Performance Analysis of Directional CSMA/CA for IEEE 802.15.3c under Saturation Environments

Meejoung Kim, Yong Sang Kim, and Wooyong Lee

In this paper, the directional carrier sense multiple access/collision avoidance (CSMA/CA) protocol in the immediate acknowledgement mode for IEEE 802.15.3c is analyzed under saturation environments. For the analysis, a sensing region and an exclusive region with a directional antenna are computed probabilistically and a Markov chain model in which the features of IEEE 802.15.3c and the effects of using directional antennas are incorporated is analyzed. An algorithm to find the maximal number of concurrently transmittable frames is proposed. The system throughput and the average transmission delay are obtained in closed forms. The numerical results show the impact of directional antennas on the CSMA/CA media access control (MAC) protocol. For instance, the throughput with a small beamwidth of antenna is more than ten times larger than that for an omnidirectional antenna. The overall analysis is verified by a simulation. The obtained results will be helpful in developing an MAC protocol for enhancing the performance of mmWave wireless personal area networks.

Keywords: Directional CSMA/CA, IEEE 802.15.3c, Millimeter-wave, WPANs, Markov chain.

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I. Introduction

Recently, spectrum utilization in the 30 GHz to 300 GHz range has drawn considerable attention because it provides a wide bandwidth for high-speed data transmission. Since the wavelengths for these frequencies are between 1 mm and 10 mm, it is referred to as the millimeter wave (mmWave) band. There are two standards for mmWave wireless personal area networks (WPANs), IEEE 802.15.3c and ECMA-387. The former is based on centralized networks [1], whilst the latter is based on distributed networks [2] aiming for more than 2 Gbps as a target data rate. In addition, the standardization of IEEE 802.11ad is ongoing, and this is based on centralized networks with a 7-Gbps target data rate [3].

The mmWave band has unique characteristics, such as its short wavelength, high frequency, large bandwidth, and high level of interaction with atmospheric constituents. Such characteristics are associated with many of its salient properties, such as supporting a high data rate, as well as problems, such as short communication coverage due to high path losses. To compensate for these drawbacks of the mmWave band, utilization of directional antennas at the physical (PHY) layer is recommended. Such antennas enable higher antenna gain over a longer transmission range by unidirectional radiation of the transmission energy.

There has been research in using directional antennas: directional neighbor discovery (D-ND) [4], resource allocation policies and scheduling algorithms [5],[6], and mmWave WPANs [7]-[9]. Analysis of the CSMA/CA protocol can be found in many papers based on various standards: IEEE 802.11 [10],[11], IEEE 802.15.4 [12], IEEE 802.15.3 [13], and 802.15.3c [14],[15]. In particular, analyses of the carrier sense multiple access/collision avoidance (CSMA/CA) protocol using directional antennas are considered in [15]. A PHY/media access control (MAC) cross-layer analytical approach was developed to evaluate the performance of IEEE 802.11 WLAN [11]. An analytical model considering hybrid multiple access was constructed to study the throughput of the IEEE 802.15.3c mmWave WPAN system [14]. In [15], a directional CSMA/CA protocol for mmWave WPANs was considered. The authors used the result from [10], which is inadequate for the IEEE 802.15.3c standard.

In this paper, the directional CSMA/CA protocol in immediate acknowledgment (Imm-ACK) mode based on IEEE 802.15.3c is considered under saturation environments. The contribution of this paper is as follows: i) a Markov chain model is presented to analyze the CSMA/CA focusing on the data transmission between devices in the contention access period based on IEEE 802.15.3c, ii) the transition probabilities in the model involve the characteristics of directional antennas via the proportions of the sensing and exclusive regions, differentiating this model from those conventionally used, iii) the numbers of interferers for transmitters' sensing and receivers' receiving and the number of concurrently transmittable frames are explicitly derived probabilistically, and iv) the characteristics of 802.15.3c such as the path loss model and parameters which support the mmWave band are used.

In section II, a preliminary overview, including the CSMA/CA protocol under IEEE 802.15.3c, is given. In section III, an algorithm to find the number of concurrently transmittable frames is proposed, and a Markov chain model considering the interactions between devices for Imm-ACK mode is presented and analyzed under saturation environments based on IEEE 802.15.3c. The throughput and average transmission delay are explicitly described in section IV, and the numerical results are presented in section V. Finally, section VI concludes the paper.

II. System Model

In this section, IEEE 802.15.3c MAC is briefly explained and the path loss model for IEEE 802.15.3c is investigated. Then, a sensing region and an exclusive region are defined by using the path loss model.

1. Overview of IEEE 802.15.3c MAC

IEEE 802.15.3c is a standard supporting the quasi-omni mode over the 60-GHz frequency band. The fundamental topology of IEEE 802.15.3c is a piconet, which consists of a piconet coordinator (PNC) and several slave devices (DEVs) within its transmission range. The neighbor discovery process is embedded in the association process between the DEVs and the PNC, with any capable DEVs being able to act as a PNC.

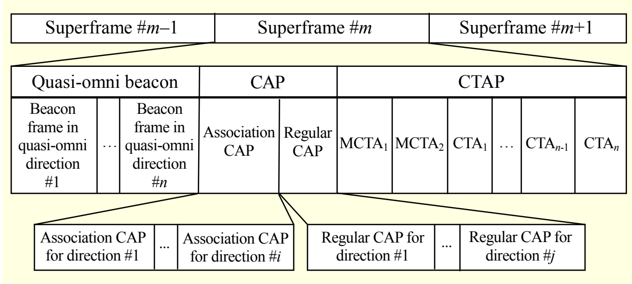

Fig. 1. Superframe structure of IEEE 802.15.3c.

The channel time in the piconet is based on the time-slotted superframe, which consists of the beacon, the contention access period (CAP), and the channel time allocation period (CTAP). The superframe structure of IEEE 802.15.3c is presented in Fig. 1. The CAP is divided into two sections: association CAP and regular CAP. The association CAP is only used by DEVs to send association request commands to the PNC, while regular CAP can be used for data exchanges among DEVs and reservation of the channel time allocations (CTAs) in CTAP between DEVs and the PNC. The medium access mechanism in the CAP is CSMA/CA whilst in the CTAP is time division multiple access (TDMA).

In the CSMA/CA access mechanism, each frame maintains two variables: *Retry_count*(*backoff stage*) and *Backoff_window* (*retry_count*). *Retry_count*(*backoff stage*) is the number of backoff times due to an unsuccessful transmission. *Backoff_window*(*retry_count*) is the number of backoff slots that a DEV must wait to attempt accessing the channel at the *retry_count*(*backoff stage*). According to the standard, the *Retry_count*(*backoff stage*) is an integer in the range of 0 to 3 and the corresponding *Backoff_window*(*retry_count*) is {7, 15, 31, 63}. *Backoff_window*(*i*) is denoted as W_i–1, where W_0=2^3 and W_i=$2^i W_0$, for notational simplicity.

IEEE 802.15.3c supports No-ACK, Imm-ACK, Delayed-ACK, and Block-ACK. In this paper, Imm-ACK mode, which is more likely to be used, is considered. Figure 2 shows the backoff procedure in the CAP period under IEEE 802.15.3c CSMA/CA with Imm-ACK mode. Since DEVs are equipped with directional antennas, CSMA/CA in Fig. 2 is performed directionally. The detailed explanation of the procedure is omitted here; this can be found in several other pieces of literature including [1].

