$\begin{bmatrix}2&0&1&2&2&0\\1&2&2&0&0&2\end{bmatrix}$	28	48	105	2,940	2048
YB [3]	$\begin{bmatrix}0&2&1&1&2&0\\2&2&1&2&0&2\end{bmatrix}$	32	64	125	4,000	512
TC [4]	$\begin{bmatrix}2&0&0&1&2&2\\2&2&1&0&0&2\end{bmatrix}$	32	64	125	4,000	512
VY [5]	$\begin{bmatrix}2&0&1&1&0&2\\0&2&1&2&2&2\end{bmatrix}$	32	64	125	4,000	512
CYV [6]	$\begin{bmatrix}2&1&2&1&2&3\\0&2&2&3&0&2\end{bmatrix}$	8	128	225	1,960	128
FVY [7]	$\begin{bmatrix}2&0&1&0&0&2\\0&2&2&1&2&2\end{bmatrix}$	24	64	135	3,240	512
ZQWL [8]	$\begin{bmatrix}0&2&2&1&2&1\\2&2&2&3&2&1\end{bmatrix}$	8	128	225	1,960	128

propagation paths between each pair of transmit and receive antennas while the system has two transmit-antennas and one receive-antenna. We observe that the eight bit-error rate curves can be separated into three groups. In the first group, codes TSC and BBH have the worst performance; BBH is slightly better than TSC. The second group contains codes CYV, FVY, and ZQWL; they each have a similar performance. Codes YB, TC, and VY in the third group have the best performance among the other codes due to their maximum value of *Det*MPD*. Since TC has a better whole distance distribution, we see that code TC is slightly better than YB and VY, though they each have the same minimum *Det*MPD* and *DS*. For example, in the *second simple-error event*, the minimum *Det*MPD* of TC is 13500 but is 8100 for YB and VY. As shown, all the curves are approximately parallel, i.e., they have the same diversity. These simulation results agree well with the values of *Det*MPD* and *DS* computed in Table 1.

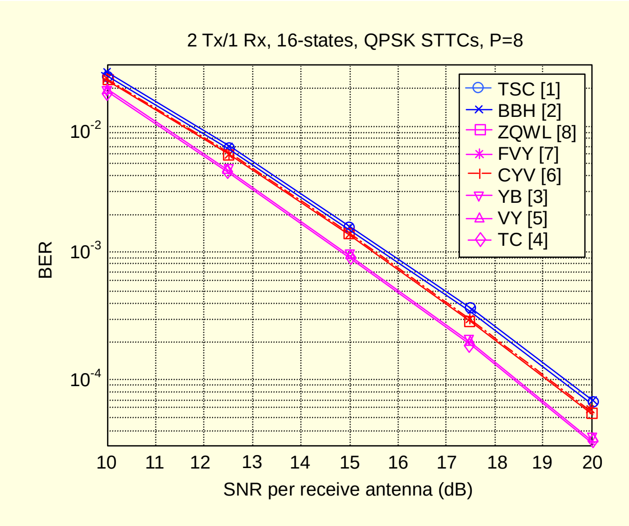

Fig. 3. Performance comparison of the eight 16-state codes listed in Table 1 for the case of eight multipaths and one receive antenna.

In order to more specifically observe the impact of propagation paths on the performance (diversity and coding gain) of SFTCs, the three representative codes TSC, TC, and CYV that are chosen, one from each of the three groups, respectively, are simulated again for the small $P=2$ and the large $P=12$. These simulation results are depicted in Fig. 4. From the analysis in section III, the correlation between adjacent subcarriers will be decreased when the number of multipaths increases from 2 to 12. In other words, the component exhibiting the constant property of the fading channel constructed by the consecutive subcarriers of OFDM blocks gradually reduces when the number of multipaths turns large and, simultaneously, when the independency between subcarriers increases. In Fig. 4, we first find the same result with example 1 in that the diversity of all the three codes is improved when the number of multipaths turns large. Also, the TC code has the best coding gain at every time, resulting from its maximum product of *Det*MPD*. Moreover, the impact of incorporating code construction with the fading channel characteristic on coding gain can be seen clearly by comparing the variation of the coding gain of the CYV code in two different channel conditions. While for the small multipath, $P=2$, the fading channels constructed by consecutive subcarriers mainly exhibit an approximately slow fading feature, and the main factor of determining the coding gain is the *Det* of the codes. Although code CYV, with a $PD=128$, is larger than TSC, the two codes have approximately the same

