

# Provably Secure Aggregate Signcryption Scheme

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An aggregate signature scheme is a digital signature scheme that allows aggregation of  $n$  distinct signatures by  $n$  distinct users on  $n$  distinct messages. In this paper, we present an aggregate signcryption scheme (ASC) that is useful for reducing the size of certification chains (by aggregating all signatures in the chain) and for reducing message size in secure routing protocols. The new ASC scheme combines identity-based encryption and the aggregation of signatures in a practical way that can simultaneously satisfy the security requirements for confidentiality and authentication. We formally prove the security of the new scheme in a random oracle model with respect to security properties IND-CCA2, AUTH-CMA2, and EUF-CMA.

**Keywords:** Identity-based cryptography, signcryption, aggregate signature, bilinear pairing.

## I. Introduction

An aggregate signature scheme is a digital signature scheme, the concept of which was first proposed by Boneh and others [1]. Aggregate signature allows aggregation of different signatures by  $n$  different users  $ID_i$  on different messages  $m_i$ . The primary objective of an aggregate signature scheme is to achieve both computation and communication efficiency. In aggregate signature, multiple signatures from various users are combined into a single compact signature. Aggregation can be used to reduce the certificate chains in public key infrastructure (PKI) settings. The aggregate signature has many real world applications ranging from traffic control to documents signed by directors of a company for official purpose.

In certain scenarios, one may need to hide the sending message so that only the receiver will be able to get back the message. In such cases, signcryption comes into the picture. Consider the scenario of an online opinion poll. The verifier has to ensure that all the concerned persons have polled their votes in an efficient way, but one may want his opinion to be secret. Only the verifier will be able to decrypt the messages and get the opinions. Consider another scenario where the directors of a company have to vote on some controversial issue. Each of them wants their vote to be hidden from others since it may disrupt the friendly atmosphere prevailing in the company. In both of these cases, aggregate signcryption can be used to increase efficiency, provide secrecy, and decrease the communication overhead. Aggregate signcryption also has applications in military communication.

The concept of public key signcryption was proposed by Zheng [2]. The idea of this kind of cryptographic primitive is to perform encryption and signature in a single logical step to obtain confidentiality, integrity, authentication, and non-repudiation more efficiently than the sign-then-encrypt approach. Identity-based cryptosystems (IBCs) were first introduced by Shamir in 1984 [3]. IBCs eliminate trust problems

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encountered in certificate-based PKIs: There is no need to bind a public key to its owner's identity since they function as a singular element. Malone-Lee [4] developed the first identity-based signcryption scheme. Selvi and others [5] proposed an identity-based threshold signcryption scheme and formally proved its security in the existing security model. Muniz and Laud [6] proposed the first strong forward-secure identity-based signcryption scheme.

Gentry and Ramzan [7] proposed an efficient identity-based aggregate signature scheme. This scheme achieves both full aggregation and also a constant number of pairing operations during signature verification. Selvi and others [8], [9] analyzed the security in some of the existing aggregate signature schemes [10]-[12] and proposed two identity-based aggregate signature schemes. However, Selvi and others [9] presented the security model for unforgeability but not the security proof. The scheme proposed in [9] cannot be considered an identity-based system because the user's public key in Selvi and others' scheme is not an identity-based public key.

Selvi and others [13] proposed the first identity-based aggregate signcryption along with a formal security model and a formal security proof. However, where the aggregate signature  $V_{\text{agg}}$  in the "IBAS-1 Unsigncrypt" algorithm is the sum of unknown signature  $V_i$ , they did not explain how to recover  $V_i$  from  $V_{\text{agg}}$ .

In our previous paper [14], a new signcryption scheme (IBRSC) was presented based on identity and ring signcryption from pairings, but the essence of IBRSC is to ensure the anonymity of a user when they need to send a message confidentially and authentically to a specific receiver. The scheme cannot allow aggregation of  $n$  distinct signatures by  $n$  distinct users on  $n$  distinct messages.

In this paper, we propose an aggregate signcryption scheme (ASC) in which signature is a modification of the aggregate signature schemes in [9]. Also, we use the added advantage that identity-based cryptosystems provide an effective remedy to the key escrow problem, which is an inherent issue in IBC. In our ASC scheme, we eliminate the interaction among the senders (signers) before the signcryption generation, which reduces the communication complexity to a large extent. However, in this scheme, we are able to achieve only partial aggregation, not full aggregation. The ASC we propose can effectively improve computation and communication efficiency and has been formally proven to satisfy confidentiality and unforgeability in the random oracle model.