2. Path Loss Model of IEEE 802.15.3c

For directional antennas, there are two models: the flat-top model that neglects the sidelobe effect and the cone plus circle model. The cone plus circle model can be used in 2D or the 3D

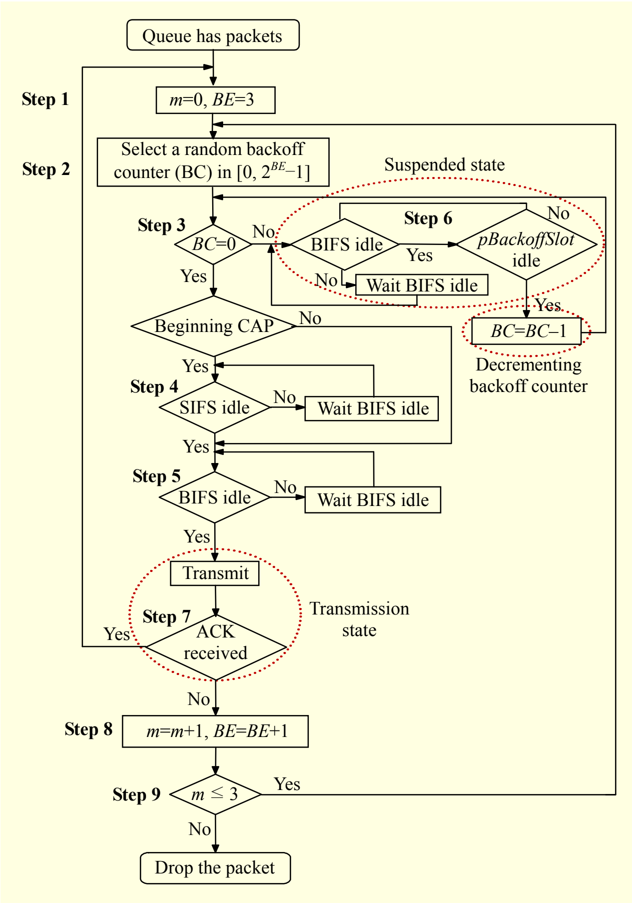

Fig. 2. CSMA/CA protocol with Imm-ACK under IEEE 802.15.3c.

cone plus sphere model that considers the sidelobe effect. Since the realistic antenna pattern is complex and does not result in a fundamental change of the capacity [16], in this paper, the cone plus circle model is employed assuming that all DEVs lie in the same plane. Then, the antenna gains with the mainlobe and sidelobe effects are defined by $G_m=2\pi\eta/\theta$ and $G_s=2\pi(1-\eta)/(2\pi-\theta)$, respectively, where η and θ ($0<\theta\leq 2\pi$) are the antenna radiation efficiency and the mainlobe beamwidth, respectively.

Let $G_T(i)$, $G_R(i)$, and $r_{i,j}$ be the antenna gains for the transmitter and receiver of frame i and the distance between the transmitter of frame i and the receiver of frame j, respectively. The average received signal power of frame i is then given by

$$P_R(i)[\text{dBm}] = G_T(i)[\text{dBi}] + G_R(i) + P_T(i) - PL(r_{i,i})[\text{dB}], \quad (1)$$

where $P_T(i)$ and $PL(r_{i,i})$ are the transmission power and path loss of frame i, respectively. The path loss model for IEEE 802.15.3c is given by

$$PL(r_{i,i})[\text{dB}] = \underbrace{PL_0}_{\text{Free space path loss at reference distance}} + \underbrace{10n\log_{10}(r_{i,i}/r_0)}_{\text{Path loss exponent at relative distance } r_{i,i}} + X_\sigma, \quad (2)$$

where r_0 and X_σ are the reference distance of 1 m and the lognormal shadowing with a mean of zero, respectively. The path loss exponent n for mmWave-based measurements is 2.0 for line-of-sigth (LOS) and 2.5 for non-LOS [17]. If shadow fading is ignored, the PL[dB] in the 60-GHz band is computed as

$$PL[\text{dB}] = 10\log_{10}\{(4\pi/\lambda)^2 r^n\}, \quad (3)$$

where λ is the wavelength of the signal given by $\lambda=c/f$. The speed of light is c and f is the frequency of the signal, which is 60 GHz in this case.

If several frames can be transmitted simultaneously, the achievable data rate of frame i is given by

$$R_i = \kappa_1 W \log_2\left\{\frac{\kappa G_T(i)G_R(i)P_T(i)r_{i,i}^{-n}}{N_0W + \sum_{i\neq j} I_{i,j}} + 1\right\}, \quad (4)$$

where κ_1, N_0, and W are a coefficient related to the efficiency of the transceiver design, the one-sided spectral density of the white Gaussian noise, and the channel bandwidth, respectively. κ is a constant proportional to $10\log_{10}(\lambda/4\pi)^2=-68.0048$ dB. $I_{i,j}$ is the interference power of frame i caused by frame j, which is given by

$$I_{i,j}[\text{dB}] = \kappa[\text{dB}] + G_T(j) + G_R(i) + P_T(j) - 10n\log_{10} r_{j,i}. \quad (5)$$

It is noted that the data rates of frames for concurrent transmissions may change according to the interference.

Let R_i^* be the average data rate of frame i during M slots when only one frame transmits at a time, which is given by $R_i^* = \kappa_1 W\log_2\{\kappa G_T(i)G_R(i)P_T(i)r_{i,i}^{-n}/N_0W+1\}/M$. If the total interference from other frames is less than the total background noise, that is, $\sum_{j\neq i} I_{j,i} \leq (M-1)N_0W$ in (4), $\log_2\{(\Gamma/N_0W+\sum_{j\neq i} I_{j,i})+1\} \geq (1/M)\log_2\{(\Gamma/N_0W)+1\}$ holds, where $\Gamma = \kappa G_T(i)G_R(i)P_T(i)r_{i,i}^{-n}$. This gives $R_i \geq R_i^*$. Therefore, to achieve a higher performance with concurrent transmission than for M serial transmissions of a frame, it is sufficient that mutual interference of each frame has to be less than the background noise, that is, $I_{j,i} \leq N_0W$, for all j, $j\neq i$, [7]. Combining this condition with (5), the following relation is obtained by

$$r_{i,j} \geq \left\{\frac{\kappa G_T(i)G_R(j)P_T(i)}{N_0W}\right\}^{1/n}, \quad \text{for all } j. \quad (6)$$

3. Sensing Region and Exclusive Region

When a DEV adopts a directional antenna in CSMA/CA, it is called directional CSMA/CA (D-CSMA/CA). During the D-CSMA/CA procedure, a transmitter has to sense the

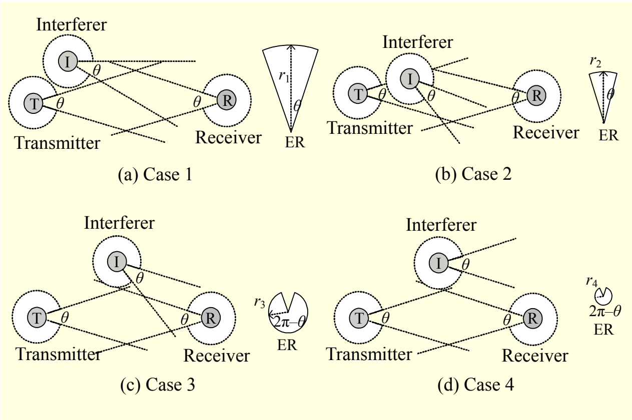

Fig. 3. Four different ER radii for directional antenna pairs.

transmissions of other transmitters to decrement its backoff counter. An area where a transmitter can sense other transmissions is known as a sensing region (SR). For a successful transmission, a transmitter-receiver pair has to be capable of interference-free communication. Thus, the concept of an exclusive region (ER) was introduced in [7], and it is defined as follows: each frame consisting of a transmitter-receiver pair has an ER around the receiver. The transmitters of all of the other frames engaging in simultaneous transmission must be located outside of the ER.

