

# The proposed “Waldmeier discontinuity”: How does it affect to sunspot cycle characteristics?

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## ABSTRACT

Recently, Svalgaard has proposed that Waldmeier introduced a discontinuity in the International Sunspot Number (*ISN*) around 1945. In this paper, we study some characteristics of the sunspot cycle using the classical *ISN* and the proposed version derived from the “Waldmeier discontinuity”. We conclude that this proposed version does not significantly improve the statistics of the characteristics of solar cycle.

**Key words.** solar activity – solar cycle – sunspot – space climate – historical records

## 1. Introduction

The most important index of solar activity is the Wolf or Zürich Sunspot Number (now called the International Sunspot Number, *ISN*) that was defined by Rudolf Wolf in the 19th century. Currently, the *ISN* is provided by the Solar Influences Data Analysis Center (SIDC) in Brussels, Belgium (Clette et al. 2007). However, Hoyt & Schatten (1998) made a comprehensive work of recovery of sunspot data and defined a new index based on these observations: Group Sunspot Number (*GSN*). This index is designed to have similar values that *ISN* had in the period 1873–1986. *GSN* only depends on the number of sunspot groups while *ISN* depends on the number of groups and the number of individual spots.

Some authors have compared both sunspot number series from different points of view: spectral characteristics (Faria et al. 2004; Li et al. 2005; Getko 2006; Li & Liang 2010) and sunspot cycle characteristics (Hathaway et al. 2002). Recently, Svalgaard (2010) has studied the ratio of *ISN* and *GSN* showing an apparent discontinuity in 1945 just when Max Waldmeier took over the production of the *ISN*. According to Svalgaard (2010), Waldmeier introduced an upward jump in the sunspot number in 1945. Therefore, Svalgaard (2010) proposed that all values before 1945 were increased by 20%. Note that when the SIDC took over the production of *ISN* (just after Waldmeier), it strove to maintain continuity with the values of Waldmeier. Therefore, it is possible to construct a Modified International Sunspot Number (*MISN*) increasing by 20% for all values of the *ISN* prior to 1945.

In this paper we study the behavior of different sunspot numbers (*GSN*, *ISN* and *MISN*) from the point of view of the solar cycle characteristics. First, we consider solar cycles (*SC*) 1–22. Then, we made the same study only for *SC* 10–22, which is the best quality period of this series (Vaquero 2007) because the annual number of days without records is zero.

## 2. Data

In the present study, we employ the usual smoothed monthly mean values (13-month running mean) of *GSN*, *ISN* and *MISN*, computed from the monthly mean sunspot numbers available from Solar Influences Data Analysis Center (SIDC, <http://sidc.oma.be/>) and National Geophysical Data Center (NGDC, <http://www.ngdc.noaa.gov/stp/solar/ssndata.html>). From these values, we computed the following parameters for each solar cycle: ascending time ( $T_a$ ), descending time ( $T_d$ ), amplitude ( $A$ ), period of the preceding cycle ( $P_{pre}$ ) and preceding cycle minimum ( $M_{pre}$ ). They are listed in Tables 1–3 (*GSN*, *ISN* and *MISN*, respectively). We use data for *SC* 1–22, i.e. the common period of this three sunspot numbers.

## 3. Results

In order to compare the three sunspot numbers (*GSN*, *ISN* and *MISN*), we are going to consider four well-known characteristics of solar cycle: “Waldmeier Effect” (the anticorrelation between the cycle amplitude and the length of the ascending phase of the cycle, Waldmeier 1939), “Amplitude-Period Effect” (the anticorrelation between the cycle amplitude and the length of the previous cycle from minimum to minimum, Chernosky 1954; Wilson et al. 1998), “Amplitude-Minimum Effect” (the correlation between cycle amplitude and the activity level at the previous minimum, Wilson et al. 1998) and “Amplitude-Descending Time Effect” (the correlation between cycle amplitude and the descending time of previous solar cycles, Du & Du 2006).

