Performance Analysis of a Two-Hop Fixed-Gain MIMO Multiuser Relay Network with End-to-End Antenna Selection

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This letter analyzes the performance of a two-hop MIMO multiuser relay network with a fixed gain amplify-and-forward protocol and antenna selection at the transmitter and receiver. A new expression for the cumulative distribution function of the highest instantaneous end-to-end signal-to-noise ratio is derived. Based on the above result, closed-form expressions for outage probability and bit error rate are presented. Also, the diversity order of the system is determined. Finally, computer simulations are compared to the analytical results, and insights and observations are provided.

Keywords: Cooperative communications, MIMO relaying systems, antenna selection, bit error rate analysis.

I. Introduction

MIMO technologies substantially improve link reliability through space diversity [1]. Relaying technology has the promise of extending network coverage without increasing transmit power [2]. Thus, the effective combination of MIMO transmission and relaying is attracting significant attention.

In MIMO relay systems, antenna selection has become a low-cost, low-complexity method to capture MIMO's essential advantages as it only requires very limited feedback (a few bits) and avoids multiple radio frequency links at nodes. A two-hop amplify-and-forward (AF) MIMO relaying system in which antenna selection is employed at both transmitter and receiver has been considered [3]-[5] where an exact bit error rate (BER) expression and diversity order of the system are obtained. However, [3]-[5] only consider an ideal-gain relaying protocol and a single-user transmission scenario. For ideal gain relaying, the relay needs to continuously monitor the channel from the source to relay. This requires substantial feedback and complexity in relay implementation.

Fixed-gain relaying simplifies the relaying operation as it only requires statistical rather than instantaneous channel state information from source to relay, and it retains most of the performance gains. To the best of our knowledge, the performance of the system assumed in [3]-[5] that instead uses a fixed-gain relaying protocol in a multiuser relay network has not been previously studied. Recently, a downlink multiuser relay network that employs opportunistic scheduling and fixed-gain relaying has been analyzed in [6]. However, the system assumes only a single antenna at each node with no antenna selection.

This letter fills this gap. Here, a fixed-gain relaying protocol [7] and opportunistic scheduling in a downlink multiuser, multiantenna relay network employing antenna selection are considered. Closed-form expressions for outage probability and average BER are provided to guide system design [8]. The diversity order of the system is also analytically determined. Finally, accuracy of the new analyses is validated by computer simulations. Different system configurations are also compared.

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II. System Model

A downlink multiuser AF fixed-gain relay network is considered where the source, relay, and each of K destinations are equipped with N_S, N_R, and N_D antennas, respectively. Antenna selection is used at each of the transmitter and receiver nodes. Therefore, the total received signal-to-noise ratio (SNR) at the k-th destination can be written as

$$\gamma_k = \frac{\gamma_1 \gamma_{2,k}}{C+\gamma_{2,k}}, \tag{1}$$

where

$$C = \frac{1}{E_{\gamma_1}[(\gamma_1+1)^{-1}]} \tag{2}$$

is a constant with a fixed-gain amplifying factor [7], in which $E_x[\cdot]$ denotes the expectation operation over the probability distribution of x, and γ_1 and $\gamma_{2,k}$ represent SNRs of the source-to-relay and relay-to-k-th destination links selected, respectively.

Opportunistic scheduling[1)] is used in a downlink multiuser relay network as given in [6]. The highest instantaneous end-to-end SNR of the selected user is given by

$$\gamma = \max_{1\le k\le K} \frac{\gamma_1 \gamma_{2,k}}{C+\gamma_{2,k}}. \tag{3}$$

It is observed from (3) that maximization of γ can be achieved by maximizing $\gamma_{2,k}$ for fixed γ_1. Therefore, (3) is equivalent to

$$\gamma = \frac{\gamma_1 \gamma_2}{C+\gamma_2}, \tag{4}$$

where $\gamma_2 = \max_{1\le k\le K} \gamma_{2,k}$.

III. Outage Probability Analysis

Theorem 1. The cumulative distribution function (CDF) of the received SNR of the system under consideration is given by

$$F_\gamma(\gamma) = 1 - 2\sum_{i=1}^{M}\binom{M}{i}\sum_{j=1}^{N}\binom{N}{j}(-1)^{(i+j)} e^{-\frac{i\gamma}{\bar\gamma_1}} \times \sqrt{\frac{ijC\gamma}{\bar\gamma_1\bar\gamma_2}} K_1\left(2\sqrt{\frac{ijC\gamma}{\bar\gamma_1\bar\gamma_2}}\right), \tag{5}$$

where $M=N_S\times N_R$, $N=N_R\times N_D\times K$, and $K_l(\cdot)$ is the l-th-order modified Bessel function of the second kind [9].