Figure 3 describes the locations and antenna directions of a receiver and an interferer candidate (I) and illustrates the radii of the ER. This includes four different cases: the I and receiver are inside of each other's radiation angle (case 1), the I is inside while the receiver is outside of each other's radiation angle (case 2), the I is outside while the receiver is inside of each other's radiation angle (case 3), and the I and receiver are outside of each other's radiation angle (case 4). Suppose that the Is in case 1 and case 4 are the same distance away from the receiver and that the same strength of signal is emitted from the Is. Then, a situation may exist in which the receiver in case 1 can detect the signal through its mainlobe, while the receiver in case 4 cannot detect the signal through its sidelobe. In other words, the I in case 1 does act, while the I in case 4 does not act, as an interferer to the receiver. To prevent interference to the receiver in case 1, the I must be further away from the receiver than the I in case 4 when the same strength of signals is emitted from the Is. In other words, the ER radius varies according to the location and antenna direction of the I. If the regions are viewed at the transmitter's side instead of the receiver's side, the radii in Fig. 3 can also be considered as the SR radii for a transmitter.

Since the required signal strengths for transmission, transmission sensing, and interfering with a receiver are different, the transmission range, the sensing region, and the exclusive region are in general different. Usually, it is assumed that the sensing region is the largest and that the transmission range is the smallest [18]. Let TR, SR_t, and ER_r be the transmission range, the sensing region of a transmitter, and the exclusive region of a receiver, respectively. Then, they are different by their signal strength thresholds. In the following, TR, SR_t, and ER_r are computed. Let G_{TM} (G_{TS}), G_{RM} (G_{RS}), and G_{IM} (G_{IS}) be the antenna gains of the mainlobe (sidelobe) of transmitters, receivers, and the other transmitters (interferers). If all DEVs have the same transmission power P_T, then TR is given by

$$TR = 10^{\{\kappa + G_{TM} + G_{RM} + P_T - P_R - PL_0[\text{dB}]\}/10n}. \quad (7)$$

Table 1. Transmission range (n=2, data rate: 1.65 Gbps, unit: m).

θ	TR (η =1)	L_{TR}	TR (η =0.9)	L_{TR}
10°	25.4720	18.0114	22.9248	16.2103
20°	12.7360	9.0057	11.4624	8.1051
30°	8.4907	6.0038	7.6416	5.4034
60°	4.2453	3.0019	3.8208	2.7017
90°	2.8302	2.0013	2.5472	1.8011
180°	1.4151	1.0006	1.2736	0.9006
360°	0.7076	0.5003	0.6368	0.4503

Table 1 shows the values of TR and the corresponding square with a side L_{TR} for different beamwidths and radiation efficiencies. This is obtained for a 1.65-Gbps (mandatory) data rate with corresponding receiver sensitivity, P_R=–55 dB, and 10 dBm for P_T.

Let $\{r_{s,i}\}_{i=1}^4$ and $\{r_{e,i}\}_{i=1}^4$ be the radii for SR_t and ER_r, respectively. Since the SR of a transmitter is independent of concurrent transmissions, whilst dependent on their locations, $\{r_{s,i}\}_{i=1}^4$ is given as (in dB)

$$r_{s,1} = 10^{\{\kappa + G_{TM} + G_{IM} + P_T - P_R - PL_0\}/10n}, \quad r_{s,2} = 10^{\{\kappa + G_{TM} + G_{IS} + P_T - P_R - PL_0\}/10n},$$
$$r_{s,3} = 10^{\{\kappa + G_{TS} + G_{IM} + P_T - P_R - PL_0\}/10n}, \quad r_{s,4} = 10^{\{\kappa + G_{TS} + G_{IS} + P_T - P_R - PL_0\}/10n}. \quad (8)$$

Here, −70 dB is used as the receiver sensitivity $P_R(i)$ which corresponds to the base data rate of 25.8 Mbps.

Since the ER of a receiver depends on concurrent transmissions and the locations of interferers, $\{\kappa G_T G_R P_T / N_0 W\}^{1/n}$ in (6) is set as the ER radius. Then, the radius for each case shown in Fig. 3 can be written as (unit: m)

$$r_{e,1} = \left(\frac{\kappa G_{TM} G_{RM} P_T}{N_0 W}\right)^{1/n}, r_{e,2} = \left(\frac{\kappa G_{TM} G_{RS} P_T}{N_0 W}\right)^{1/n},$$
$$r_{e,3} = \left(\frac{\kappa G_{TS} G_{RM} P_T}{N_0 W}\right)^{1/n}, r_{e,4} = \left(\frac{\kappa G_{TS} G_{RS} P_T}{N_0 W}\right)^{1/n}. \quad (9)$$

III. Analysis of Directional CSMA/CA Protocol

1. Expected Numbers of Transmitters in Sensing Region and Exclusive Region

In this paper, it is assumed that a piconet is an $L\times L$ square room and that $2N$ DEVs consisting of N transmitters and N receivers are randomly distributed in this piconet. It is assumed that a DEV cannot transmit and receive simultaneously, that each DEV is equipped with a directional antenna, and that the antennas of all transmitter-receiver pairs are directed towards their peers by the ND process and the beamforming technique. It is assumed that each transmitting DEV has a queue of length K, the frame is generated by a Poisson process with a traffic arrival rate λ (frame/T_{slot}), the load of a frame is constant, and the generated frames are transmitted to one specific peer until the flows is ended. Here, T_{slot} is the duration of the unit backoff slot. Because the saturation environment is considered, there are N frames contending for the channel from N distinct transmitter-receiver pairs at any time and each transmitting DEV can contain up to K frames in its queue.

It is known that the probability density function (pdf) of the distance between two DEVs, $f(x)$, is given as [19]

$$f(x)=\begin{cases} f_1(x)=\dfrac{2x}{L^2}\left(\dfrac{x^2}{L^2}-\dfrac{4x}{L}+\pi\right), & \text{if } 0\le x\le L,\\ f_2(x)=\dfrac{2x}{L^2}\left\{\begin{matrix}4\sqrt{\dfrac{x^2}{L^2}-1}-\left(\dfrac{x^2}{L^2}+2-\pi\right)\\ -4\tan^{-1}\sqrt{\dfrac{x^2}{L^2}-1}\end{matrix}\right\}, & \text{if } L<x\le\sqrt{2}L.\end{cases}\tag{10}$$

