Block-Mode Lattice Reduction for Low-Complexity MIMO Detection

Kwonhue Choi, Hannah Kim, Sooyoung Kim, and Young-il Kim

We propose a very-low-complexity lattice-reduction (LR) algorithm for multi-input multi-output detection in time-varying channels. The proposed scheme reduces the complexity by performing LR in a block-wise manner. The proposed scheme takes advantage of the temporal correlation of the channel matrices in a block and its impact on the lattice transformation matrices during the LR process. From this, the proposed scheme can skip a number of redundant LR processes for consecutive channel matrices and performs a single LR in a block. As the Doppler frequency decreases, the complexity reduction efficiency becomes more significant.

Keywords: MIMO, lattice reduction, time varying fading.

I. Introduction

For multi-input multi-output (MIMO) systems, the lattice-reduction (LR)-aided detection (LRAD) scheme is considered as a good design choice since it can achieve full diversity with a favorable complexity [1]-[3]. However, the overall computational load for LR could be still one of the important issues in the receiver design, especially in the time-varying channels where we should frequently perform LR for consecutive channel matrices.

In a time-varying channel, the lattice transformation matrices of LR frequently remain the same or different only at small portion of the elements for adjacent channel matrices due to the temporal correlation. Adaptive LR techniques were proposed by using this property in [4], [5], where high complexity reduction could be achieved. Our investigation results reveal that substantial redundant calculations still exist, especially in slow-fading channels. We develop a block-mode LR algorithm that needs a single LR process in a block consisting of several consecutive channel matrices and eliminates a large number of redundant calculations.

II. Basic Concept and Conventional Complexity-Reduced LRAD Schemes

Consider an $N\times L$ MIMO system with N transmit antennas and L receiving antennas. Let $\mathbf{H}$ denote the channel matrix whose dimension is $L\times N$, and its elements are i.i.d zero-mean complex Gaussian random variables. A lattice-reduced channel matrix by LR, $\mathbf{H}'$, can be represented by [1]

$$\mathbf{H}'=\mathbf{HP}, \tag{1}$$

where $\mathbf{P}$ is an $N\times N$ lattice transformation matrix which is integer unimodular. With LRAD, the received signal vector $\mathbf{y}$ is expressed by

$$\mathbf{y}=\mathbf{Hs}+\mathbf{n}=(\mathbf{HP})(\mathbf{P})^{-1}\mathbf{s}+\mathbf{n}=\mathbf{H}'\mathbf{z}+\mathbf{n}, \tag{2}$$

Table 1. Comparison of conventional LR schemes in time-varying channel.

Brute-force LR	Adaptive LR [4]	Adaptive LR [5]
Get $\mathbf{H}_i$ Perform LR with $\mathbf{P}_i=\mathbf{I}$ $i=i+1$	Get $\mathbf{H}_i$ Perform LR with $\mathbf{P}_i=\mathbf{P}_{i-1}$ $i=i+1$	Get $\mathbf{H}_i$ Estimate (3) (4), check (5) Perform LR with $\mathbf{P}_i=\mathbf{P}_{i-1}$ only if not satisfying (5) $i=i+1$

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Table 2. LR results for sequence of time-varying channel matrices.