Proof. The CDF of the overall received SNR in (1) is

$$P\left(\frac{\gamma_1\gamma_2}{C+\gamma_2} < \gamma\right) = \int_0^\infty P\left(\gamma_1 < \frac{C\gamma + y\gamma}{y}\right) f_{\gamma_2}(y)dy. \tag{6}$$

To derive the CDF of γ, the CDF of γ_1 and the probability density function (PDF) of γ_2 are required. These are given by [5]

$$F_{\gamma_1}(x) = 1 + \sum_{i=1}^{M}\binom{M}{i}(-1)^i e^{-\frac{ix}{\bar\gamma_1}}, \tag{7}$$

$$f_{\gamma_2}(y) = \frac{N}{\bar\gamma_2}\sum_{j=0}^{N}\binom{N-1}{j}(-1)^j e^{-\frac{(j+1)x}{\bar\gamma_2}}, \tag{8}$$

where $\bar\gamma_1$ and $\bar\gamma_2$ denote average SNRs of each transmitter-receiver antenna pair for the source-to-relay and relay-to-k-th destination hops, respectively. Substituting (7) and (8) into (6), applying the binomial expansion, yields

$$F_\gamma(\gamma) = 1 - \sum_{i=1}^{M}\binom{M}{i}\sum_{j=1}^{N-1}\binom{N-1}{j}(-1)^{i+j}\exp\left(-\frac{i\gamma}{\bar\gamma_1}\right) \times \frac{N}{\bar\gamma_2}\int_0^\infty e^{-\frac{iC\gamma}{y\bar\gamma_1} - \frac{(j+1)y}{\bar\gamma_2}} dy. \tag{9}$$

Finally, applying (3.324.1) of [10] and performing algebraic manipulations, (5) is obtained. □

Remark. Substituting $\gamma=\gamma_{th}$ into (5) leads to a closed-form expression for system outage probability, where γ_{th} denotes the SNR threshold needed to avoid system outage.

IV. Average BER Analysis

In this section, we derive a new closed-form expression for average BER. When quadrature phase-shift keying is utilized at the source, the conditional BER is given by $P_e(\gamma) = Q(\sqrt{\gamma})$. The average BER is obtained by the CDF-based method [11], that is,

$$\bar P_e = -\int_0^\infty F_\gamma(\gamma) P_e'(\gamma) d\gamma, \tag{10}$$

where $P_e'(\gamma) = -\frac{1}{2\sqrt{2\pi}} e^{-\frac{\gamma}{2}} \gamma^{-\frac{1}{2}}$ denotes the derivative of $P_e(\gamma)$. Substituting (5) into (10), with the help of (3.381.4) and (6.621.3) of [10], and through algebraic manipulation, leads to the following closed-form expression for average BER:

$$\bar P_e = \frac{1}{2} - \frac{1}{2}\sum_{i=1}^{M}\binom{M}{i}\sum_{j=1}^{N}\binom{N}{j}(-1)^{i+j} \times \frac{ijC\sqrt{\bar\gamma_1}}{\bar\gamma_2\sqrt{(2i+\bar\gamma_1)^3}} e^{\frac{ijC}{\bar\gamma_2(2i+\bar\gamma_1)}} \times \left(K_1\left(\frac{ijC}{\bar\gamma_2(2i+\bar\gamma_1)}\right) - K_0\left(\frac{ijC}{\bar\gamma_2(2i+\bar\gamma_1)}\right)\right). \tag{11}$$

V. Diversity Order Analysis

First, we determine the value of constant C. Applying the PDF of γ_1 given by

1) Fairness is not guaranteed among users. This may be addressed by the proportional fairness algorithm. However, this topic is beyond the scope of this letter.

$$f_{\gamma_1}(x)=\frac{M}{\overline{\gamma}_1}\sum_{i=0}^{M}\binom{M-1}{i}(-1)^i e^{-\frac{(i+1)x}{\overline{\gamma}_1}} \tag{12}$$

into (2), with the help of (3.353.4) of [10] and (5.1.7) of [9], yields

$$C=\frac{\overline{\gamma}_1}{M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^i e^{\frac{i+1}{\overline{\gamma}_1}}E_1\left(\frac{i+1}{\overline{\gamma}_1}\right)}, \tag{13}$$

where $E_1(\cdot)$ denotes the exponential integral function [9]. In addition, the power series of the exponential function and the exponential integral function can be written, respectively, as

$$\exp(z)=\sum_{l=0}^{\infty}\frac{z^l}{l!}, \tag{14}$$

$$E_1(z)=-\varphi(1)-\ln z-\sum_{n=1}^{\infty}\frac{(-1)^n z^n}{nn!}, \tag{15}$$

where $\varphi(\cdot)$ denotes the digamma function [10]. Using (14) and (15), C for fixed-gain relaying can be approximated by

$$C\approx\frac{1}{M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^{i+1}\ln(i+1)\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1})}. \tag{16}$$