## II. Preliminaries

### 1. Computation Assumptions

There are some computation assumptions about

preliminaries related to an ASC, such as bilinear pairing, the bilinear Diffie-Hellman (BDH) problem, the decisional bilinear Diffie-Hellman (DBDH) problem, and the discrete logarithm (DL) problem. A bilinear pairing is a map  $\hat{e}: G_1 \times G_1 \rightarrow G_2$  with the bilinearity, non-degeneracy, and computability properties, where  $G_1$  is an additive cyclic group and  $G_2$  is a multiplicative cyclic group of the same order  $q$ . Bilinearity means that given elements  $P, R, Q \in G_1$ , then  $\hat{e}(P + Q, R) = \hat{e}(P, R) \hat{e}(Q, R)$  and  $\hat{e}(P, Q + R) = \hat{e}(P, Q) \hat{e}(P, R)$ . In particular, for  $a, b \in \mathbb{Z}_q^*$ ,  $\hat{e}(P^a, Q^b) = \hat{e}(P, Q)^{ab} = \hat{e}(P^{ab}, Q) = \hat{e}(P, Q^{ab})$ . Non-degeneracy means that there exist  $P, Q \in G_1$ , such that  $\hat{e}(P, Q) \neq I_{G_2}$ , where  $I_{G_2}$  is the identity element of  $G_2$ . Computability means that there exists an efficient algorithm to compute  $\hat{e}(P, Q)$  for all  $P, Q \in G_1$ . The above properties can be derived from Weil or Tate pairing on an elliptic curve over a finite field [15]. For any probabilistic polynomial time algorithm  $\mathcal{A}$ , the BDH problem in  $G_1$  is to compute  $\hat{e}(g, g)^{abc}$ , and the advantage in solving the BDH problem is defined as  $\text{Adv}_{\mathcal{A}}^{\text{BDH}} = \Pr[\mathcal{A}(g, g^a, g^b, g^c) = \hat{e}(P, P)^{abc} \mid a, b, c \in \mathbb{Z}_q^*]$ . The DBDH problem is to decide if  $w = \hat{e}(P, P)^{abc}$ . And the advantage in solving DBDH problem is defined as  $\text{Adv}_{\mathcal{A}}^{\text{DBDH}} = \left| \Pr[\mathcal{A}(g, g^a, g^b, g^c, w, \hat{e}(P, P)^{abc}) = 1] - \Pr[\mathcal{A}(g, g^a, g^b, g^c, w) = 1] \right|$ . The DL problem is to find  $x$ . And the advantage in solving the DL problem is defined as  $\text{Adv}_{\mathcal{A}}^{\text{DL}} = \Pr[\mathcal{A}(g, h) = x]$  when  $g$  and  $h$  are given.

### 2. Framework of ASC

An ASC consists of the following probabilistic polynomial time algorithms.

**Setup( $k$ ).** Given the security parameter of the system  $k$ , the private key generator (PKG) generates the set of public parameters  $\pi$  and the master secret key  $s$  of the system.

**Key Extract( $\text{ID}_i$ ).** Given an identity  $\text{ID}_i$ , the PKG, using the set of public parameters  $\pi$  and the master secret key  $s$ , computes the corresponding private key  $\langle s_i, d_i \rangle$ , which is transmitted to  $\text{ID}_i$  in a secure way, and the public key  $\langle X_i, q_i \rangle$ .

**Signcrypt( $m_i, X_i, d_i, \text{ID}_i, \text{ID}_B$ ).** Let  $\mathcal{M}$  be the message space,  $\mathcal{W}$  - the signcrypt message space, and  $\mathcal{R}$  - the space of senders. We will identify any member  $X \in \mathcal{R}$  by its identity  $\text{ID}_X$ .

For any  $m_i \in \mathcal{M}$ ,  $i$  ( $1 \leq i \leq n$ ) ( $n \in \mathbb{Z}^+$ ) is an arbitrary fixed integer, the algorithm **Signcrypt( $m_i, X_i, d_i, \text{ID}_i, \text{ID}_B$ )** is defined as follows:

The sender  $\text{ID}_i$  having a private key  $\langle s_i, d_i \rangle$  runs this algorithm to generate a signcryption on message  $m_i$  that will be aggregated and send it to a receiver with identity  $\text{ID}_B$ . The output is a ciphertext  $\sigma_i \in \mathcal{W}$ .

**Aggregate( $\{\sigma_i, \text{ID}_i\}_{i=1, \dots, n}$ ).** Let  $\mathcal{W}$  be the aggregate signcrypt message space. Given a set of  $n$  signcrypts  $\{\sigma_i\}_{i=1, \dots, n}$  and the corresponding identity  $\text{ID}_i$ , this algorithm outputs the final

aggregate signcryption  $\sigma_{\text{agg}} \in \mathcal{W}'$ .

**Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ). For any  $\sigma_{\text{agg}} \in \mathcal{W}'$  and a receiver with identity  $ID_B$  with the private key  $\langle s_B, d_B \rangle$ , the algorithm **Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ) is defined as follows:

The receiver  $ID_B$  receives  $\sigma_{\text{agg}}$  and runs **Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ). If  $\sigma_{\text{agg}}$  is a valid aggregate signcryption from  $\{ID_i\}_{i=1,\dots,n}$  to  $ID_B$ , then the output is a plaintext  $\{m_i\}_{i=1,\dots,n}$  ( $m_i \in \mathcal{M}$ ); otherwise, the output is “Invalid.”

### 3. Formal Security Model for Aggregate Signcryption

Three security properties that are desired out of any ASC scheme are *message confidentiality*, *ciphertext authentication*, and *signature non-repudiation*.

**Definition 1.** An ASC is said to be semantically secure against indistinguishability under adaptive chosen ciphertext attack (IND-ASC-CCA2) if no probabilistic polynomial time adversary  $\mathcal{A}$  has a non-negligible advantage in the following game.

*Start.* The simulator  $C$  runs **Setup**( $k$ ) and sends the set of public parameters  $\pi$  to the adversary  $\mathcal{A}$ .

*Phase 1.* The adversary  $\mathcal{A}$  makes a polynomially bounded number of queries to the simulator  $C$ .

–*Keygen queries.* The adversary  $\mathcal{A}$  produces an identity  $ID_i$  and obtains the corresponding secret key of  $ID_i$ .