Since the radii of SR and ER can be larger than the maximal distance of the room, $r_{s,i}$ and $r_{e,i}$ are reset as $\min(r_{s,i},\sqrt{2}L)$ and $\min(r_{e,i},\sqrt{2}L)$, respectively, in the following equations. Since the ERs for the cases shown in Fig. 3 are not mutually exclusive and the cases are independent, the probability of ER_r for a receiver, which is the proportion of ER_r to the area of an $L\times L$ room, is obtained as

$$\begin{aligned}P_{\text{ER}_\text{r}}=&\left[\begin{matrix}\left(\frac{\theta}{2\pi}\right)^2\int_0^{r_{e,1}}f(x)dx+\left(\frac{\theta}{2\pi}\right)\left(1-\frac{\theta}{2\pi}\right)\sum_{i=2}^{3}\int_0^{r_{e,i}}f(x)dx\\ +\left(1-\frac{\theta}{2\pi}\right)^2\int_0^{r_{e,4}}f(x)dx\end{matrix}\right]\\ &-\left[\begin{matrix}\left(\frac{\theta}{2\pi}\right)^3\left(1-\frac{\theta}{2\pi}\right)\left\{\prod_{i=1,2}\int_0^{r_{e,i}}f(x)dx+\prod_{i=1,3}\int_0^{r_{e,i}}f(x)dx\right\}\\ +\left(\frac{\theta}{2\pi}\right)^2\left(1-\frac{\theta}{2\pi}\right)^2\left\{\prod_{i=1,4}\int_0^{r_{e,i}}f(x)dx+\prod_{i=2,3}\int_0^{r_{e,i}}f(x)dx\right\}\\ +\left(\frac{\theta}{2\pi}\right)\left(1-\frac{\theta}{2\pi}\right)^3\left\{\prod_{i=2,4}\int_0^{r_{e,i}}f(x)dx+\prod_{i=3,4}\int_0^{r_{e,i}}f(x)dx\right\}\end{matrix}\right]\\ &+\left[\begin{matrix}\left(\frac{\theta}{2\pi}\right)^4\left(1-\frac{\theta}{2\pi}\right)^2\prod_{i=1,2,3}\int_0^{r_{e,i}}f(x)dx\\ +\left(\frac{\theta}{2\pi}\right)^3\left(1-\frac{\theta}{2\pi}\right)^3\left\{\prod_{i=1,2,4}\int_0^{r_{e,i}}f(x)dx+\prod_{i=1,3,4}\int_0^{r_{e,i}}f(x)dx\right\}\\ +\left(\frac{\theta}{2\pi}\right)^2\left(1-\frac{\theta}{2\pi}\right)^4\prod_{i=2,3,4}\int_0^{r_{e,i}}f(x)dx\end{matrix}\right]\\ &-\left(\frac{\theta}{2\pi}\right)^4\left(1-\frac{\theta}{2\pi}\right)^4\prod_{i=1}^{4}\int_0^{r_{e,i}}f(x)dx.\end{aligned}\tag{11}$$

To compute P_{SR_t}, replace $\{r_{e,i}\}_{i=1}^4$ with $\{r_{s,i}\}_{i=1}^4$ in (11).

In the process of D-CSMA/CA in Imm-ACK mode, a DEV cannot decrement its backoff counter if it senses the transmissions of other transmitters, a collision occurs if other transmitters interfere with it at each stage and the transmission fails if the frame cannot be transmitted to the final stage. Let $E(K_{\text{SR}_\text{t}}^N)$, $E(K_{\text{ER}_\text{r}}^N)$, and E_{con}^N be the expected number of transmitters influencing the decrement for the tagged transmitter's (TTx) backoff counter, the expected number of interferers for the tagged receiver (TRx), and the expected number of DEVs that can interfere with the transmission of the tagged transmitter-receiver pair when N frames exist, respectively. Then, they are given by

$$\begin{aligned}&E(K_{\text{SR}_\text{t}}^N)=(N-1)P_{\text{SR}_\text{t}},\ E(K_{\text{ER}_\text{r}}^N)=(N-1)P_{\text{ER}_\text{r}},\\ &E_{\text{con}}^N=E(K_{\text{SR}_\text{t}}^N)+E(K_{\text{ER}_\text{r}}^N)-E(K_{\text{SR}_\text{t}\cap\text{ER}_\text{r}}^N),\end{aligned}\tag{12}$$

where $E(K_{\text{SR}_\text{t}\cap\text{ER}_\text{r}}^N)$ is the average number of transmitters in both of SR_t and ER_r, which is given by $E(K_{\text{SR}_\text{t}\cap\text{ER}_\text{r}}^N)=(N-1)\cdot P_{\text{SR}_\text{t}\cap\text{ER}_\text{r}}$ with $P_{\text{SR}_\text{t}\cap\text{ER}_\text{r}}=P_{\text{SR}_\text{t}}P_{\text{ER}_\text{r}}$.

2. Number of Concurrently Transmittable Frames

An algorithm is proposed to find the number of concurrently transmittable frames. Since the algorithm is recursive, initially N_1 is set to N. The algorithm is as follows.

Algorithm 1. Number of concurrently transmittable frames.

1. Select a tagged frame from N_1 frames.
2. Construct a group G_1 that consists of the tagged frame and the frame influencing the transmission of the tagged frame. Then, $|G_1|=\lceil E_{\text{con}}^{N_1}+1\rceil$. Note that the transmissions of frames in G_1 may influence other frames that are not in G_1, the portion of such frames is approximately $\lceil E_{\text{con}}^{N_1}+1\rceil\cdot P_{\text{SER}}$, where $P_{\text{SER}}=P_{\text{SR}_\text{t}}+P_{\text{ER}_\text{r}}-P_{\text{SR}_\text{t}}P_{\text{ER}_\text{r}}$.
3. Let N_2 be the number of frames that are not affected by the transmissions of frames in G_1. Then, this is given by $N_2=\lceil\{N_1-(\lceil E_{\text{con}}^{N_1}+1\rceil)\}-\lceil E_{\text{con}}^{N_1}+1\rceil\cdot P_{\text{SER}}\rceil$.

4. Choose a frame from N_2 frames randomly and construct G_2, which consists of the chosen frame and the frames influencing the transmission of the chosen frame. Then, $|G_2| = \lceil E_{\text{con}}^{N_2} + 1 \rceil$ with $E_{\text{con}}^{N_2} = (N_2 - 1) \cdot P_{\text{SER}}$.
5. Similarly, generate G_3 with $|G_3| = \lceil E_{\text{con}}^{N_3} + 1 \rceil$, where $N_3 = \lceil \{N_2 - \lceil E_{\text{con}}^{N_2} + 1 \rceil\} - \lceil E_{\text{con}}^{N_2} + 1 \rceil \cdot P_{\text{SER}} \rceil$.
6. Repeat this procedure until N_k<1.

From the algorithm, assume that $\{G_i\}_{i=1}^{k}$ is generated. Then, the frames from each k disjoint group can be transmitted concurrently.

3. Analysis of Directional CSMA/CA Protocol

By observing the system at the end of the busy period and idle slots, an embedded Markov chain can be constructed for a DEV.