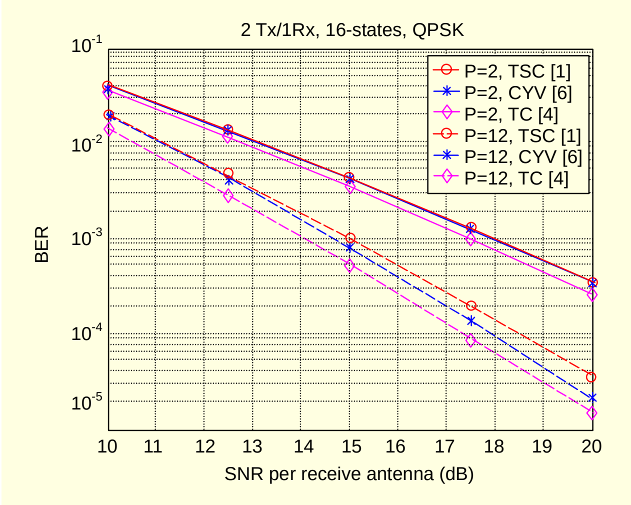

Fig. 4. Performance comparison of codes TSC, CYV, and TC in two channel environments.

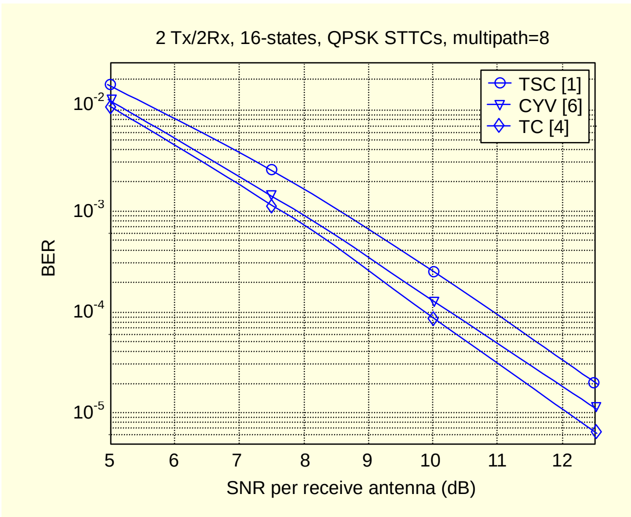

Fig. 5. Performance comparison of codes TSC, CYV, and TC with two receive antennas.

coding gain because CYV has a smaller *Det*=8 than TSC, while CYV is slightly better than TSC. However, when *P*=12, the independency of the fading channels becomes stronger, and the code CYV with a large *PD* can obtain more coding gain so that the coding gain of CYV is more than TSC. As shown in Fig. 4, the gap of the performance curves of TSC and CYV becomes wider when *P*=12.

Evidently, the coding gain of SFTCs is affected by both code construction and the channel characteristics. But the channel characteristics are usually not available at the transmitter and are time varying. Hence, we propose that the optimization of SFTCs is to maximize the minimum product of *Det* and *MPD*. The above simulation results have well confirmed our theoretical analysis. For the case of two receive antennas, the simulations were performed and demonstrated in Fig. 5. Identical results are achieved.

Example 3. Coding gain of 4-state SFTCs versus code constructions over MIMO-OFDM Systems

This example investigates the impact of the constructions of 4-state SFTCs and the channel multipaths on the coding gain.

Table 2. Two transmit antennas, 4-states, QPSK STTCs.