–*Signcrypt queries.*  $\mathcal{A}$  produces a message  $m_i \in \mathcal{M}$ , a signer identity  $ID_{i_s}$  and a target identity  $ID_B$ . Then,  $C$  returns the signcrypted ciphertext  $\sigma_i = \text{Signcrypt}(m_i, X_i, d_i, ID_{i_s}, ID_B)$  to  $\mathcal{A}$ , where the private key  $d_i$  is generated by querying the *Keygen* oracle.

–*Unsigncrypt queries.*  $\mathcal{A}$  produces a receiver identity  $ID_B \notin \{ID_i\}_{i=1,\dots,n}$  and an aggregate signcryption  $\sigma_{\text{agg}}$ . The simulator  $C$  generates the private key  $s_B$  by querying the *Keygen* oracle.  $C$  returns the result of **Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ) to  $\mathcal{A}$ . The result returned is  $\perp$  if  $\sigma$  is an invalid signcrypted ciphertext from  $\{ID_i\}_{i=1,\dots,n}$  to  $ID_B$ .

*Selection.*  $\mathcal{A}$  produces two messages sets  $m_{i0}$  and  $m_{i1}$  with equal length from the message space  $\mathcal{M}$ , identities  $\{ID_i\}_{i=1,\dots,n}$  and a final receiver identity  $ID_B^* \notin \{ID_i\}_{i=1,\dots,n}$ , and sends them to  $C$ . The adversary  $\mathcal{A}$  must not have queried the private key corresponding to  $ID_B^* \notin \{ID_i\}_{i=1,\dots,n}$  in the first phase.

*Challenge.* The simulator  $C$  chooses randomly a bit  $b^* \leftarrow \{0, 1\}$  and obtains the challenge aggregate signcryption  $\sigma_{\text{agg}}^*$  by running  $\sigma_i^* = \text{Signcrypt}(m_{ib^*}, X_i, d_i, ID_{i_s}, ID_B^*)$  and **Aggregate**( $\{\sigma_i^*, ID_i\}_{i=1,\dots,n}$ ), and returns  $\sigma_{\text{agg}}^*$  to  $\mathcal{A}$ .

*Phase 2.*  $\mathcal{A}$  is allowed to make polynomially bounded number of new queries as in *Phase 1* with the restrictions that it should not have made the *Unsigncrypt queries* for the unsigncryption of  $\sigma_{\text{agg}}^*$  or the *Keygen queries* for the private keys of  $ID_B^*$ .

*Response.*  $\mathcal{A}$  outputs a bit  $b'$  and wins the game if  $b' = b^*$ . The advantage of  $\mathcal{A}$  is defined as

$$\text{Adv}_{\mathcal{A}}^{\text{IND-ASC-CCA2}} = |2\Pr[b' = b^*] - 1|.$$

**Definition 2.** An ASC is said to be existentially ciphertext unforgeable under adaptive chosen message outsider attack, or AUTH-ASC-CMA2 secure, if no probabilistic polynomial time adversary  $\mathcal{A}$  has a non-negligible advantage in the following game.

*Start.* The simulator  $C$  runs **Setup**( $k$ ) and sends the set of public parameters  $\pi$  to the adversary  $\mathcal{A}$ .

*Query.* The adversary  $\mathcal{A}$  makes a polynomially bounded number of queries to the simulator  $C$ . The attack may be conducted adaptively, and allows the same queries as in the IND-ASC-CCA2 game, namely, *Keygen queries*, *Signcrypt queries*, and *Unsigncrypt queries*.

*Forgery.*  $\mathcal{A}$  produces a new aggregate signcryption  $\sigma_{\text{agg}}$  sent from a set  $\{ID_i\}_{i=1,\dots,n}$  of  $n$  users on messages  $\{m_i\}_{i=1,\dots,n}$  to a final receiver  $ID_B \notin \{ID_i\}_{i=1,\dots,n}$ , where the private keys of the users in  $\{ID_i\}_{i=1,\dots,n}$  was not queried in query phase and  $\sigma_i$  is not the output of a previous query to the *Signcrypt queries*.

*Outcome.* The adversary  $\mathcal{A}$  wins the game if  $\perp$  is not returned by **Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ).

**Definition 3.** An ASC is said to be existentially signature unforgeable against chosen message insider attack, or EUF-ASC-CMA secure, if no probabilistic polynomial time adversary  $\mathcal{A}$  has a non-negligible advantage in the following game.

*Start.* The simulator  $C$  runs **Setup**( $k$ ) and sends the set of public parameters  $\pi$  to the adversary  $\mathcal{A}$ .

*Query.* The adversary  $\mathcal{A}$  makes a polynomially bounded number of queries to the simulator  $C$ . The attack may be conducted adaptively, and allows the same queries as in the IND-ASC-CCA2 game, namely, *Keygen queries*, *Signcrypt queries*, and *Unsigncrypt queries*.

*Forgery.* The adversary returns a recipient identity  $ID_B$  and a ciphertext  $\sigma_i$ .

*Outcome.* The adversary  $\mathcal{A}$  wins the game if: Under the private key of  $ID_B$ , the ciphertext  $\sigma_i$  is decrypted as a signed message  $(ID_{i_s}, \hat{m}_i, \hat{V}_i)$  that satisfies  $ID_{i_s} \neq ID_B$ ,  $ID_{i_s} \in \{ID_i\}_{i=1,\dots,n}$ ;  $\perp$  is not returned by **Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ); provided that: (1) the private key of the user  $ID_{i_s} \in \{ID_i\}_{i=1,\dots,n}$  was not queried in query phase; (2)  $\sigma_i$  is not the output of a previous query to the *Signcrypt queries* that involved  $m_i$ ,  $ID_{i_s}$  and recipient  $ID_B'$ , and resulted in a ciphertext  $\sigma_i'$  whose decryption under the private key of  $ID_B'$  is the claimed forgery  $(ID_{i_s}, \hat{m}_i, \hat{V}_i)$ .