Let $n(t)$ and $b(t)$ be the values of stage and a DEV's backoff counter at an observed time t. Then, the process $\{(n(t), b(t))\}$ is a 2D discrete-time Markov chain specifying the state of the head packet in the DEV's queue at the observed point and the corresponding state space is denoted by

$$\Omega = \{(n(t), b(t)) \mid 0 \le n(t) \le m, 0 \le b(t) \le W_{n(t)} - 1, \; n(t) \text{ and } b(t) \text{ are integers}\}. \quad (13)$$

To simplify the notation, the transition probabilities $P((i_1, j_1) \mid (i_0, j_0))$ are used instead of $P(n(t+T) = i_1, b(t+T) = j_1 \mid n(t) = i_0, b(t) = j_0)$, where T can be either T_{slot}, which is given by *pBackoffSlot*, or the duration of a busy period. As illustrated in Fig. 4, the Markov chain model for a DEV involves the following one-step transition probabilities:

$$\begin{aligned}
&P\{(i,j) \mid (i,j+1)\} = 1 - P_{\text{b,bo}}, \; j \in [0, W_{i-1} - 1], i \in [0, m] \\
&P\{(i,j) \mid (i,j)\} = P_{\text{b,bo}}, \qquad j \in [0, W_{i-1} - 1], i \in [0, m] \\
&P\{(0,j) \mid (i,0)\} = (1 - P_{\text{b,bo}} - p_{\text{col}}) / W_0, j \in [0, W_0 - 1], i \in [0, m-1] \\
&P\{(i,j) \mid (i-1,0)\} = p_{\text{col}} / W_i, \; j \in [0, W_i - 1], i \in [1, m] \\
&P\{(0,j) \mid (m,0)\} = 1 / W_0, \qquad j \in [0, W_0 - 1] \\
&P\{\text{drop} \mid (m,0)\} = p_{\text{col}},
\end{aligned} \quad (14)$$

where $P_{\text{b,bo}}$ and p_{col} are the probability that the shared channel is observed to be busy when a DEV is in the backoff state and the collision probability, respectively. Both probabilities can be obtained by considering the interactions of DEVs over the shared channel, which are given in the following subsection.

It is noted that all of the states in Ω are positive-recurrent and that the system is stable. Therefore, there exist stationary probabilities $\{b_{i,j}, b_{\text{drop}} : j \in [0, W_i - 1], i \in [1, m]\}$ of the Markov chain that satisfies the following equations:

$$\mathbf{bP} = \mathbf{b} \text{ and } \sum_{i=0}^{m} \sum_{j=0}^{W_0 - 1} b_{i,j} + b_{\text{drop}} = 1, \quad (15)$$

where $\mathbf{b} = (b_{0,0}, b_{0,1}, \cdots, b_{0,W_0-1}, b_{1,0}, b_{1,1}, \cdots, b_{1,W_1-1}, \cdots, b_{m,0}, b_{m,1}, \cdots, b_{m,W_m-1}, b_{\text{drop}})$, and $\mathbf{P}$ is the transition probability matrix when Ω is ordered lexicographically, which is given by

$$\mathbf{P} = \begin{pmatrix} A & B_1 & \mathbf{0} & \cdots & \cdots & \cdots & \mathbf{0} \\ B_0 & C & B_2 & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ B_0 & \mathbf{0} & C & B_3 & \mathbf{0} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & C & B_{m-1} & \mathbf{0} \\ B_0 & \mathbf{0} & \cdots & \cdots & \mathbf{0} & C & \mathbf{0} \\ D & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad (16)$$

where A, B_0, B_i ($i \in [1, m-1]$), C, and D are given by

$$A = \begin{pmatrix} P_{\text{b,bo}} + \alpha & \alpha & \alpha & \cdots & & \cdots & \alpha \\ 1 - P_{\text{b,bo}} & P_{\text{b,bo}} & 0 & & \cdots & \cdots & 0 \\ 0 & 1 - P_{\text{b,bo}} & P_{\text{b,bo}} & & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & & 0 & 1 - P_{\text{b,bo}} & P_{\text{b,bo}} \end{pmatrix},$$

$$B_0 = \begin{pmatrix} \alpha & \alpha & \cdots & \alpha \\ 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix}, \quad B_i = \begin{pmatrix} \frac{p_{\text{col}}}{W_i} & \frac{p_{\text{col}}}{W_i} & \cdots & \frac{p_{\text{col}}}{W_i} \\ 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} P_{\text{b,bo}} & 0 & \cdots & 0 \\ 1 - P_{\text{b,bo}} & P_{\text{b,bo}} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 - P_{\text{b,bo}} & P_{\text{b,bo}} \end{pmatrix}, \quad (17)$$

$$D = \begin{pmatrix} \frac{1}{W_0} & \frac{1}{W_0} & \cdots & \frac{1}{W_0} \end{pmatrix}, \text{ and } \alpha = \frac{1 - p_{\text{col}} - P_{\text{b,bo}}}{W_0}.$$

By using the first part of (15), the following relation between the stationary probabilities can be derived:

$$\begin{aligned}
&b_{i,0} = \{p_{\text{col}} / (1 - P_{\text{b,bo}})\}^i b_{0,0}, i \in [1, m], \\
&b_{i,j} = (W_i - j) / W \cdot \{p_{\text{col}} / (1 - P_{\text{b,bo}})\}^i b_{0,0}, i \in [0, m], j \in [1, W_i - 1], \\
&b_{\text{drop}} = p_{\text{col}} \{p_{\text{col}} / (1 - P_{\text{b,bo}})\}^m b_{0,0}.
\end{aligned} \quad (18)$$

Therefore, substituting (18) into the 2nd part of (15), $b_{0,0}$ is obtained as

$$b_{0,0} = \left[1 + \gamma_1 + \gamma_2 + p_{\text{col}} \left(\frac{p_{\text{col}}}{1 - P_{\text{b,bo}}} \right)^m + \frac{W_0 - 1}{2} \right]^{-1}, \quad (19)$$

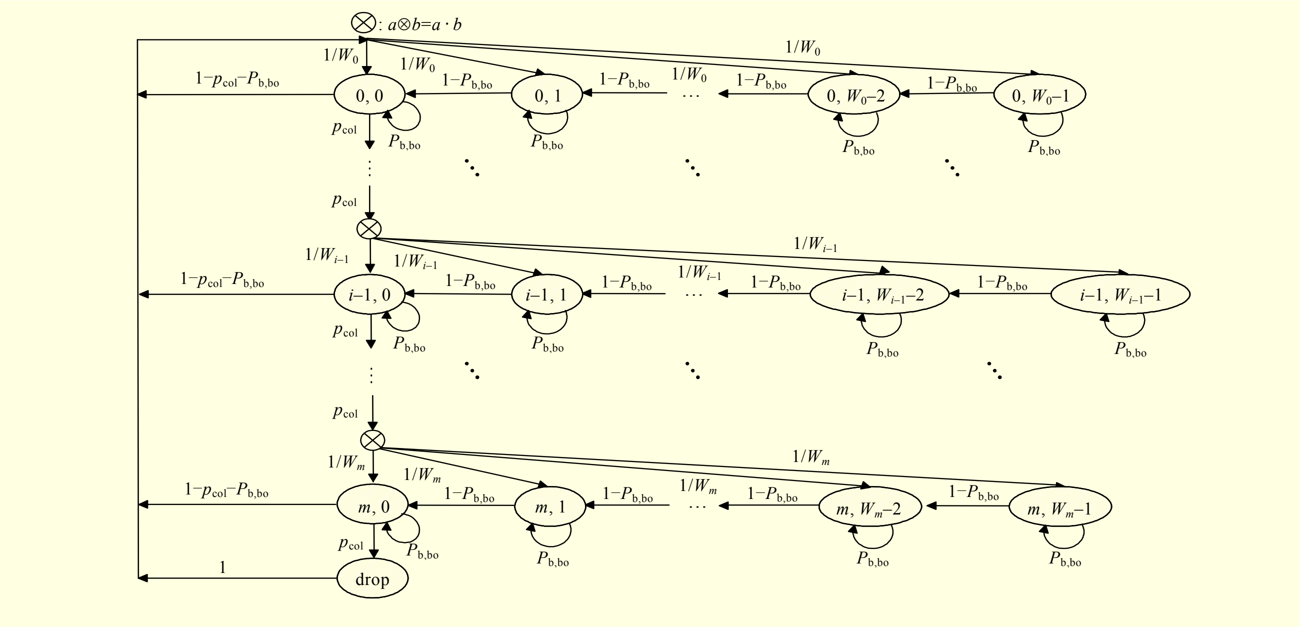

Fig. 4. Markov chain model for CSMA/CA protocol with Imm-ACK mode.