i	1	2	3	4	5	6	7	8	…
$\mathbf{H}_i$	$\begin{bmatrix}-0.07 & -1.20\\ 0.06 & -0.47\end{bmatrix}$	$\begin{bmatrix}-0.08 & -1.20\\ 0.06 & -0.47\end{bmatrix}$	$\begin{bmatrix}-0.09 & -1.19\\ 0.06 & -0.47\end{bmatrix}$	$\begin{bmatrix}-0.10 & -1.19\\ 0.07 & -0.48\end{bmatrix}$	$\begin{bmatrix}-0.11 & -1.19\\ 0.07 & -0.48\end{bmatrix}$	$\begin{bmatrix}-0.12 & -1.18\\ 0.08 & -0.48\end{bmatrix}$	$\begin{bmatrix}-0.15 & -1.17\\ 0.08 & -0.48\end{bmatrix}$	$\begin{bmatrix}-0.16 & -1.17\\ 0.08 & -0.49\end{bmatrix}$	…
$\mathbf{H}'_i$	$\begin{bmatrix}-0.07 & -0.78\\ 0.06 & -0.83\end{bmatrix}$	$\begin{bmatrix}-0.08 & -0.70\\ 0.06 & -0.86\end{bmatrix}$	$\begin{bmatrix}-0.09 & -0.63\\ 0.06 & -0.88\end{bmatrix}$	$\begin{bmatrix}-0.10 & -0.56\\ 0.07 & -0.91\end{bmatrix}$	$\begin{bmatrix}-0.11 & -0.60\\ 0.07 & -0.86\end{bmatrix}$	$\begin{bmatrix}-0.12 & -0.54\\ 0.08 & -0.88\end{bmatrix}$	$\begin{bmatrix}-0.15 & -0.41\\ 0.08 & -0.91\end{bmatrix}$	$\begin{bmatrix}-0.16 & -0.51\\ 0.08 & -0.84\end{bmatrix}$	…
$\mathbf{P}_i$	$\begin{bmatrix}1 & -6\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -6\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -6\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -6\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -5\\ 0 & 1\end{bmatrix}$	$\begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}$	…

where $\mathbf{s}$ and $\mathbf{n}$ are the transmitted symbol and complex Gaussian noise vectors, respectively. The transmitted symbol vector $\mathbf{s}$ is transformed to $\mathbf{z} = \mathbf{P}^{-1}\mathbf{s}$. In the receiver, we multiply $(\mathbf{H}')^{-1}$ by $\mathbf{y}$ and quantize the result to get $\hat{\mathbf{z}}$. Then, the estimate of the transmitted symbol, $\hat{\mathbf{s}}$, can be found by multiplying $\mathbf{P}$, that is $\hat{\mathbf{s}} = \mathbf{P}\hat{\mathbf{z}}$. A detailed process to get ($\mathbf{H}'$, $\mathbf{P}$) pair from $\mathbf{H}$ can be found in [1], [6].

Table 1 shows the comparison of the basic operational principles of various LR schemes in a time-varying channel. Let $\mathbf{H}_i$ denote the i-th channel matrix in a time domain that is supposed to go through an LR operation. There is a temporal correlation between two consecutive channel matrices, that is, $\mathbf{H}_{i-1}$ and $\mathbf{H}_i$. Based on this, the adaptive LR schemes in [4], [5] first estimate the near lattice-reduced channel matrix $\hat{\mathbf{H}}'_i$ by using the LR results of the previous channel matrix as

$$\hat{\mathbf{H}}'_i = \mathbf{H}_i \mathbf{P}_{i-1}, \tag{3}$$

where $\mathbf{P}_{i-1}$ is the unimodular matrix obtained by performing LR for the ($i-1$)th channel $\mathbf{H}_{i-1}$. Then, the LR process is performed on $\hat{\mathbf{H}}'_i$. This way, many complexities in the LR process can be reduced.

In addition, the adaptive LR in [5] even ignores the LR process on $\hat{\mathbf{H}}'_i$ itself if there are sufficient temporal correlations. It measures the orthogonality of $\hat{\mathbf{H}}'_i$ using the orthogonality defect factor defined as

$$\Theta(\hat{\mathbf{H}}'_i) = \frac{\prod_{m=1}^{N} \left\| \hat{\mathbf{h}}'_m \right\|^2}{\det[(\hat{\mathbf{H}}'_i)^H \hat{\mathbf{H}}'_i]}, \tag{4}$$

where $\hat{\mathbf{h}}'_m$ is the m-th column vector of $\hat{\mathbf{H}}'_i$. Then, the LR operation is performed on $\hat{\mathbf{H}}'_i$ only if the following condition is not satisfied:

$$1/\alpha < \Theta(\hat{\mathbf{H}}'_i)/\Theta(\mathbf{H}'_{i-1}) < \alpha, \tag{5}$$

where α is a predetermined constant by the Doppler frequency (fading rate).

As summarized in Table 1, the conventional adaptive LR in [4] performs LR at every channel matrices, although the complexity is reduced. The other conventional adaptive LRAD scheme in [5] eliminates some of the unnecessary LR processes, but Θ should be calculated and checked at every channel matrix, that is, the computations of (4) and (5).