Note that also other correlations have been described in the literature. We can cite, for example, that (a) the amplitude of *SC*  $n$  is inversely related to the period of the *SC*  $n$  (Charbonneau & Dikpati 2000; Hathaway et al. 2002) and (b) the amplitude of *SC*  $n$  is related to the period of the *SC*  $n - 3$  (Solanki et al. 2002; Du et al. 2006).

**Table 1.** Solar cycle parameters used in this study for *GSN*.

<i>SC</i>	<i>GSN</i>					
	$T_a$ (months)	Rise rate (months <sup>-1</sup> )	$T_d$ (months)	$A$	$P_{pre}$ (months)	$M_{pre}$
1	71	1.0	59	71.3		4.6
2	41	2.6	69	106.5	130	2.5
3	48	1.7	57	79.5	110	5.8
4	43	2.1	145	90.5	105	0
5	20	2.6	107	51.1	188	3.5
6	75	0.4	78	31.5	127	0
7	81	0.8	47	64.4	153	0.1
8	40	2.9	76	116.8	128	5.2
9	64	1.5	85	93.2	116	8.3
10	58	1.5	79	85.8	149	1.7
11	42	2.4	97	99.9	137	4.6
12	63	1.1	71	68.2	139	1.8
13	47	2.0	96	96	134	5.6
14	49	1.3	89	64.6	143	2.2
15	49	2.3	73	111.3	138	1.7
16	58	1.4	62	81.6	122	6.6
17	43	2.9	83	125.1	120	4.4
18	40	3.6	81	145.2	126	7.9
19	47	4.0	75	186.1	121	3.1
20	72	1.5	69	109.3	122	10
21	40	3.9	80	154.2	141	12.8
22	44	3.5		153	120	11.1

**Table 2.** Solar cycle parameters used in this study for *ISN*.

<i>SC</i>	<i>ISN</i>					
	$T_a$ (months)	Rise rate (months <sup>-1</sup> )	$T_d$ (months)	$A$	$P_{pre}$ (months)	$M_{pre}$
1	75	1.2	59	86.5		8.4
2	40	2.9	69	115.8	134	11.2
3	35	4.5	76	158.5	109	7.2
4	41	3.4	122	141.2	111	9.5
5	82	0.6	65	49.2	163	3.2
6	70	0.7	83	48.7	147	0
7	79	0.9	48	71.5	153	0.1
8	40	3.7	76	146.9	127	7.3
9	55	2.4	94	132	116	10.6
10	50	2.0	85	98	149	3.2
11	41	3.4	100	140.3	135	5.2
12	60	1.2	74	74.6	141	2.2
13	47	1.9	96	87.9	134	5
14	49	1.3	89	64.2	143	2.7
15	49	2.2	72	105.4	138	1.5
16	56	1.4	65	78.1	121	5.6
17	43	2.8	82	119.2	121	3.5
18	39	3.9	83	151.8	125	7.7
19	47	4.3	79	201.3	122	3.4
20	49	2.3	91	110.6	126	9.6
21	42	3.9	81	164.5	140	12.2
22	34	4.7	82	158.5	123	12.3

### 3.1. Waldmeier effect

We consider the Waldmeier Effect comparing the amplitude and the ascending time for each solar cycle. Figure 1 (upper panels) shows the relationship between both parameters for *SC* 1–22 (left panel) and *SC* 10–22 (right panel).