Name	Generation matrix	Det	PD	MPD	Det* MPD	DS
TSC [1]	$\begin{bmatrix} 0\,0\,2\,1 \\ 2\,1\,0\,0 \end{bmatrix}$	4	4	9	36	64
BBH [2]	$\begin{bmatrix} 2\,0\,1\,3 \\ 2\,2\,0\,1 \end{bmatrix}$	8	8	15	120	16
TC [4]	$\begin{bmatrix} 0\,2\,1\,2 \\ 2\,1\,2\,0 \end{bmatrix}$	4	24	35	196	8

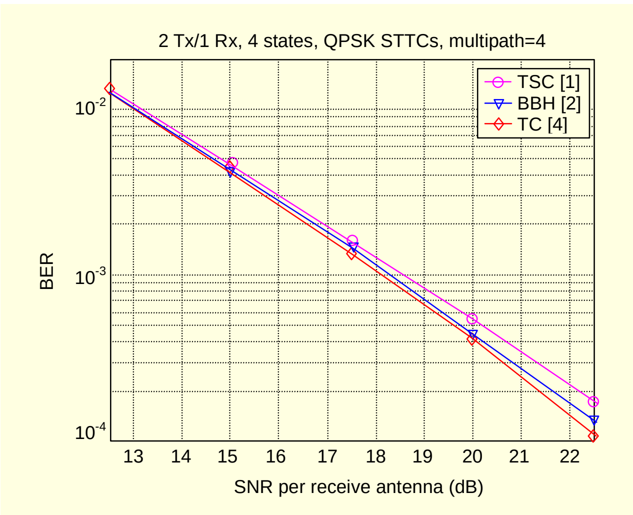

Fig. 6. Performance comparison of 4-state codes TSC, BBH, and TC with four multipaths.

Differing from 16-state codes, 4-state codes have the same minimum values of rank, *r*=2, and *symbol-wise Hamming distance*, $\delta = 2$. The three representative 4-state codes TSC, BBH and TC are computed, and the corresponding *Det*, *PD*, *MPD* and *Det***MPD*, and *DS* are listed in Table 2. Two simulations are carried out for *P*=4 and *P*=12, and the results are shown in Figs. 6 and 7, respectively. Since code TC has the maximum minimum value of *Det***MPD*=144 in the two cases, it exhibits the best performance despite its small *Det*=4, validating our design criterion.

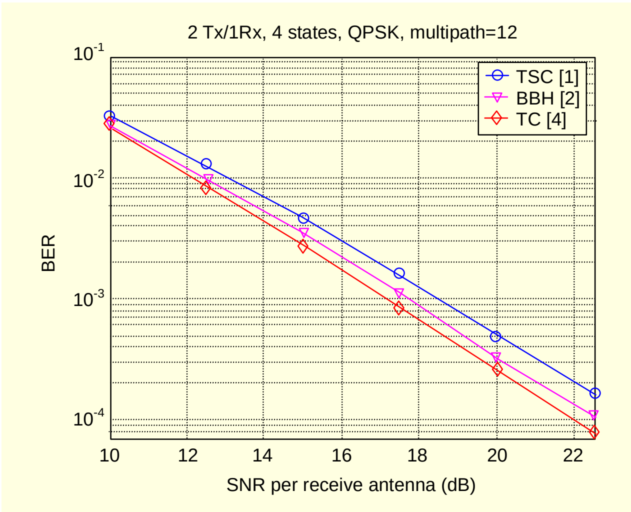

Fig. 7. Performance comparison of 4-state codes TSC, BBH, and TC with twelve multipaths.

Example 4. Robustness of SFTCs versus code constructions over MIMO-OFDM systems

This example studies the robustness of SFTCs for various channel environments in MIMO-OFDM systems. To highlight the main problem, we assume that the channel between each pair of transmit and receive antennas has only two propagation paths with an equal average normalized power of 1/2. The time delay denotes the time difference arrived at the receiver of the second path relative to the first path. The three 16-state codes of TSC [1], CYV [6], and TC [4] are simulated for delay=200 ns ($4T_s$) and delay=600 ns ($12T_s$), respectively, where T_s=50 ns. Figure 8 shows the simulation results. We observe that the diversity is improved significantly when the delay of the multipath increases, and that the space-frequency trellis code with a large product of *Det***MPD* exhibits better coding gain than the others. Since the TC code has the maximum minimum value of *Det***MPD*=4000, it exhibits the best performance in the two channel situations. All the simulation results can be well explained by considering the correlation of the subcarriers of OFDM. When the delay turns large, the correlation between adjacent subcarriers will be reduced so that the diversity can be improved. For the case of two receive antennas, the simulations are also achieved and are demonstrated in Fig. 9, which illustrates consistent results with Fig. 8. These results lead us to the conclusion that the SFTCs designed according to our criterion in this paper are robust against the time delay of multipaths, and the larger the product of *Det*MPD* for SFTCs, the better the robustness.