Our investigation results reveal that some redundant calculations still remain. Table 2 illustrates an example of LR results for a sequence of time-varying channel matrices. For ease of illustration, we consider the case of a 2×2 real-valued MIMO channel. It is shown that the first four $\mathbf{P}_i$s ($\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$) are identical to each other as are the next three $\mathbf{P}_i$s ($\mathbf{P}_5$, $\mathbf{P}_6$, $\mathbf{P}_7$). Although we note that $\mathbf{H}'_i$ continuously changes as $\mathbf{H}_i$ changes, the calculation of $\mathbf{H}'_i$ in a block is straightforward as long as we can confirm that $\mathbf{P}_i$ is constant within the block. By the relation in (1), we can get $\mathbf{H}'_i$ by simply multiplying $\mathbf{H}_i$ to $\mathbf{P}_i$.

III. Proposed Scheme

Motivated by the above investigation, we propose a modified complexity reduction scheme. In the proposed scheme, to determine if $\mathbf{P}_i$ does not change for a certain block of consecutive channel matrices, we perform LR only once in a block with K consecutive channel matrices. More specifically, we first perform an LR process for the first channel matrix in a block consisting of K consecutive channel matrices. For example, if LR is performed for $\mathbf{H}_i$, then all the other channel matrices of $\mathbf{H}_{i+1}$ to $\mathbf{H}_{i+K}$ are buffered with their corresponding received symbol vectors without any LR processes. Next, we perform LR for $\mathbf{H}_{i+K}$ and compare $\mathbf{P}_{i+K}$ with $\mathbf{P}_i$. Depending on whether they are the same or not, one of two modes are activated, that is, the block and sequential modes.

First, the block mode is enabled if $\mathbf{P}_{i+K}$ and $\mathbf{P}_i$ are the same. In this case, it is highly probable that $\mathbf{P}_i$ to $\mathbf{P}_{i+K}$s are all equal. For this reason, LR operations for a block of $K-1$ channel matrices are eliminated by just setting $\mathbf{P}_{i+j} = \mathbf{P}_i$, for $1 \le j \le K-1$. Then, the corresponding lattice-reduced channel matrices, $\mathbf{H}'_{i+j}$s, are simply obtained by performing the matrix multiplication given in (1). Therefore, this block mode reduces

Step 1. Set i=1.
Step 2. Perform LR for $\mathbf{H}_i$
Step 3. Buffer $\mathbf{H}_{i+j}$s for $1 \le j \le K$ and the received signal vectors.
Step 4. Perform LR for $\mathbf{H}_{i+K}$ with initial setting of $\mathbf{P}_{i+K}$=$\mathbf{P}_i$.
IF $\mathbf{P}_{i+K}$=$\mathbf{P}_i$: block mode
Step 5. FOR j=1,..., K–1 DO
Set $\mathbf{P}_{i+j}$=$\mathbf{P}_i$
Compute $\mathbf{H'}_{i+j}$=$\mathbf{H}_{i+j}\,\mathbf{P}_{i+j}$. (6)
END FOR
ELSE: sequential mode
Step 6. FOR j=1,..., K–1 DO
Perform LR for $\mathbf{H}_{i+j}$ with initial setting of $\mathbf{P}_{i+j}$=$\mathbf{P}_{i+j-1}$. (7)
END FOR
END IF
Step 7. Set i=i+K and go to Step 3.

Fig. 1. Procedure of proposed complexity-reduced LR.

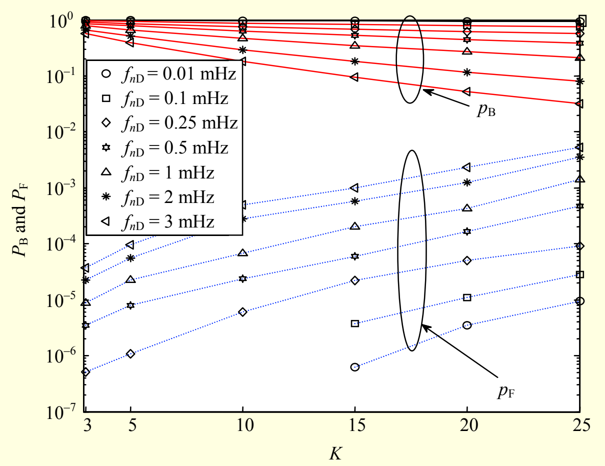

Fig. 2. Occurrence rate and failure rate in the block mode.

the total amount of calculation required for a whole LR operation for each channel matrix into just a single matrix multiplication.