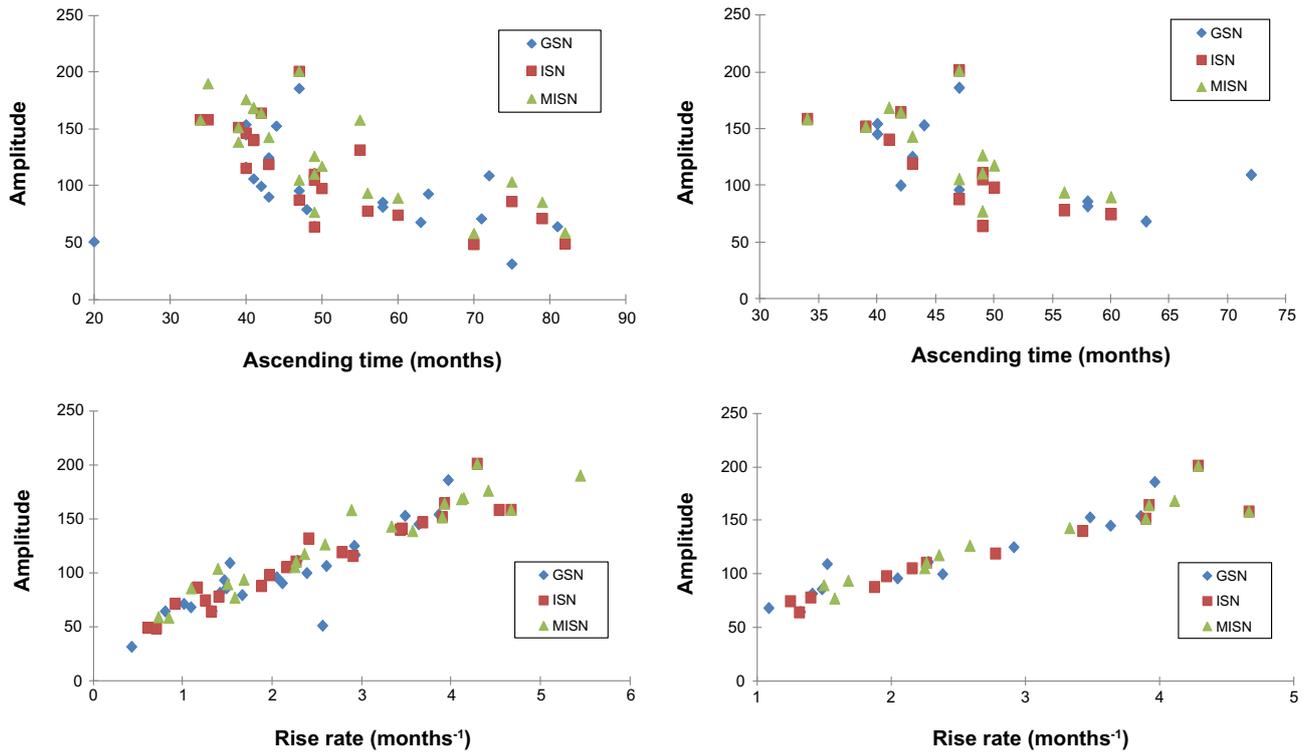
Using data from *SC* 1–22, we obtain a high linear Pearson coefficient for *ISN* and *MISN* (–0.741 and –0.758, respectively). However, we obtain a low correlation coefficient for

*GSN* (–0.381). These results vary significantly if we use data from the *SC* 10–22, correlation coefficients were slightly lower for *ISN* and *MISN* (–0.669 and –0.666, respectively) and a higher correlation coefficient for *GSN* (–0.509).

We note that the “Waldmeier Effect” is very weak in the *GSN*, as was already shown by Hathaway et al. (2002), and there are no significantly different values when we use *ISN* or *MISN*.

**Table 3.** Solar cycle parameters used in this study for *MISN*.

<i>SC</i>	<i>MISN</i>					
	$T_a$ (months)	Rise rate (months <sup>-1</sup> )	$T_d$ (months)	$A$	$P_{pre}$ (months)	$M_{pre}$
1	75	1.4	60	103.8		10.1
2	39	3.6	69	138.9	135	13.4
3	35	5.4	76	190.3	108	8.6
4	41	4.1	122	169.4	111	11.5
5	82	0.7	65	59	163	3.8
6	70	0.8	83	58.4	147	0
7	79	1.1	48	85.8	153	0.1
8	40	4.4	76	176.3	127	8.8
9	55	2.9	94	158.3	116	12.7
10	50	2.4	85	117.6	149	3.8
11	41	4.1	100	168.4	135	6.2
12	60	1.5	74	89.6	141	2.7
13	47	2.2	96	105.5	134	6
14	49	1.6	89	77.1	143	3.2
15	49	2.6	72	126.5	138	1.8
16	56	1.7	65	93.8	121	6.7
17	43	3.3	82	143	121	4.2
18	39	3.9	83	151.8	125	9.3
19	47	4.3	79	201.3	122	3.4
20	49	2.3	91	110.6	126	9.6
21	42	3.9	81	164.5	140	12.2
22	34	4.7	82	158.5	123	12.3


