Computationally Efficient 2-D DOA Estimation Using Two Parallel Uniform Linear Arrays

Hailin Cao, Lisheng Yang, Xiaoheng Tan, and Shizhong Yang

ABSTRACT—A new computationally efficient algorithm-based propagator method for two-dimensional (2-D) direction-of-arrival (DOA) estimation is proposed, which uses two parallel uniform linear arrays. The algorithm takes advantage of the special structure of the array which enables 2-D DOA estimation without pair matching. Simulation results show that the proposed algorithm achieves very accurate estimation at a computational cost 4 dB lower than that of standard methods.

Keywords—Propagator method, direction of arrival (DOA), uniform linear array, smart antennas.

I. Introduction

Over the last several decades, direction-of-arrival (DOA) estimation of signals impinging on an array of sensors, which is applied in many fields, such as radar, sonar, wireless communications, seismic data processing, and so on [1]-[3], has been widely studied to increase the resolution capability and to reduce the computational cost. The most popular methods of DOA estimation, such as MUSIC and ESPRIT, are highly computationally intensive because they require eigenvalue decomposition (EVD) of the cross spectral matrix or singular value decomposition (SVD) in the received complex-valued signal subspace. Macros proposed the propagator method (PM) [4], which does not use any EVD or SVD for one-dimensional (1-D) DOA estimation. Wu and others extended the PM to a 2-D DOA estimation problem using two parallel uniform linear arrays (ULAs) [5]. However, Wu's method requires an additional pair matching between the 2-D azimuth and elevation angle estimation, which needs a linear computational load of $O(2NMp)$, where N is the number of sensor elements, M is the number of data snapshots, and p is the source number. Kikuchi [6] and Li [7] also have proposed different methods for pair matching in different scenarios.

In this letter, a novel 2-D DOA algorithm based on PM using two parallel ULAs is presented. The proposed algorithm achieves very accurate estimation by taking advantage of a special structure of the array and requires no pair matching. Also, the computational intensity involved in the proposed algorithm is very low.

II. System Model and Problem Formulation

The proposed algorithm utilizes special array geometry instead of that of standard parallel ULAs. Consider the arrays in Fig.1. There are two ULAs with spacing d. One array comprises $(N+1)$ elements, and the other comprises N elements. We set up three subarrays of N elements using this structure. These subarrays, J, H, and L, coordinate as $(d*i,0)$, $(d*(i-0.5),d)$, and $(d*(i+0.5),d)$ for $0 \le i \le N-1$, respectively. Suppose that there are p narrowband incoherent sources with the same wavelength impinging on the arrays. We assume that the k-th source has an elevation angle θ_k and azimuth angle ϕ_k. The noise is temporally and spatially white Gaussian with variance σ^2. Thus, the observed signals at J, H, and L subarrays can be written as

$$J(t) = AS(t) + N_j(t), \tag{1}$$

$$H(t) = A\Psi_h S(t) + N_h(t), \tag{2}$$

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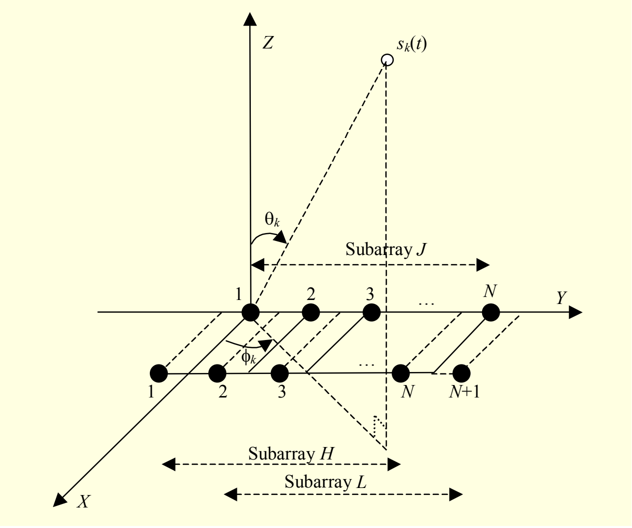

Fig. 1. Parallel array configuration for 2-D DOA estimation.