Second, in the sequential mode which is enabled if $\mathbf{P}_{i+K}$ and $\mathbf{P}_i$ are not the same, we perform separate LR operations for each of $K-1$ channel matrices, one by one, but we reduce the computations for an LR operation by using $\mathbf{P}_i$ as an initial state in LR iterations. This is equivalent to performing LR on $\hat{\mathbf{H}}'_i$ given in (3). Consequently, the overall procedure is described as shown in Fig. 1.

Let us define p_B as the occurrence rate of the block modes, that indicates the probability of $\mathbf{P}_i = \mathbf{P}_{i+K}$. Then, if we properly set the value of K, we can roughly estimate the average computation complexity per channel matrix as

$$C_{\text{proposed}} \approx p_B \times C_{\text{one matrix mult}} + (1-p_B) \times C_{\text{sequential}} \qquad (8)$$

$$\approx (1-p_B) \times C_{\text{sequential}} \qquad (9)$$

where $C_{\text{one matrix mult}}$ denotes the computation complexity required for one matrix multiplication in (6) and $C_{\text{sequential}}$ denotes the average computation complexity of one LR operation in the sequential mode in (7). As $C_{\text{sequential}}$ is typically much larger than $C_{\text{one matrix mult}}$, C_{proposed} can be further approximated as (9). This estimate implies that we can effectively reduce the computational complexity to the fraction ($1-p_B$) of the computation complexity of one LR operation in sequential mode. In the next section, the simulation results reveal that if we properly set K in the practical Doppler frequency range, p_B is significant, and thus the complexity reduction by the proposed scheme is significant.

IV. Simulation Results

All the LRAD schemes investigated in this letter utilize LLL algorithm as a baseline [6], [7]. We use a 3×3, that is, N=L=3, MIMO system with 16-QAM. As a time-varying channel model, all the elements in the channel matrix are i.i.d, and we use the 2-ray Jakes fading model with various values of the maximum Doppler shift, f_D [8]. Let us define the normalized Doppler frequency as $f_{nD} = f_D T_D$, where T_D is the time interval of two consecutive channel matrices. In the following simulations, we consider f_{nD} range of 10^{-5} to 3×10^{-3} Hz.[1)]

In Fig. 2, we plot the occurrence rate of the block mode, p_B. In addition, we plot the failure rate in the block mode, p_F , that indicates

$$p_F = \Pr\,[\mathbf{P}_i \ne \mathbf{P}_{i+j}\,, \exists j \text{ for } 1 \le j \le K \,|\, \text{block mode}] \qquad (10)$$

for block length K and f_{nD}. In accordance with our intuition, p_B decreases and p_F increases as K increases. However, we note that the proposed scheme effectively activates the block modes even with a considerably large K. For example, with K=10 and f_{nD}=50 mHz, almost 60% of the channel matrices are processed by the block mode while maintaining p_F below 3×10^{-5}. Note that even with failure occurring in the block modes, the LR result obtained by the block mode is very similar to the exact LR result, and thus, the detection performance is almost the same.

Figure 3 compares the computational complexities of various LRAD schemes in terms of average number of the required complex multiplications n_m and divisions n_d at each channel matrix, respectively. The number of iterations in an LR process varies depending on the channel conditions, resulting in difficulties in deriving a compact closed-form for the complexity of the algorithm. Therefore, we count the number of computations whenever we meet the multiplication or division operations during the simulation and estimate the average. As to setting of K, we consider two kinds of cases. In the first case, we apply the optimal Ks which are separately optimized at each f_{nD} via simulation and thus are different at

1) This range covers various combinations of f_Ds and T_Ds. For example, if T_D=1 μs, the above range corresponds to f_D of 10 Hz to 3,000 Hz, and if T_D= 20 μs, f_D of 0.5 Hz to 150 Hz.