## III. ASC

We propose an ASC scheme in this section. We follow the

framework of ASC that we presented in section II.2.

**Setup**( $k$ ). Given the security parameter of the system  $k$ , the PKG chooses two groups,  $G_1$  and  $G_2$ , of the same prime order  $q$ , the generator  $g$  of  $G_1$  and a bilinear map  $\hat{e}: G_1 \times G_1 \rightarrow G_2$ . The PKG then chooses four cryptographic hash functions  $H_0: \{0, 1\}^* \times G_1 \rightarrow \mathbb{Z}_q^*$ ,  $H_1: G_2 \rightarrow \{0, 1\}^l \times \mathbb{Z}_q^*$ ,  $H_2: \{0, 1\}^l \times \{0, 1\}^* \times G_1 \times G_1 \times \{0, 1\}^* \times G_1 \rightarrow \mathbb{Z}_q^*$ ,  $H_3: \{0, 1\}^l \times \{0, 1\}^* \times G_1 \times \mathbb{Z}_q^* \times G_1 \times \{0, 1\}^* \times G_1 \rightarrow \mathbb{Z}_q^*$ , and  $\mathcal{M} = \{0, 1\}^l$ , where  $l \in \mathbb{Z}^+$  is arbitrarily fixed. The PKG chooses its secret key  $s \in \mathbb{Z}_q^*$ . The set of system public parameters is  $\pi = \{q, G_1, G_2, \hat{e}, g, g^s, H_0, H_1, H_2, H_3\}$ .

**Keygen**( $ID_i$ ). For an identity  $ID_i \in \{0, 1\}^*$ , the algorithm does the following:

- (i) Chooses a random  $x_i \in \mathbb{Z}_q$  and computes  $X_i = g^{x_i}$ ,  $q_i = H_0(ID_i, X_i)$ ;
- (ii) Computes  $s_i = X_i^s$ ,  $d_i = (x_i + sq_i) \bmod q$ ;
- (iii) The PKG sends the corresponding private key  $\langle s_i, d_i \rangle$ , which is transmitted in a secure way, and the public key  $\langle X_i, q_i \rangle$  to the user  $ID_i$ .

**Signcrypt**( $m_i, X_i, d_i, ID_i, ID_B$ ). To send a message  $m_i$  to the receiver  $ID_B$  with the public key  $\langle X_B, q_B \rangle$ , the sender  $ID_i$  with the private key  $\langle s_i, d_i \rangle$  and the public key  $\langle X_i, q_i \rangle$  does the following:

- (i) Chooses a random  $r_i \in \mathbb{Z}_q^*$  and computes  $W_i = g^{r_i}$ ,  $w_i = \hat{e}(g^s, X_B)^{r_i}$ ;
- (ii) Lets  $h_{1i} = H_1(w_i)$ ;  $h_{2i} = H_2(m_i, ID_i, X_i, w_i, ID_B, X_B)$ ;
- (iii) Lets  $h_{3i} = H_3(m_i, ID_i, X_i, h_{2i}, w_i, ID_B, X_B)$ ;
- (iv) Computes  $V_i = (r_i h_{2i} + h_{3i} d_i) \bmod q$ ;
- (v) Sets  $c_i = (m_i || V_i) \oplus h_{1i}$ ;  $Z_i = g^{V_i}$ ;
- (vi) Outputs  $\sigma_i = \langle c_i, W_i, Z_i, X_i \rangle$  as the signcrypton of  $ID_i$  on message  $m_i$ .

**Aggregate**( $\{\sigma_i, ID_i\}_{i=1, \dots, n}$ ). On input, a set of  $n$  signcryptons  $\sigma_i = \langle c_i, W_i, Z_i, X_i \rangle$ ,  $i=1$  to  $n$ , and the corresponding identity  $ID_i$  (such that  $\forall i=1$  to  $n$ ,  $\sigma_i$  is the signcrypton on message  $m_i$  by  $ID_i$ ):

- (i) Let  $Z_{\text{agg}} = \prod_{i=1}^n Z_i$ ;
- (ii) Output the final aggregate signcrypton  $\sigma_{\text{agg}} = \langle \{c_i, W_i, X_i, ID_i\}_{i=1, \dots, n}, Z_{\text{agg}} \rangle$ .

The aggregation can be done by any of the senders or by any third party.

**Unsigncrypt**( $\sigma_{\text{agg}}, s_B, d_B$ ). On input, the aggregate signcrypton  $\sigma_{\text{agg}} = \langle \{c_i, W_i, X_i, ID_i\}_{i=1, \dots, n}, Z_{\text{agg}} \rangle$ , the receiver with identity  $ID_B$ , the public key  $\langle X_B, q_B \rangle$ , and the private key  $\langle s_B, d_B \rangle$ , do the following:

- (i) Compute  $w_i = \hat{e}(W_i, s_B)$ , recover  $m_i || V_i = c_i \oplus H_1(w_i)$ ;
- (ii) For  $i=1$  to  $n$ , compute  $h_{2i} = H_2(m_i, ID_i, X_i, w_i, ID_B, X_B)$ ,  $h_{3i} = H_3(m_i, ID_i, X_i, h_{2i}, w_i, ID_B, X_B)$ ;

- (iii) Check if  $Z_{\text{agg}} = g^{\sum_{i=1}^n V_i}$ , and

$$Z_{\text{agg}} = \prod_{i=1}^n (W_i)^{h_{2i}} \prod_{i=1}^n (X_i)^{h_{3i}} (g^s)^{\sum_{i=1}^n q_i h_{3i}}.$$

If the check succeeds, the output is  $m_i$ , which has been decrypted; otherwise, the output reads “Invalid.”