**Fig. 1.** Upper panels: scatter plot of the classical Waldmeier Effect for cycles 1–22 (left panel) and 10–22 (right panel). Lower panels: scatter plot of a variation of the Waldmeier Effect for cycles 1–22 (left panel) and 10–22 (right panel).

A variation of the “Waldmeier Effect” is the correlation between the cycle amplitude and the rise rate (Cameron & Schüssler 2007; Dikpati et al. 2008; Karak & Choudhuri 2011). We can also consider this variation of the “Waldmeier Effect”. Figure 1 (lower panels) shows the relationship between both parameters for *SC* 1–22 (left panel) and *SC* 10–22 (right panel).

We obtain very high correlation coefficients for *SC* 1–22 using *ISN*, *MISN* and *GSN* (0.956, 0.951 and 0.878, respectively). If we use data of *SC* 10–22, we obtain similar correlation coefficients for *ISN* and *MISN* (0.954 and 0.942, respectively) and higher for *GSN* (0.949). We note that the correlation in *MISN* is slightly weaker than in *ISN*.

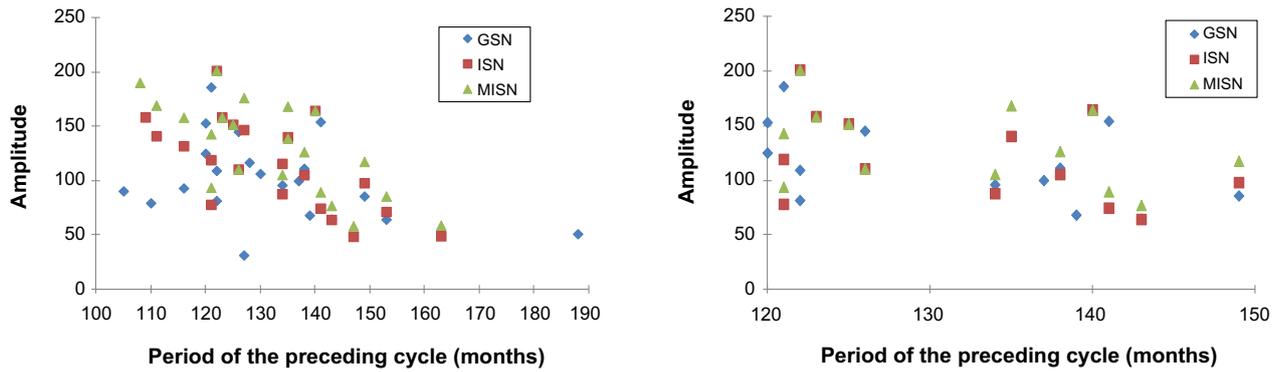


Fig. 2. Scatter plot of the Amplitude-Period Effect for cycles 1–22 (left panel) and 10–22 (right panel).