$$L(t) = A\Psi_l S(t) + N_l(t), \quad t = 1, 2, \cdots, M, \tag{3}$$

where $S(t) = \left[s_1(t) \ s_2(t) \ \cdots \ s_p(t) \right]^{\mathrm{T}}$,

$$N_k(t) = \left[n_{k,1}(t) \ n_{k,2}(t) \ \cdots n_{k,N}(t) \right]^{\mathrm{T}}, \quad k = j, h, l,$$

$$A = \left[a_1(\theta_1, \phi_1) \ a_2(\theta_2, \phi_2) \ \cdots \ a_p(\theta_p, \phi_p) \right],$$

$$a_i(\theta_i, \phi_i) = \left[1 \ e^{j2\pi(d/\lambda)\sin(\theta_i)\sin(\phi_i)} \ \cdots \ e^{j2\pi(d/\lambda)(N-1)\sin(\theta_i)\sin(\phi_i)} \right]^{\mathrm{T}}, \quad i = 1, 2, \cdots, p.$$

The $p \times p$ diagonal matrices Ψ_h and Ψ_l contain information regarding the elevation angle θ_i and the azimuth angle ϕ_i, which can be represented by

$$\left.\begin{aligned} \Psi_k &= \mathrm{diag}(e^{ju_{kx_1} + u_{ky_1}}, e^{ju_{kx_2} + u_{ky_2}}, \cdots, e^{ju_{kx_p} + u_{ky_p}}), \\ u_{kx_i} &= 2\pi(d_{kx}/\lambda)\sin\theta_i \cos\phi_i, \\ u_{ky_i} &= 2\pi(d_{ky}/\lambda)\sin\theta_i \sin\phi_i, \\ & \qquad k = h, l; \quad i = 1, 2, \cdots, p \end{aligned}\right\}, \tag{4}$$

where $d_{hy} = (-d/2)$, $d_{ly} = (d/2)$, and $d_{hx} = d_{lx} = d$, as shown in Fig.1.

Suppose that

$$W = (H + L)/\sqrt{2}, \tag{5}$$

then we have

$$W = A\Psi S + \bar{N}, \tag{6}$$

$$\bar{N} = (N_h + N_l)/\sqrt{2}, \tag{7}$$

where $\bar{N}$ has the same variance σ^2 as N_h and N_l. Considering the special structure of the array in Fig. 1, Ψ is a $p \times p$ diagonal matrix , which can be written as

$$\begin{aligned} \Psi = \sqrt{2}\mathrm{diag}(&\cos(2\pi(d/(2\lambda))\sin\theta_1 \sin\phi_1)e^{j2\pi(d/\lambda)\sin\theta_1\cos\phi_1}, \\ &\cos(2\pi(d/(2\lambda))\sin\theta_2 \sin\phi_2)e^{j2\pi(d/\lambda)\sin\theta_2\cos\phi_2}, \cdots, \\ &\cos(2\pi(d/(2\lambda))\sin\theta_p \sin\phi_p)e^{j2\pi(d/\lambda)\sin\theta_p\cos\phi_p}). \end{aligned} \tag{8}$$

The information of the elevation angle θ_i and the azimuth angle ϕ_i for source i is involved in the amplitude components and phase components of the diagonal elements in (8). As a result, the estimated elevation and azimuth angles can be calculated as

$$\hat{\phi}_i = \tan^{-1}\left(\frac{2\cos^{-1}(\mathrm{amplitude}(\Psi_{i,i})/\sqrt{2})}{\mathrm{phase}(\Psi_{i,i})} \right), \tag{9}$$

$$\hat{\theta}_i = \sin^{-1}\left(\frac{\mathrm{phase}(\Psi_{i,i})}{2\pi(d/\lambda)\cos\hat{\phi}_i} \right). \tag{10}$$

Obviously, there is a one-to-one correspondence between the estimated elevation angle $\hat{\theta}_i$ and the azimuth angle $\hat{\phi}_i$ ($i = 1, 2, \cdots, p$). Thus, the proposed algorithm can estimate the elevation and azimuth angles without pair matching. With the data matrices J and W, the estimation of the diagonal elements of matrix Ψ can be solved by employing the PM [4], the steps of which are very similar to those in [6]. The main difference between Wu's method and the proposed method is the computational intensity caused by different dimensions of data matrix. From the definition in [4], we can see that the proposed method has the number of multiplications in the order of $2NMp$ while Wu's method has the order of $3NMp$ in calculating Ψ. As previously mentioned, Wu's method needs an additional computational intensity of $2NMp$ for pair matching. Regarding major computational cost, the proposed method has the number of multiplications in the order of $2NMp$, where Wu's method requires a computational load at $O(5NMp)$, and the direct ESPRIT method needs $O(N^3 + 3N^2M)$ in the EVD. With the increase of the array size or the number of snapshots, the advantage in computational intensity of the proposed method becomes more obvious.