## IV. Analysis of Proposed Method

### 1. Correctness

Correctness of the unsigncrypton algorithm:

$$\begin{aligned} w_i &= \hat{e}(W_i, s_B) \\ &= \hat{e}(g^{r_i}, X_B^s) = \hat{e}(g^s, X_B)^{r_i}, \\ Z_i &= g^{V_i} = g^{r_i h_{2i} + h_{3i} d_i} \\ &= (g^{r_i})^{h_{2i}} g^{h_{3i} (x_i + sq_i)} \\ &= (W_i)^{h_{2i}} (X_i)^{h_{3i}} (g^s)^{q_i h_{3i}}, \\ Z_{\text{agg}} &= \prod_{i=1}^n Z_i = \prod_{i=1}^n ((W_i)^{h_{2i}} (X_i)^{h_{3i}} (g^s)^{q_i h_{3i}}) \\ &= \prod_{i=1}^n (W_i)^{h_{2i}} \cdot \prod_{i=1}^n (X_i)^{h_{3i}} \cdot (g^s)^{\sum_{i=1}^n q_i h_{3i}}. \end{aligned}$$

### 2. Proof of Confidentiality

**Theorem 1.** Assume there is an IND-ASC-CCA2 adversary  $\mathcal{A}$  that is able to distinguish two valid ciphertexts during the game defined in Definition 1 with a non-negligible advantage and asks *Keygen queries*, *Signcrypt queries*, and *Unsigncrypt queries*; then, there exists a simulator  $C$  that can solve an instance of the DBDH problem with a non-negligible advantage.

*Proof.* The proof proceeds in the IND-ASC-CCA2 game. Assume that the simulator  $C$  receives a random DBDH problem instance  $(g, g^a, g^b, g^c, w)$ ; then, the goal is to decide whether  $w = \hat{e}(g, g)^{abc}$  or not.  $C$  will run the adversary  $\mathcal{A}$  as a subroutine and act as the adversary's challenger in the IND-ASC-CCA2 game.

*Start.* The simulator  $C$  sets the master public key  $g^s = g^a$  and gives the system public parameters to  $\mathcal{A}$ .

*Phase 1.* The simulator  $C$  will set the random oracles of  $H_0$ ,  $H_1$ ,  $H_2$ ,  $H_3$ , *Keygen*, *Signcrypt*, and *Unsigncrypt*. To maintain consistency, the simulator  $C$  will maintain four lists:  $H_0$ -List,  $H_1$ -List,  $H_2$ -List, and  $H_3$ -List, which will be detailed later.  $C$  will also simulate all oracles required during the game and control the  $H_0$  random oracle. The adversary  $\mathcal{A}$  outputs and threatens to attack the identity  $ID_B$ .

*$H_0$  Oracle.* When the  $H_0$  oracle is queried with  $ID_i \in \{0, 1\}^*$ ,  $C$  does the following: checks the  $H_0$ -List  $\langle ID_i, X_i, q_i, x_i \rangle$ ,  $x_i \in \mathbb{Z}_q^*$ ,

if  $ID_i = ID_B$ , selects a new random  $\lambda_i \in \mathbb{Z}_q^*$ ; sets  $X_i = g^{\lambda_i}$ ,  $q_i = \lambda_i$ , adds the tuple  $\langle ID_i, X_i, q_i, * \rangle$  to the  $H_0$ -List; and returns  $q_i$ . Otherwise,  $C$  selects a new random  $\lambda_i \in \mathbb{Z}_q^*$ ,  $x_i \in \mathbb{Z}_q$ , sets  $X_i = g^{x_i}$ ,  $q_i = \lambda_i$ , adds the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  to the  $H_0$ -List, and returns  $q_i$ .

**$H_1$  Oracle.** When the  $H_1$  oracle is queried with an input  $w_i$ ,  $C$  checks the  $H_1$ -List. If there exists a tuple  $\langle w_i, h_{1i} \rangle$  in  $H_1$ -List,  $C$  returns  $h_{1i}$ . Otherwise,  $C$  selects a new random  $h_{1i} \in \mathbb{Z}_q^*$ , adds the tuple  $\langle w_i, h_{1i} \rangle$  to the  $H_1$ -List, and returns  $h_{1i}$ .

**$H_2$  Oracle.** When the  $H_2$  oracle is queried with an input  $(m_i, ID_i, X_i, w_i, ID_B, X_B)$ ,  $C$  checks the  $H_2$ -List. If there exists a tuple  $\langle m_i, ID_i, X_i, w_i, ID_B, X_B, h_{2i} \rangle$  in the  $H_2$ -List,  $C$  returns  $h_{2i}$ . Otherwise,  $C$  chooses a new random  $h_{2i} \in \mathbb{Z}_q^*$ , adds the tuple  $\langle m_i, ID_i, X_i, w_i, ID_B, X_B, h_{2i} \rangle$  to  $H_2$ -List, and returns  $h_{2i}$ .