The diversity order of the system is defined as

$$d=-\lim_{\gamma\to\infty}\frac{\log P_{\text{out}}}{\log\gamma}, \tag{17}$$

where P_{out} denotes system outage probability.

Theorem 2. The diversity order of the system under consideration is $\min\{M,N\}$.

Proof. First, we set $\overline{\gamma}_2=\mu\overline{\gamma}_1$ where μ is a constant. Using power series of $K_1(z)$ from (8.446) of [10] given by

$$K_1(z)=\frac{1}{z}+\sum_{j=0}^{\infty}\frac{\left(\frac{z}{2}\right)^{2j+1}}{j!(j+1)!}\left[\ln\frac{z}{2}-\frac{1}{2}\psi(j+1)-\frac{1}{2}\psi(j+2)\right] \tag{18}$$

and (14), substituting (16) into (5) with $\gamma=\gamma_{\text{th}}$ yields the system outage probability as

$$P_{\text{out}}=1-2\sum_{i=1}^{M}\binom{M}{i}\sum_{k=1}^{N}\binom{N}{k}(-1)^{(i+k)}\sum_{l=0}^{\infty}\frac{i^l\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_1}\right)^l}{l!}$$
$$\times\sqrt{\frac{ik(\rho\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1}))\gamma_{\text{th}}}{\mu}}\left\{\frac{1}{2}\sqrt{\frac{\mu}{ik(\rho\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1}))\gamma_{\text{th}}}}\right.$$
$$+\sum_{j=0}^{\infty}\frac{\left(\sqrt{\frac{ik(\rho\overline{\gamma}_1^{-1}+(o\overline{\gamma}_1^{-1}))\gamma_{\text{th}}}{\mu}}\right)^{2j+1}}{j!(j+1)!}$$
$$\left.\times\left(\ln\sqrt{\frac{ik(\rho\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1}))\gamma_{\text{th}}}{\mu}}-\frac{1}{2}\psi(j+1)-\frac{1}{2}\psi(j+2)\right)\right\}, \tag{19}$$

where $\rho=\left(M\sum_{i=0}^{M-1}\binom{M-1}{i}(-1)^{i+1}\ln(i+1)\right)^{-1}$. Through algebraic manipulation, we obtain

$$P_{\text{out}}$$
$$=\sum_{i=1}^{M}\binom{M}{i}(-1)^i\sum_{l=1}^{\infty}\frac{\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_1}\right)^l}{l!}-\sum_{i=1}^{M}\binom{M}{i}\sum_{k=1}^{N}\binom{N}{k}(-1)^{(i+k)}\sum_{l=0}^{\infty}\frac{i^l\left(-\frac{\gamma_{\text{th}}}{\overline{\gamma}_1}\right)^l}{l!}$$
$$\times\sum_{j=0}^{\infty}\frac{\left(\frac{ik(\rho\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1}))\gamma_{\text{th}}}{\mu}\right)^{j+1}}{j!(j+1)!}$$
$$\times\left\{\ln\frac{ik\left(\rho\overline{\gamma}_1^{-1}+o(\overline{\gamma}_1^{-1})\right)\gamma_{\text{th}}}{\mu}-\psi(j+1)-\psi(j+2)\right\}. \tag{20}$$

Substituting (20) into (17), using the identity $\sum_{i=1}^{M}\binom{M}{i}(-1)^i i^L=0$, for $L=1,\cdots,M-1$, and $\lim_{z\to\infty}z\ln(1+\frac{1}{z})=1$, we obtain the diversity order of the system as $d=\min\{M,N\}$.

VI. Numerical Results

In the following, Rayleigh fading channels are assumed. In Figs. 1 and 2, we set SNR threshold γ_{th}=0 dB and (N_t, N_R, N_D, K)=(2, 2, 2, 2). Figures 1 and 2 show the system outage probability and average BER, respectively, for a balanced link ($\overline{\gamma}_2=\overline{\gamma}_1$) and an unbalanced link ($\overline{\gamma}_2=2\overline{\gamma}_1$). It is observed in both cases that Monte Carlo simulations agree closely with the derived analytical results.