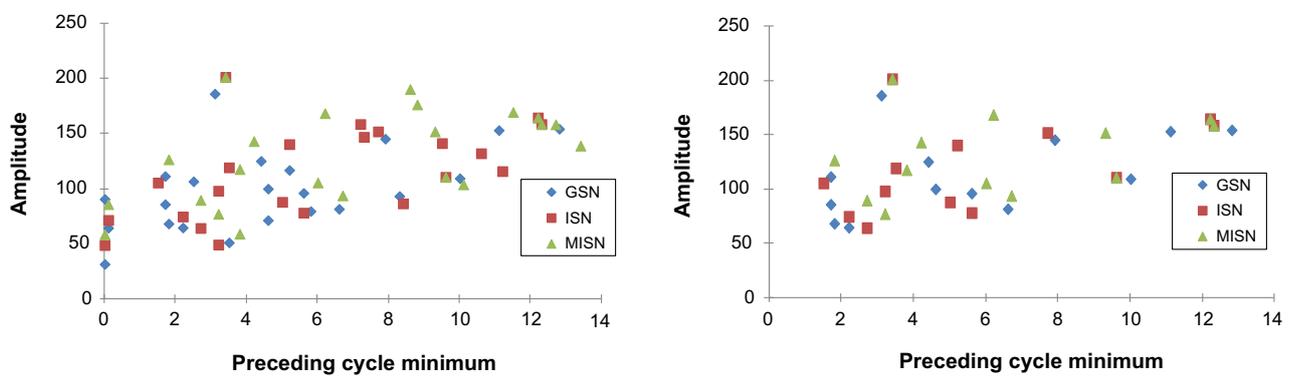


Fig. 3. Scatter plot of the Amplitude-Minimum Effect for cycles 1–22 (left panel) and 10–22 (right panel).

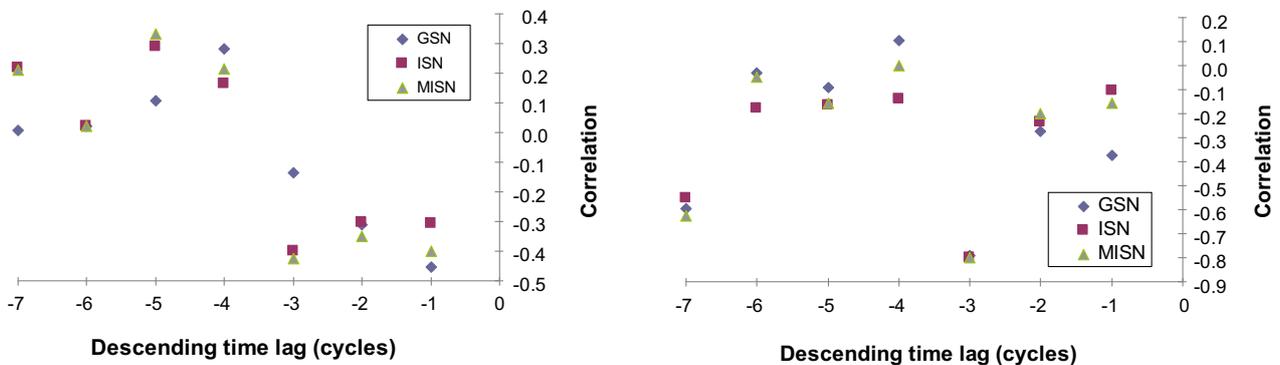


Fig. 4. Cross-correlation coefficient between amplitude and descending time at lags from one to seven cycles for cycles 1–22 (left panel) and 10–22 (right panel).

### 3.2. Amplitude-period effect

The second studied characteristic is the Amplitude-Period Effect, that is considered by comparing the amplitude of a solar cycle and the period of the preceding solar cycle. Figure 2 shows the relationship between both parameters for SC 1–22 (left panel) and SC 10–22 (right panel).

Using data from SC 1–22, we obtain a high correlation coefficient for ISN and MISN (−0.669 and −0.719, respectively). Moreover, we obtain a low correlation coefficient for GSN (−0.358). However, correlation coefficients were lower for ISN and MISN (−0.419 and −0.376, respectively) and higher for GSN (−0.512) with data from the SC 10–22.

Therefore, “Amplitude-Period Effect” is very weak in the GSN, as was already shown by Hathaway et al. (2002) and Vaquero & Trigo (2008). Results for ISN and MISN are very similar. Note that Vaquero & Trigo (2008) concluded that this relationship is only strongly significant in a statistical sense during the first half of the historical record of ISN. Moreover, it is considerably less significant for the GSN.