III. Simulations

Suppose there are two uncorrelated incident sources with directions, (30°, 55°) and (35°, 60°). The sensor array in Fig. 1 is used with N=16 and d=λ/2. The estimation results at the first source are shown in Fig. 2, with each point independently run 500 times with 200 snapshots of each run. From the simulation

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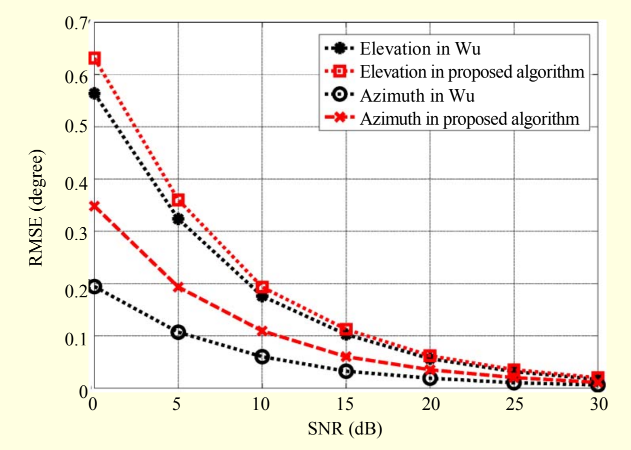

Fig. 2. RMSE for the first source versus SNR.

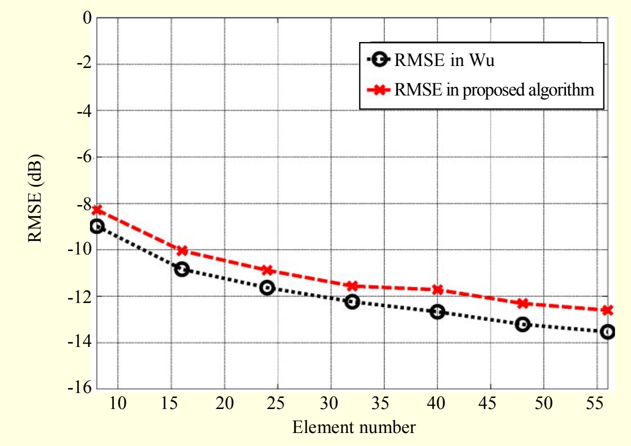

Fig. 3. RMSE for the first source versus *N*.

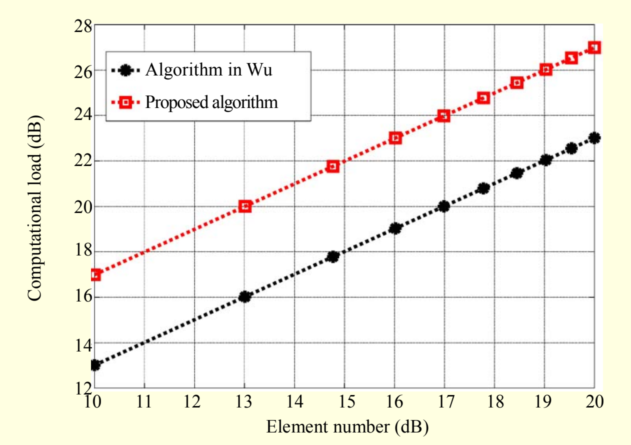

Fig. 4. Computational intensity versus *N*.

results, it can be seen that the new algorithm achieves very accurate estimation as does Wu's method. This test was repeated for various *N* values at signal-to-noise ratio (SNR) of 15 dB. Corresponding root mean square errors (RMSEs) are plotted in Fig. 3. It can be concluded that both the new algorithm and Wu's method achieve very accurate estimation when *N* is sufficiently large. We have not included the results for the second source which showed similar performance. The computational intensity of the proposed method and Wu's method versus *N* is plotted in Fig. 4. It can be observed that computational load of Wu's method is 4 dB higher than that of the proposed method.

IV. Conclusion

A computationally efficient algorithm for DOA estimation using two parallel ULAs has been proposed. The algorithm is based on PM and a special array structure. The analysis and simulations demonstrated that the proposed algorithm achieves very accurate estimates with very low computational intensity. This makes the proposed algorithm more suitable for real-time digital signal processing implementations.

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