Next, a specific example is provided to illustrate theorem 2. We assume the following downlink relay network scenario is applicable to current cellular networks: a multiple-antenna source, a single-antenna relay, and multiple destinations each equipped with a single antenna.

Figure 3 plots average BER for different combinations of the number of source antennas and users. Again, it is observed that simulations very closely match the new fixed-gain relaying system analysis. It is also clear that the diversity order, as observable by the slope of the average BER, that curves in higher SNR regions (as expected given theorem 2) is equal to $\min\{N_S, K\}$. From Fig. 3, comparing the cases of (N_S=2, K=4), (N_S=4, K=2), and (N_S=2, K=2), it is clear that multiuser diversity order cannot be improved by the addition of only source antennas or only users. This observation is in contrast to the case of multiuser systems in the absence of relaying as

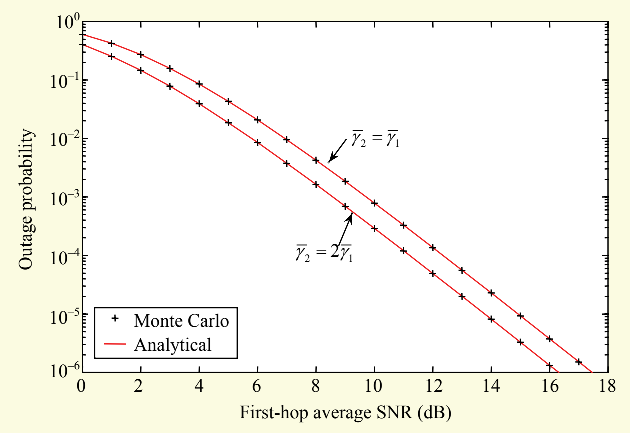

Fig. 1. System outage probability for balanced and unbalanced links.

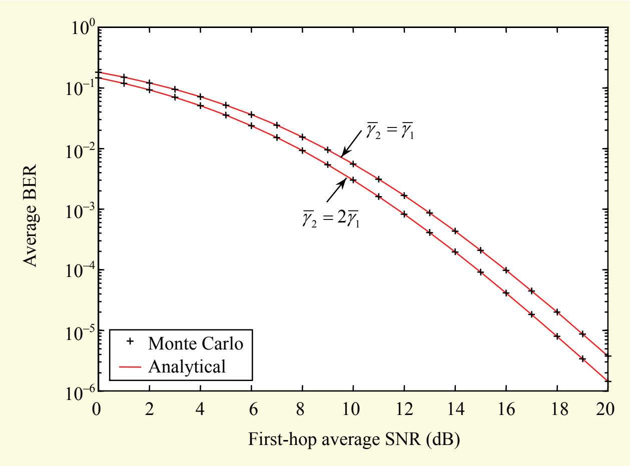

Fig. 2. Average BER for balanced and unbalanced links.

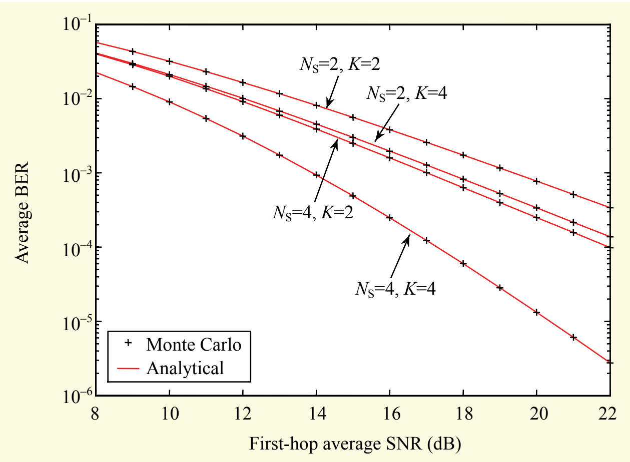

Fig. 3. System outage probability for different system configurations.

reported in [12]. In other words, the number of transmit antennas must be at least equal to the number of users to obtain full multiuser diversity order for systems as analyzed in Fig. 3.

VII. Conclusion

Closed-form expressions for outage probability and average BER were derived for fixed-relaying two-hop multiuser systems that utilize antenna selection and were used to study performance as a function of relative link power imbalance. Diversity order analysis also shows how multiuser diversity is influenced by the number of source antennas and users in a downlink multiuser MIMO relay network, which is shown to have behavior that is not analogous to point-to-point systems.

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