### 3.3. Amplitude-minimum effect

The Amplitude-Minimum Effect has also been considered. Figure 3 represents the solar amplitude versus the activity

**Table 4.** Summary of correlation coefficients obtained in this study. Single asterisk indicates a 95% significant level. Double asterisks indicate a 99% significant level.

	<i>SC</i>	<i>ISN</i>	<i>MISN</i>	<i>GSN</i>
Waldmeier Effect	1–22	−0.741**	−0.758**	−0.381*
	10–22	−0.669**	−0.666**	−0.509*
Modified Waldmeier Effect	1–22	0.956**	0.951**	0.878**
	10–22	0.954**	0.942**	0.949**
Amplitude-Period Effect	1–22	−0.669**	−0.719**	−0.358
	10–22	−0.419	−0.376	−0.512*
Amplitude-Minimum Effect	1–22	0.559*	0.536*	0.542*
	10–22	0.468*	0.311	0.479*
Amplitude-Descending Time Effect	1–22	−0.399	−0.424	−0.136
	10–22	−0.799**	−0.801**	−0.791**

level at the previous minimum for *SC* 1–22 (left panel) and *SC* 10–22 (right panel).

We obtain low correlation coefficients between amplitude and the activity level at the previous minimum for *SC* 1–22 using *ISN*, *MISN* and *GSN* (0.559, 0.536 and 0.542, respectively). If we use data of *SC* 10–22, we obtain lower correlation coefficients for *ISN*, *MISN* and *GSN* (0.468, 0.311 and 0.479, respectively). Note that the lowest value of the correlation coefficient is obtained for *MISN*.

### 3.4. Amplitude-descending time effect

In this section, we analyze the relationship between the cycle amplitude and the descending time of previous *SC*. Note that the amplitude of a solar cycle was found to be well correlated (correlation coefficient equal to  $-0.811$ ) with the descending time three cycles earlier, in smoothed monthly mean sunspot numbers for *SC* 8–23 (Du & Du 2006). Therefore, this relationship could be useful for long-term solar activity predictions.

However, we do not find any significant correlation using data of *SC* 1–22 and lags from  $-1$  to  $-7$  (Fig. 4, left panel) for all considered sunspot numbers. In agreement with Du & Du (2006), we found a negative correlation ( $-0.799$ ) for lag equal to  $-3$  using *ISN* and data of *SC* 10–22 (Fig. 4, right panel). Moreover, we found almost the same result for *MISN* and *GSN* ( $-0.801$  and  $-0.791$ , respectively) for *SC* 10–22.

## 4. Conclusions

We have computed the correlation coefficients between some parameters that are related to several characteristics of the solar cycle. These results are summarized in Table 4. According to our analysis, the following conclusions have been reached:

1. The “Waldmeier Effect” and the “Amplitude-Period Effect” are much more apparent in the *ISN* and *MISN* than in *GSN*. The “Amplitude-Minimum Effect” and the “Amplitude-Descending Time Effect” are equally apparent in the *ISN*, *MISN* and *GSN*.
2. The use of *MISN* instead of *ISN* slightly improves the correlation coefficient of the Waldmeier Effect and Amplitude-Period Effect. However, it slightly worsens the correlation coefficient of the Amplitude-Minimum Effect and Amplitude-Descending Time Effect.

3. If only *SC* 10–22 are considered, then the correlation coefficients greatly worsen for all studied characteristics, except for the Amplitude-Descending Time Effect.
4. The Amplitude-Descending Time Effect must not be considered a tool for long-term solar activity prediction because the correlation between the amplitude of a solar cycle and the descending time three cycles earlier vanished when data of *SC* 1–22 were used. Note that the exact relationship might be nonstationary. A similar conclusion may hold for Amplitude-Period Effect (see Table 4).
5. The use of *MISN* does not significantly improve the statistics of the characteristics of solar cycle.

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