where γ_1 and γ_2 are given by

$$\gamma_1=\frac{1}{(1-P_{\text{b,bo}}-p_{\text{col}})}\left\{p_{\text{col}}-\left(\frac{p_{\text{col}}}{1-P_{\text{b,bo}}}\right)^m\right\},$$

$$\gamma_2=\frac{1}{2}\left[\begin{array}{l}\dfrac{2W_0p_{\text{col}}}{1-P_{\text{b,bo}}-2p_{\text{col}}}\left\{1-\left(\dfrac{2p_{\text{col}}}{1-P_{\text{b,bo}}}\right)^m\right\}\\ -\dfrac{p_{\text{col}}}{1-P_{\text{b,bo}}-p_{\text{col}}}\left\{1-\left(\dfrac{p_{\text{col}}}{1-P_{\text{b,bo}}}\right)^m\right\}\end{array}\right],$$

respectively. Substituting (19) into (18), the stationary probabilities $\{b_{i,j}, b_{\text{drop}} : j\in[0,W_i-1], i\in[0,m]\}$ are obtained.

4. Interactions of Devices over the Shared Channel

Assuming that N frames are contending for the channel, when a transmitter-receiver pair tries to communicate, there are two viewpoints from which the channel is considered busy: the transmitter and the receiver. From the transmitter's viewpoint, the channel is considered busy when the transmitter senses that it is. From the receiver's viewpoint, the channel is considered busy when other transmitters interfere with the receiver. In the former case, the decrement of the backoff counter of a transmitter is suspended during a busy period, while it is decremented in the latter case, unless the interferer is located in SR_t. A collision occurs if the transmitters in SR_t or the interferers in ER_r send frames simultaneously. If a DEV's backoff counter reaches zero or it selects a zero backoff counter, it has to wait for an idle BIFS before it transmits. Therefore, there should be idle slots between two busy periods.

Let P_b and τ be the probabilities that a shared channel is busy at a given observation point and that a DEV is in the transmittable state, respectively. Then, τ is given by $\sum_{i=0}^{m} b_{i,0}$ and can be decomposed according to the previous channel state, either idle, or busy. Because there are no consecutive busy periods, a DEV transmits a frame with a conditional probability p_{tx} only if the previous channel state was idle. Therefore, p_{tx} is given by $p_{\text{tx}}=\tau/(1-P_\text{b})$. In this case, P_b is calculated as follows. Let $E(B)$ and $E(I)$ be the average number of consecutive busy periods and idle slots, respectively. Since there is at least one idle slot after a busy period, it follows that $E(B)$=1. Let p_c be the probability that a collision occurs in a time slot when a transmitter transmits. Since a collision occurs either because of other transmitters in SR_t or because of interferers in ER_r and the collision occurs in either data frame or ACK, p_c and p_{col} can be written as

$$p_\text{c}=1-(1-p_{\text{tx}})^{E_{\text{con}}^N} \quad \text{and} \quad p_{\text{col}}=p_\text{c}+(1-p_\text{c})p_\text{c}. \quad (20)$$

Then $E(I)$ is obtained as

$$E(I)=\sum_{i=0}^{\infty} iP(I=i)=\frac{1}{1-(1-p_\text{c})^{(E(K_{\text{SR}_\text{t}}^N)+1)/E_{\text{con}}^N}}. \quad (21)$$

In (21), $P(I{=}i){=}\left\{1{-}(1{-}p_{\text{tx}})^{E(K_{\text{SR}_\text{t}}^N)+1}\right\}\cdot\left\{1{-}(1{-}p_{\text{tx}})^{E(K_{\text{SR}_\text{t}}^N)+1}\right\}^i$ is used. Therefore, P_b can be calculated as

$$P_\text{b}=\{1-(1-p_\text{c})^{(E(K_{\text{SR}_\text{t}}^N)+1)/E_{\text{con}}^N}\}/\{2-(1-p_\text{c})^{(E(K_{\text{SR}_\text{t}}^N)+1)/E_{\text{con}}^N}\}. \quad (22)$$

IV. Performance Analysis

To evaluate the performance of D-CSMA/CA over the shared channel, the throughput and average transmission delay of a frame are considered as performance measures.

1. Throughput

Let ψ_1 be the probability of a successful transmission of a data frame of the TTx when some of the transmitters affecting the TTx-TRx pair are transmitting. Because the transmitters sensing TTx's transmission are frozen, they cannot transmit in subsequent timeslots until TTx's transmission finishes. In this case, the transmission is successful if the transmitters in the area $ER_{\mathrm{r}} - (SR_{\mathrm{t}} \cap ER_{\mathrm{r}})$ do not transmit in the subsequent $\lceil E(T_{\mathrm{payload}}) / T_{\mathrm{slot}} \rceil - 1$ slots. Therefore, ψ_1 is computed as

$$\psi_1 = \frac{p_{\mathrm{tx}}(1-p_{\mathrm{tx}})^{E_{\mathrm{con}}^N} \left\{ (1-p_{\mathrm{tx}})^{E(K_{\mathrm{ER_r}}^N) - E(K_{\mathrm{SR_t \cap ER_r}}^N)} \right\}^{\lceil E(T_{\mathrm{payload}})/T_{\mathrm{slot}} \rceil - 1}}{1-(1-p_{\mathrm{tx}})^{E_{\mathrm{con}}^N + 1}}. \quad (23)$$

Similarly, the probability of successful transmission of ACK from the TRx when some of the transmitters affecting the TTx-TRx pair are transmitting, ψ_2, can be obtained by

$$\psi_2 = \frac{p_{\mathrm{tx}}(1-p_{\mathrm{tx}})^{E_{\mathrm{con}}^N} \left\{ (1-p_{\mathrm{tx}})^{E(K_{\mathrm{ER_r}}^N) - E(K_{\mathrm{SR_t \cap ER_r}}^N)} \right\}^{\lceil T_{\mathrm{ACK}}/T_{\mathrm{slot}} \rceil - 1}}{1-(1-p_{\mathrm{tx}})^{E_{\mathrm{con}}^N + 1}}. \quad (24)$$

Let P_{as} be the probability that a successful transmission of data and an ACK frame of a transmitter-receiver pair under the condition that some frames among $E_{\mathrm{con}}^N + 1$ are transmitting. Let P_{asuc} and P_{acol} be the probabilities of a successful transmission and collision among the group consisting of the $E_{\mathrm{con}}^N + 1$ frames, respectively. Then, they are given by

$$P_{\mathrm{as}} = \psi_1 \cdot \psi_2, \; P_{\mathrm{asuc}} = P_{\mathrm{b}} P_{\mathrm{as}}, \; \text{and} \; P_{\mathrm{acol}} = P_{\mathrm{b}}(1-P_{\mathrm{as}}). \quad (25)$$

In this case, the throughput Th_g achieved by $E_{\mathrm{con}}^N + 1$ frames is defined as

$$Th_g = \frac{P_{\mathrm{asuc}} E(P)}{(1-P_{\mathrm{b}})T_{\mathrm{slot}} + P_{\mathrm{asuc}} E(T_{\mathrm{suc}}) + P_{\mathrm{acol}} E(T_{\mathrm{col}})}, \quad (26)$$

where $E(P)$ is the average payload, which is given by the amount of data that can be transmitted for the $E(T_{\mathrm{payload}}) = T_{\mathrm{headers}} + E(T_{\mathrm{data}})$. $E(T_{\mathrm{suc}})$ and $E(T_{\mathrm{col}})$ are given by $E(T_{\mathrm{suc}}) = BIFS + E(T_{\mathrm{payload}}) + SIFS + T_{\mathrm{ACK}}$ and $E(T_{\mathrm{col}}) = BIFS + E(T_{\mathrm{payload}}) + T_{\mathrm{ACK\_timeout}}$, respectively. Here, T_{ACK} and $T_{\mathrm{ACK\_timeout}}$ are the time for receiving ACK and the waiting time for ACK for unsuccessful transmission, respectively.