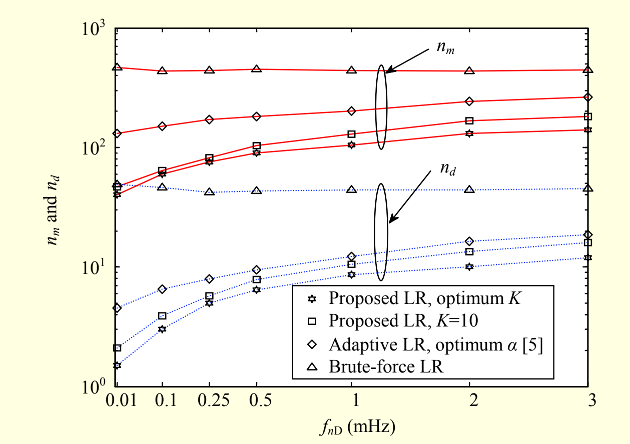

Fig. 3. Computational complexities of various LR schemes.

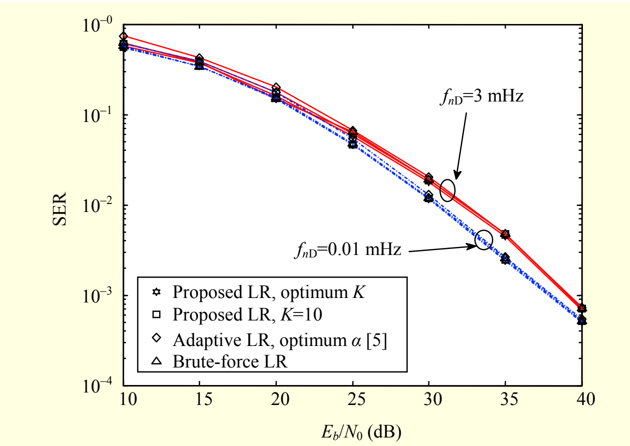

Fig. 4. SER performance of various LRAD schemes.

each $f_{n\mathrm{D}}$. It may be difficult to use the optimum K values adaptively in a real system. For this reason, we include the second case when we apply a constant global suboptimum value of K irrespective of $f_{n\mathrm{D}}$.

In Fig. 3, we note that the proposed LR scheme using optimum K requires much less complexity compared to the conventional LR schemes. For example, with $f_{n\mathrm{D}}$=0.5 mHz, the proposed LR scheme requires 20.4% (=90/440) multiplications and 14.9 % (=6.4/43) divisions of the brute-force LR, 50% (=90/180) multiplications, and 68.1% (=6.4/9.4) divisions of the adaptive LR scheme [5]. As we intuitively expect, it is shown that the complexity reduction becomes more significant in a low Doppler frequency region. We also plot the result for the case when the proposed scheme employs a constant value of K=10 irrespective of $f_{n\mathrm{D}}$. Even though there are slight increases of the computations, there still exists a significant gap between the proposed scheme with a constant K and the adaptive LR scheme. Considering the basic detection processing delay and the overall hardware dimension of the recent broadband communication systems, the detection latency and required memory size by the block length of 10 would be sufficiently feasible.

The symbol error rate (SER) performances of various LRAD schemes are compared in Fig. 4. For the adaptive LR scheme in [5], we properly set α in (5) so that it results in the maximum complexity reduction while achieving the same performance to the brute-force LR scheme. As shown in Fig. 4, the LRAD using the proposed scheme shows the same SER performance without any performance degradation. It is also shown that performance does not change even if we do not use the optimum K values but use a constant global suboptimum value (K=10) for the proposed scheme.

V. Conclusion

We presented a computationally efficient LR scheme for time-varying channels. Computational redundancy is significantly eliminated by using the block-mode LR that requires only a single matrix multiplication per channel matrix. The idea of the proposed block-mode concept can also be extended to frequency-selective fading channels, where we usually use multi-carrier transmissions such as MIMO OFDM. In this case, the block-mode LR can be applied to multiple subcarriers experiencing correlated fading.

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