**$H_3$  Oracle.** When the  $H_3$  oracle is queried with an input  $(m_i, ID_i, X_i, h_{2i}, w_i, ID_B, X_B)$ ,  $C$  checks the  $H_3$ -List. If there exists a tuple  $\langle m_i, ID_i, X_i, h_{2i}, w_i, ID_B, X_B, h_{3i} \rangle$  in the  $H_3$ -List,  $C$  returns  $h_{3i}$ . Otherwise,  $C$  chooses a new random  $h_{3i} \in \mathbb{Z}_q^*$ , adds the tuple  $\langle m_i, ID_i, X_i, h_{2i}, w_i, ID_B, X_B, h_{3i} \rangle$  to the  $H_3$ -List, and returns  $h_{3i}$ .

**Keygen Oracle.** When  $\mathcal{A}$  makes a *Keygen query* with  $ID_i$  as the input,  $C$  checks the  $H_0$ -List to verify whether or not there is an entry for  $ID_i$ . If the  $H_0$ -List does not contain an entry for  $ID_i$ , return  $\perp$ . Otherwise, if  $ID_i = ID_B$ ,  $C$  recovers the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  from the  $H_0$ -List and returns  $\langle X_i, q_i, *, * \rangle$ ; if  $ID_i \neq ID_B$ ,  $C$  recovers the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  from the  $H_0$ -List and returns  $\langle X_i, q_i, s_i, d_i \rangle$ , where  $s_i = (g^a)^{x_i} = (g^{x_i})^a = (X_i)^a$ , and  $d_i \in \mathbb{Z}_q$  is randomly selected.

**Signcrypt Oracle.** When  $\mathcal{A}$  makes a *Signcrypt query* with  $ID_i$  as the input,  $C$  checks the  $H_0$ -List to verify whether or not there is an entry for  $ID_i$ . If the  $H_0$ -List does not contain an entry for  $ID_i$ , return  $\perp$ . Otherwise,  $C$  executes *Signcrypt*( $m_i, X_i, d_i, ID_i, ID_B$ ) as usual and returns what the *signcrypt* algorithm returns.

**Unsigncrypt Oracle.** When  $\mathcal{A}$  makes an *Unsigncrypt query* with  $\sigma_{agg} = \langle c_i, W_i, X_i, ID_i \rangle_{i=1, \dots, n}, Z_{agg}$  and the receiver with identity  $ID_B$ ,  $C$  first verifies whether or not there are entries for  $ID_i$  ( $ID_i \neq ID_B$ ) and  $ID_B$  in  $H_0$ -List and there is an entry of the form  $\langle ID_i, X_i, q_i, \lambda_i \rangle$ . If at least one of these conditions is not satisfied,  $C$  returns  $\perp$ . Otherwise,  $C$  executes *Unsigncrypt*( $\sigma_{agg}, s_B, d_B$ ) in the normal way and returns what the *Unsigncrypt* algorithm returns.

**Challenge.** After getting sufficient training,  $\mathcal{A}$  submits two equal length messages  $m_{i0}$  and  $m_{i1}$ .  $C$  randomly chooses a bit  $b^* \leftarrow \{0, 1\}$  and obtains the challenge signcrypt ciphertext by running *Signcrypt*( $m_{ib^*}, X_i, d_i, ID_i, ID_B$ ) and *Aggregate*( $\{\sigma_i^*, ID_i\}_{i=1, \dots, n}$ ), then returns  $\sigma_{agg}^*$  to  $\mathcal{A}$ .

**Phase 2.** This phase is similar to *Phase 1*. However, in *Phase 2*,  $\mathcal{A}$  cannot ask for *Unsigncrypt* on the challenge aggregate signcrypt  $\sigma_{agg}^* = \langle c_i, W_i, X_i, ID_i \rangle_{i=1, \dots, n}, Z_{agg}^*$  or the *Keygen*

*queries* for the secret keys of  $ID_B$ .

**Output.** After  $\mathcal{A}$  has made a sufficient number of queries,  $\mathcal{A}$  returns its guess: a bit  $b'$ . If  $b' = b^*$ , then  $C$  outputs 1 as the answer to the DBDH problem. Otherwise, it outputs 0. Since the adversary is denied access to the *Unsigncrypt* oracle with the challenge signcrypt, for  $\mathcal{A}$  to find that  $\sigma_i$  is not a valid ciphertext,  $\mathcal{A}$  should have queried the  $H_1$  Oracle with  $w_i = \hat{e}(W_i, s_B)$ . Here,  $s_B$  is the private key of the receiver, and it is  $(X_B)^a = (g^b)^a = g^{ab}$ . Also,  $C$  has set  $W_i = g^c$ . We have  $w_i = \hat{e}(W_i, s_B) = \hat{e}(g^c, g^{ab}) = \hat{e}(g, g)^{abc}$ .  $\square$

### 3. Proof of Authentication

**Theorem 2.** The ASC proposed is secure against any probabilistic polynomial-time AUTH-ASC-CMA2 adversary  $\mathcal{A}$  under the random oracle model if the DL problem is hard in  $G_1$ .