Let Th_{g_i} be the throughput obtained by (26) for N_i, $i = 1, \cdots, k$, instead of N in (20) through (25), where k is the number of concurrently transmittable frames obtained in section III.2. Then, the total throughput is given as

$$Th = \sum_{i=1}^{k} Th_{g_i}. \quad (27)$$

2. Average Transmission Delay

In this paper, the processing delay and queuing delay are considered. The queuing delay of a frame is the sojourn time in the queue before the directional CSMA/CA process begins, while the processing delay is the period between the instant that the frame becomes the head frame in the queue and the instant that the frame transmission is either completed or dropped.

Let $w(i, j)$ and $E(i, j)$ be the number of waiting backoff slots before the transmission and the average required time to transmission of a frame when the DEV is in state (i, j), respectively. Since the sojourn time in state (i, j) depends on P_{b}, $w(i, j)$ is given by

$$\begin{aligned} w(i,0) &= 1 + \chi_{\{\text{PNC uses } BIFS\}} + P_{\mathrm{b}}\{\chi_{\{\text{PNC uses } BIFS\}} + 1 + l\}, \\ w(i,j) &= w(i,j-1) + 1 + P_{\mathrm{b}}\{\chi_{\{\text{PNC uses } BIFS\}} + 1 + l\}, \\ & \quad j \in [0, W_i - 1], i \in [0, m], \end{aligned} \quad (28)$$

where χ_A is an indicator function taking the value 1 if A is true, or 0 if it is false, and l is the load duration represented by T_{slot}. Then, the average waiting time slots before transmission, $E(W)$, and the average processing delay, $E_{\mathrm{p}}(D)$, are given by

$$\begin{aligned} E(W) &= \sum_{i=0}^{m} \sum_{j=0}^{W_i - 1} w(i,j) \Big/ \sum_{k=0}^{m} W_k, \\ E_{\mathrm{p}}(D) &= \sum_{i=0}^{m} \sum_{j=0}^{W_i - 1} E(i,j) \Big/ \sum_{k=0}^{m} W_k, \end{aligned} \quad (29)$$

respectively, where $E(i,j) = w(i,j) \cdot T_{\mathrm{slot}} + P_{\mathrm{asuc}} E(T_{\mathrm{suc}}) + P_{\mathrm{acol}} E(T_{\mathrm{col}})$, $j \in [0, W_i - 1], i \in [0, m]$.

Since the load of a frame in the queue is assumed as a constant, the service time for the load is constant if all DEV use the same data rate for their frame transmissions. Therefore, the queueing model can be considered as M/D/1/K. In [20], the average queue length and the average waiting time of a frame for a M/D/1/K queue were analyzed, which are given by

$$\begin{aligned} E(Q) &= K - \sum_{k=0}^{K-1} b_k / (1 + \rho b_{K-1}), \\ E_{\mathrm{q}}(D) &= \{K - 1 - (\sum_{k=0}^{K-1} b_k - K)/(1 + \rho b_{K-1})\} E_{\mathrm{p}}(D), \end{aligned} \quad (30)$$

respectively, where $\rho = \lambda E_{\mathrm{p}}(D)$, $b_0 = 1$, and

$$b_n = \sum_{k=0}^{n} (-1)^k (n-k)^k e^{(n-k)\rho} \rho^k / k, \text{ for } n \geq 1.$$

Therefore, the average transmission delay is given by $E(D) = E_{\mathrm{p}}(D) + E_{\mathrm{q}}(D)$.

Since TR can be less than the maximal distance of the piconet, the transmitter-receiver pairs that cannot communicate with each other can exist. Let P_{ntx} be the probability of such transmitter-receiver pairs. Then, it is given by

$$P_{\text{ntx}}=\begin{cases}\int_{TR}^{L} f_1(x)dx+\int_{L}^{\sqrt{2}L} f_2(x)dx, & \text{if } 0<TR\le L,\\ \int_{TR}^{\sqrt{2}L} f_2(x)dx, & \text{if } L<TR\le\sqrt{2}L,\\ 0, & \text{if } TR>\sqrt{2}L.\end{cases} \quad (31)$$

V. Numerical Results

In this section, the numerical results are presented. Matlab 7.7 was used for the analysis and simulation. The set of parameters is based on the IEEE 802.15.3c standard and is as follows: $T_{\text{SF\_max}}$=65,535, T_{CAP}=6,553, T_{beacon}=50, *SIFS*=2.5, *BIFS*=6.5, T_{slot}=6.5, *pCCADectTime*=4, $T_{\text{ACK}}=T_{\text{slot}}$, $T_{\text{ACK\_timeout}}=SIFS+T_{\text{slot}}$ (μs), n=2, W=1,728 MHz, P_T=10 mW, P_R=−55 dB, and N_0=−91.9 dB, which is the average of the noise. The simulation was executed 200 times. In general, the length of a CAP is assumed as 1/10 of the length of a superframe. Therefore, 1,000 slot times which corresponds to the length of a CAP was used as the duration for each simulation. All of the assumptions and other parameters described in sections II through IV are used. The numerical results were obtained using a 1.65-Gbps data rate, and L=10 m. Two backoff slots (T_{slot}) were used as l, which corresponds to 21.45×10^3 bits for the 1.65-Gbps data rate. In this simulation environment, each DEV transmits 10^5 frames during 200 simulations. Two different radiation efficiencies are considered: η=0.9 and 1. In the Appendix, it is shown that p_{col} is unique and found exhaustively for the analytical results.

Figure 5 compares the total throughputs Th according to θ and η. Th increases as θ decreases and η increases. For instance, the throughputs for θ=10° and 360° are 1.3862 Gbps and 0.1051 Gbps, respectively, when the number of devices is 60 and η=1. Throughput using a directional antenna with θ=10° is ten times larger than that for an omni-directional antenna. This is due to the transmission range and spatial reuse of directional antennas. Figure 6 compares the average processing transmission delays $E_p(D)$ and the average numbers of time slots used in backoff stage $E(W)$ according to θ. $E_p(D)$ decreases as θ decreases for the same reason as Th. Trends for $E_p(D)$ and $E(W)$ are identical. This is due to the fact that a longer stay in the backoff stage causes a larger transmission delay. Figure 7 compares the queueing delays, total delays, and average queue length for two pairs of K and λ: K=5, λ=0.5 and K=10, λ=1. This shows that $E_q(D)$ and $E(D)$ increase as K, λ, and θ increase, and the trend of both delays is identical. There are fewer frames in the queue for the smaller θ. Figure 8 shows the number of frames that are concurrently transmittable with and without CSMA/CA, $th_{\text{ub,c}}$ and $th_{\text{ub,s}}$, respectively, and the number of frames transmitted concurrently and successfully, obtained by algorithm 1 and by simulation. It shows that the number of concurrently successfully transmitted frames increases as the number of DEVs and η increase. In [8], the authors considered the number of concurrently transmittable frames for scheduling, which is $th_{\text{ub,s}}$ in the figure. For the scheduling, the numbers in [9] were compared with a different parameter setting. Figure 9 compares the drop probabilities and P_{ntx} according to θ and η. Despite the fact that there are dropped frames in the analytical results, they are negligible. For instance, for the worst case in the figure, η=0.9, θ=20°, and 50 frames, the drop probability is 29.31×10^{-4}. In the simulation, no frames are dropped for the considered parameters.