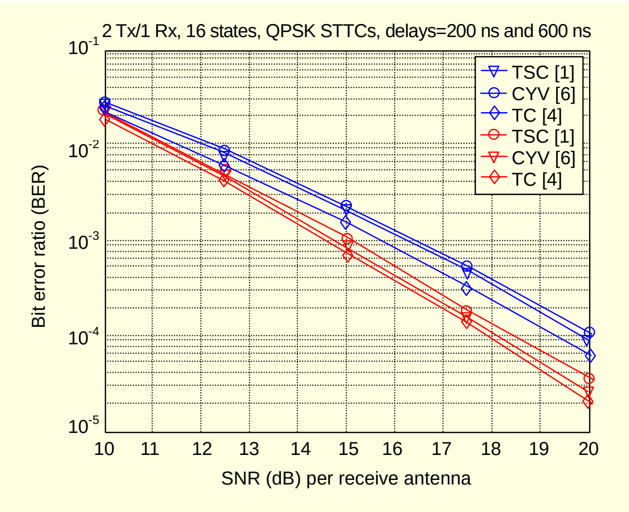

Fig. 8. Impact of the time delay of multipaths on the performances of SFTCs TSC, CYV, and TC.

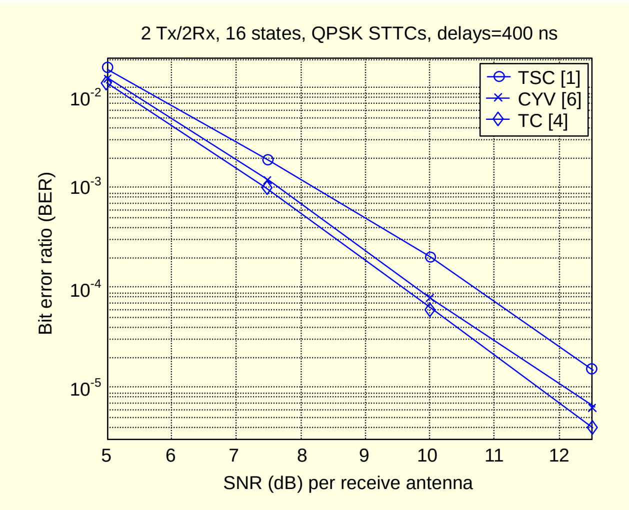

Fig. 9. Performance comparison of SFTCs TSC, CYV and TC with time delay=400 ns for two receive antennas.

Example 5. Impact of the channel model on the performance over MIMO-OFDM systems

Up to now, we have assumed that the wireless propagation channel, established by a pair of transmit and receive antennas, has the same average power per tap, i.e., $E[|h_{i,j}(l)|^2]=1/P$, named as a uniform power channel model. But this may be far from what exists in real scenarios. Actually, based on a large number of experimental measurements, it is found that the power delay profile is an exponential function, which is expressed as

$$E\left[\left|h_{i,j}(l)\right|^2\right]=a_l,\ \ l=0,1,\ldots,P-1 \tag{25}$$

and

$$\begin{aligned} a_l &= a_0\,\mathrm{e}^{-lT_s/\varepsilon} \\ \sum_{l=0}^{P-1} a_l &= 1, \end{aligned} \tag{26}$$

where ε is the average rms delay spread, T_s is the sampling interval, and a_0 is the average power of the first tap, which has the relation

$$a_0=\frac{1-\mathrm{e}^{-T_s/\varepsilon}}{1-\mathrm{e}^{-PT_s/\varepsilon}}.$$