**Proof.** On getting a DL problem instance  $(g, W_r = g^{r_r})$  and  $(g, g^{d_r})$  as a challenge in the AUTH-ASC-CMA2 game defined in Definition 2, the simulator  $C$  uses  $\mathcal{A}$  to solve the DL problem. The goal of  $C$  is to determine  $r_r$  and  $d_r$ . The simulator  $C$  gives the system public parameters to  $\mathcal{A}$ .  $\mathcal{A}$  knows  $g^{d_r}$  from computing  $W_r(g^s)^{q_r} = (g^{d_r})^{q_r} = g^{r_r + s q_r} = W_r(g^s)^{q_r}$ . The proof of Theorem 2 is similar to the proof of Theorem 1, with some changes given in the following random oracles.

**$H_0$  Oracle.** When the  $H_0$  oracle is queried with an input  $ID_i \in \{0, 1\}^*$ ,  $C$  checks the  $H_0$ -List; if the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  exists,  $C$  returns  $q_i$ . Otherwise,  $C$  selects new random  $\lambda_i \in \mathbb{Z}_q^*$ ,  $x_i \in \mathbb{Z}_q$ , sets  $X_i = g^{x_i}$ ,  $q_i = \lambda_i$ , adds the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  to  $H_0$ -List, and returns  $q_i$ .

**Keygen Oracle.** When  $\mathcal{A}$  makes a *Keygen query* with  $ID_i$  as the input,  $C$  checks the  $H_0$ -List to verify whether or not there is an entry for  $ID_i$ . If the  $H_0$ -List does not contain an entry for  $ID_i$ , return  $\perp$ . Otherwise, if  $ID_i \in \{ID_i\}_{i=1, \dots, n}$  (the corresponding senders identities set),  $C$  recovers the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  from the  $H_0$ -List and returns  $\langle X_i, q_i, *, * \rangle$ ; if  $ID_i \notin \{ID_i\}_{i=1, \dots, n}$  (the corresponding senders identities set),  $C$  recovers the tuple  $\langle ID_i, X_i, q_i, x_i \rangle$  from the  $H_0$ -List and returns  $\langle X_i, q_i, s_i, d_i \rangle$ , where  $s_i = (g^s)^{x_i}$ , and  $d_i \in \mathbb{Z}_q$  is randomly selected.

**Forgery.**  $\mathcal{A}$  chooses the corresponding senders identities set  $\{ID_i\}_{i=1, \dots, n}$  and the receiver identity  $ID_B$  and outputs a forged signcrypt  $\sigma_r^* = \langle c_r^*, W_r^*, Z_r^*, X_r^* \rangle$  on message  $m_r^*$  from  $ID_r \in \{ID_i\}_{i=1, \dots, n}$  to  $C$ . The simulator  $C$  retrieves the entry corresponding to  $ID_B$  in the  $H_0$ -List and uses  $s_B$  to execute *Unsigncrypt*( $\sigma_{agg}^*, s_B, d_B$ ). If  $\sigma_r^*$  is a valid signcrypt from  $ID_r$  to receiver  $ID_B$ , that is, a message  $m_r^*$  is returned by the *Unsigncrypt* algorithm, then  $C$  applies the oracle replay technique to produce two valid signcrypts  $\sigma_r' = (c_r', W_r', Z_r',$



$X_r'$ ) and  $\sigma_r'' = (c_r'', W_r'', Z_r'', X_r'')$  on message  $m_r$  from the  $ID_r$  to receiver  $ID_B$ . This is achieved by running the turing machine again with the same random tape but with a different hash value.  $C$  obtains the 'signatures'  $V_r' = r_r h_{2r}' + h_{3r}' d_r$  and  $V_r'' = r_r h_{2r}'' + h_{3r}'' d_r$  with  $h_{2r}' \neq h_{2r}''$  and  $h_{3r}' \neq h_{3r}''$ .  $C$  successfully computes  $r_r$  and  $d_r$ . Indeed, from  $V_r' = r_r h_{2r}' + h_{3r}' d_r$  and  $V_r'' = r_r h_{2r}'' + h_{3r}'' d_r$ , we have

$$r_r = \frac{V_r' h_{3r}'' - V_r'' h_{3r}'}{h_{2r}' h_{3r}'' - h_{2r}'' h_{3r}'}, \quad h_{2r}' h_{3r}'' - h_{2r}'' h_{3r}' \neq 0,$$

$$d_r = \frac{V_r' h_{2r}'' - V_r'' h_{2r}'}{h_{3r}' h_{2r}'' - h_{3r}'' h_{2r}'}, \quad h_{3r}' h_{2r}'' - h_{3r}'' h_{2r}' \neq 0. \quad \square$$

#### 4. Proof of Non-repudiation

**Theorem 3.** The ASC proposed is secure against any probabilistic polynomial-time EUF-ASC-CMA adversary  $\mathcal{A}$  under the random oracle model if the DBDH problem is hard in  $G_1$ .

*Proof.* The proof of Theorem 3 is similar to the proof of Theorem 2, with some changes in the following random oracles. We only provide the changes.

*Keygen Oracle.* If the  $H_0$ -List does not contain an entry for  $ID_b$ ,  $C$  return  $\perp$ . Otherwise, if  $ID_i = ID_r$  (this corresponding identity  $ID_r$  is called the "guessed" sender),  $C$  recover the tuple  $\langle ID_b, X_b, q_b, x_i \rangle$  from the  $H_0$ -List and return  $\langle X_b, q_b, *, * \rangle$ . If  $ID_i \neq ID_r$ , recover the tuple  $\langle ID_b, X_b, q_b, x_i \rangle$  from the  $H_0$ -List and return  $\langle X_b, q_b, s_i, d_i \rangle$ , where  $s_i = (g^s)^{x_i}$ , and  $d_i \in \mathbb{Z}_q$  is randomly selected.