VI. Conclusion

In this paper, a Markov chain model is presented to analyze the directional CSMA/CA with Imm-ACK mode focusing on the data transmission between devices in the contention access period based on IEEE 802.15.3c. The transition probabilities in

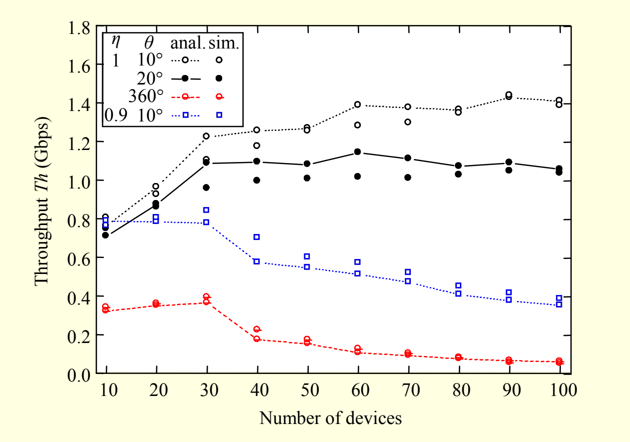

Fig. 5. Comparison of throughputs.

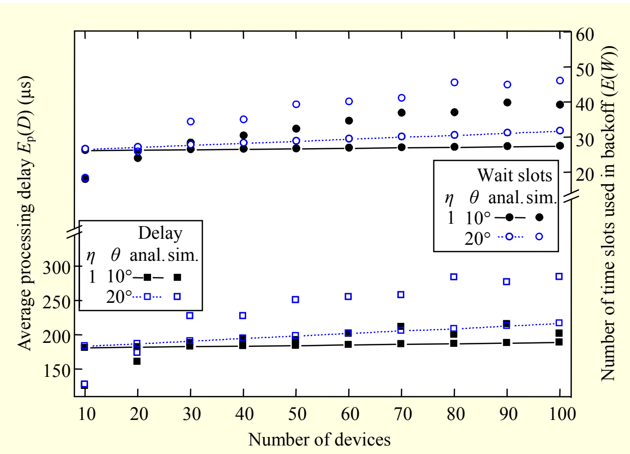

Fig. 6. Average processing delays vs. average number of time slots used in backoff.

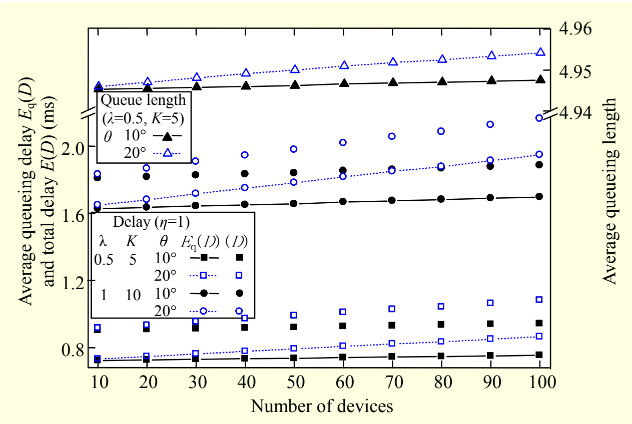

Fig. 7. Average queueing and total delays vs. average queue length.

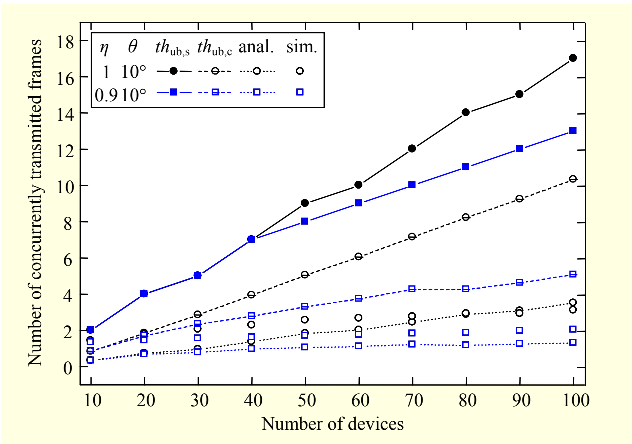

Fig. 8. Average numbers of concurrently transmittable and transmitted frames.

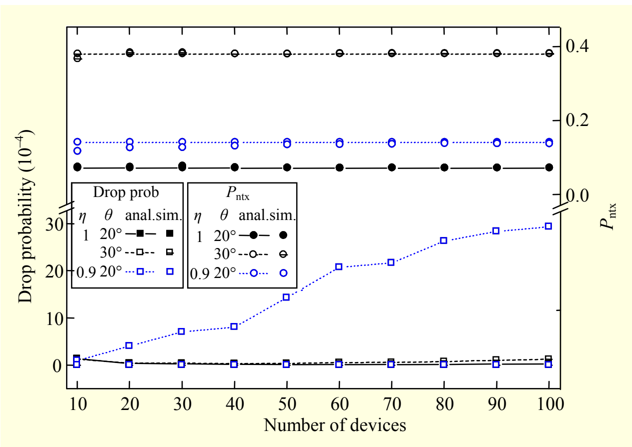

Fig. 9. Drop probabilities vs. P_{ntx}.

the model involve the characteristics of directional antennas and the features of 802.15.3c. In addition, the numbers of interferers for transmitters' sensing and receivers' receiving, and the number of concurrently transmittable frames when devices are equipped with directional antennas are derived probabilistically. The analysis is verified by simulation and the numerical results show the effects of using directional antennas in communications requiring a high data rate.

In the near future, this model will be extended to non-saturation environments and a more practical model by eliminating the restrictions imposed in this paper, for example, the use of identical transmission power and the non-existence of obstacles. Furthermore, efficient scheduling algorithms will be developed for the reservation period based on the values derived to enhance the performance of mmWave WPANs.

Appendix. Proof of existence and uniqueness of p_{col}

p_{tx} and p_c are nonlinear functions of the two unknowns τ and p_{col}, which can be solved by a numerical technique. Now it is proven that this system has a unique solution. From (21), $p_{tx} = 1-(1-p_{col})^{1/2E^N_{con}}$ is obtained. Let $f(p_{col}) = p_{tx} = 1-(1-p_{col})^{1/2E^N_{con}}$. Then, $f(0)=0$, $f(1)=1$, and f is a monotonically increasing function. On the other hand, (20) and (23) are functions of p_{col}. By setting them as $g(p_{col})$ and $h(p_{col})$, respectively, $g(p_{col}) = \tau/\{1-h(p_{col})\}$ is obtained. Since $h(0)=0$ and $h(1)=1/2$, $g(0)=\tau>0$ and $g(1)=2\tau<1$ are derived. Since g is a monotone, $g(0)>f(0)$, and $g(1)>f(1)$, there exists p_{col} uniquely that satisfies $f(p_{col})=g(p_{col})$.

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