Substituting (25) and (26) into (14), the correlation between subcarriers is given as

$$R_{HH}(k,k+n)=\frac{1-\mathrm{e}^{-T_s/\varepsilon}}{1-\mathrm{e}^{-PT_s/\varepsilon}}\sum_{l=0}^{P-1}\mathrm{e}^{-lT_s/\varepsilon}\,\mathrm{e}^{j2\pi nl/K}. \tag{27}$$

For a comparison of (27) with (14), a numeric result is provided with $\varepsilon=50$ ns and $P=12$ as an example, which corresponds to a typical office environment and is specified as Hiperlan/2 model A in Europe's wireless LAN standard. As illustrated in Fig. 10, the correlation between subcarriers becomes higher than the uniform power channel model. From our analysis, we expect

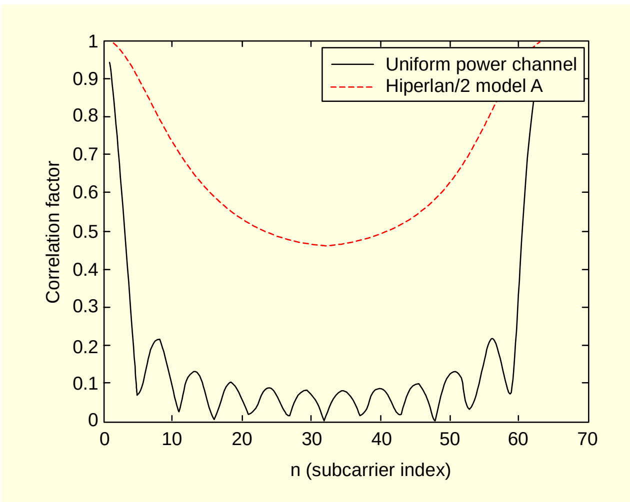

Fig. 10. Comparison of the correlation of the uniform power channel with Hiperlan/2 model A.

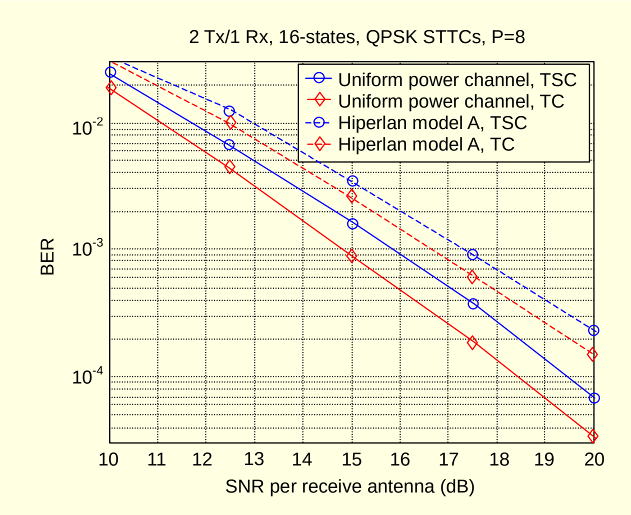

Fig. 11. Performance comparison of TSC and TC codes in different channel models.

that the performance should degrade in real scenarios.

Instead of the uniform power channel model, the simulations of 16-state, two-transmit antennas, QPSK TSC and TC codes in Hiperlan/2 model A are carried out, depicted in Fig. 11. As shown, the performance degrades significantly in Hiperlan/2 model A. Similar simulation results can be found in [16].

VI. Conclusion

In this paper, the basic issues such as the design criterion of SFTCs and their robustness were investigated. The main idea is to decompose the correlated fading channel constructed by the consecutive subcarriers of OFDM into two independent components: one corresponds to the slow fading channel, the other to the fast fading channel. Hence, the design problem of SFTCs is to optimize the code in order to obtain a good performance simultaneously on both the slow fading and fast fading channels. In particular, we showed that the coding gain is mainly dominated by the minimum product of both the determinant and modified product distances. We also found that the SFTCs designed according to our criterion are robust against the time delay of the multipaths.

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