Eventually,  $\mathcal{A}$  returns a forgery, consisting of a ciphertext  $\sigma_i$  and a recipient identity  $ID_B$ .  $C$  decrypts the ciphertext for  $ID_B$  (by invoking its own decryption oracle), which causes the plaintext forgery  $(ID_b, m_b, V_i)$  to be revealed. Note that if  $C$  has made the correct guess, that is,  $ID_i = ID_r$ , then  $ID_B \neq ID_r$  and the decryption works.

If  $\sigma_i$  is a valid signcryption from  $ID_i$  to receiver  $ID_B$ , that is, a message  $m_i$  is returned by the *Unsigncrypt* algorithm, then  $C$  applies the oracle replay technique to produce two valid signed messages  $(ID_b, m_b, V_i)$  and  $(ID_b, m_b, V_i')$  on a message  $m_i$  from the  $ID_i$  to receiver  $ID_B$ . This is achieved by running the turing machine again with the same random tape but with a different hash value.  $C$  obtains the 'signatures'  $V_r' = r_r h_{2r}' + h_{3r}' d_r$  and  $V_r'' = r_r h_{2r}'' + h_{3r}'' d_r$  with  $h_{2r}' \neq h_{2r}''$  and  $h_{3r}' \neq h_{3r}''$ .  $\square$

#### 5. Efficiency

The primary objective of the aggregate signature scheme is to achieve both computation and communication efficiency. Using aggregate signature schemes, signatures from different

Table 1. Ciphertext size.

Scheme	Ciphertext size for sender
ASC	$ M  +  Z_q^*  + 3 G_1 $
AS and BF	$ M  +  Z_q^*  + 4 G_1 $

users on different messages can be aggregated into a single compact signature. We eliminate the interaction among the senders (signers) before signcryption generation, which reduces the communication complexity to a large extent. Also, our ASC is more efficient than the sign-then-encrypt approach. We have compared our scheme (ASC) with the sign-then-encrypt scheme (AS and BF) in Table 1, where the aggregate signature scheme (AS) is proposed in [9] and the encryption scheme (BF) is proposed in [15]. In the comparison table,  $|M|$  represents the length of a message.  $|G_1|$  is the number of  $G_1$  elements.

The major parameters involved in our ASC scheme are the computation costs for *Signcrypt* and *Unsigncrypt* operations. For computation cost, we mainly consider the number of pairing  $\hat{e}$  computations performed, as they are the costliest operations involved. In our ASC scheme, the sender only performs one pairing  $\hat{e}$  computation, and the receiver only performs  $n$  pairings  $\hat{e}$  computations. In the IBAS scheme of Selvi and others [13], each sender only performs one pairing  $\hat{e}$  computation, but the receiver performs  $2n+3$  pairings  $\hat{e}$  computations. Otherwise, the *Unsigncrypt* algorithm in Selvi and others' IBAS scheme [13] does not explain how to recover the unknown users' signatures from the aggregate signature. Thus, our ASC is more efficient than the IBAS scheme of Selvi and others [13].

We conducted five experiments for one pairing  $\hat{e}$  operation using a pairing-based cryptography library [16], running each experiment 1,000 times. The results of the five experiments were 8.258133 s, 8.265714 s, 8.243305 s, 8.394295 s, and 8.384370 s. Therefore, the average time of one pairing  $\hat{e}$  operation is about 0.008309 s.

#### V. Conclusion

We studied an ASC built upon the identity-based aggregate signature scheme proposed by Selvi and others [9]. Our proposed ASC was formally proven to be secure with respect to its IND-CCA2, AUTH-CMA2, and EUF-CMA security properties in a random oracle model. The ASC does not need the interaction among the signers, which is a requirement in existing efficient aggregate signature schemes, and it is efficient in pairing  $\hat{e}$  computations.

## Appendix:

In Table A1, we summarize all the notations of our paper.

Table A1. Definition and theorem.

Name	Content
Definition 1	An ASC is said to be semantically secure against indistinguishability under adaptive chosen ciphertext attack (IND-ASC-CCA2) if no probabilistic polynomial time adversary $\mathcal{A}$ has a non-negligible advantage in the following game.
Definition 2	An ASC is said to be existentially ciphertext unforgeable under adaptive chosen message outsider attack, or AUTH-ASC-CMA2 secure, if no probabilistic polynomial time adversary $\mathcal{A}$ has a non-negligible advantage in the following game.
Definition 3	An ASC is said to be existentially signature unforgeable against chosen message insider attack, or EUF-ASC-CMA secure, if no probabilistic polynomial time adversary $\mathcal{A}$ has a non-negligible advantage in the following game.
Theorem 1	Assume there is an IND-ASC-CCA2 adversary $\mathcal{A}$ that is able to distinguish two valid Ciphertexts during the game defined in definition 1 with non-negligible advantage and asking <i>Keygen queries</i> , <i>Signcrypt queries</i> , and <i>Unsigncrypt queries</i> , then there exists a simulator $\mathcal{C}$ that can solve an instance of the DBDH problem with non-negligible advantage.
Theorem 2	The ASC proposed is secure against any probabilistic polynomial-time AUTH-ASC-CMA2 adversary $\mathcal{A}$ under the random oracle model if the DL problem is hard in $G_1$ .
Theorem 3	The ASC proposed is secure against any probabilistic polynomial-time EUF-ASC-CMA adversary $\mathcal{A}$ under the random oracle model if the DBDH problem is hard in $G_1$ .

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