

# Sweep-Based Plausible Elastic Deformations

Seung-Hyun Yoon, Choong-Gyoo Lim, and Myung-Soo Kim

**ABSTRACT**—We present a simple and efficient technique for a plausible elastic deformation of three-dimensional objects. An elastic sweep surface is constructed by interpolating key cross sections with positions, orientations, and boundary shapes determined by physical simulation of simple mass-spring systems. The deformable parts of an object are approximated by the elastic sweep surfaces, and the vertices of the deformable parts are bound to nearby sweep surfaces. As an external force is applied, the corresponding parts of an object change their shapes elastically. We demonstrate the effectiveness of our technique and show its real-time performance on mesh objects.

**Keywords**—Sweep surface, elastic deformation.

## I. Introduction

Sweeps are a procedural modeling technique for the representation of three-dimensional tubular objects. When an area or a volume moves along a prescribed trajectory in three-dimensional space, the swept volume of this moving object is generated and its boundary is a sweep surface.

In some recent studies [1]–[3], sweeping has been interpreted from a slightly different point of view. Instead of being used as a modeling tool for generating new objects, sweeps have been employed as control mechanisms for the modification of existing shapes. Hyun and others [1] introduced a technique for human body modeling and deformation using sweep surfaces. Yoon and Kim [3] extended this technique to the freeform deformation of three-dimensional objects [4]. In this letter, we

further extend these techniques to the plausible elastic deformation of three-dimensional objects.

## II. Elastic Sweep Surface

In this section, we describe our sweep surface model which is elastically deformable. We employ a sweep surface which is generated by interpolating key cross sections  $\{X_i\}$  as follows:

$$S(\theta, t) = C(t) + R(t) \cdot O_t(\theta) \\ = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} r_{11}(t) & r_{12}(t) & r_{13}(t) \\ r_{21}(t) & r_{22}(t) & r_{23}(t) \\ r_{31}(t) & r_{32}(t) & r_{33}(t) \end{bmatrix} \cdot \begin{bmatrix} r(\theta, t) \cos \theta \\ r(\theta, t) \sin \theta \\ 0 \end{bmatrix},$$

where  $O_t(\theta)$  represents a star-shaped cross-sectional closed curve using a scalar radius function  $r(\theta, t)$ ; and  $C(t)$  and  $R(t)$  describe its position and orientation, respectively. Each key cross section  $X_i$  is associated with a local transformation  $T_{i-1,i}$  (represented by a  $4 \times 4$  matrix) from the previous key cross section  $X_{i-1}$ . Thus, when a key cross section  $X_j$  changes its position and orientation, all the subsequent key cross sections  $X_k$  ( $k > j$ ) are automatically updated by a series of relative transformations. This arrangement facilitates intuitive deformations such as bending and twisting.

**Combining key cross sections with mass-spring systems.** We introduce three types of mass-spring systems to achieve independent elastic changes to the position, orientation and radii of a key cross section. An elastic change to the position of a key cross section  $X_i$  is realized by a positional mass-spring system, as shown in Fig. 1(a). The origin of  $X_i$  oscillates around its initial position during the simulation process. Although this positional mass-spring system generates a position in a three-dimensional space, the corresponding changes are limited to the straight line that connects the initial position with the current one; thus, it can be represented as a one-dimensional distance.

We also develop a rotational mass-spring system which can

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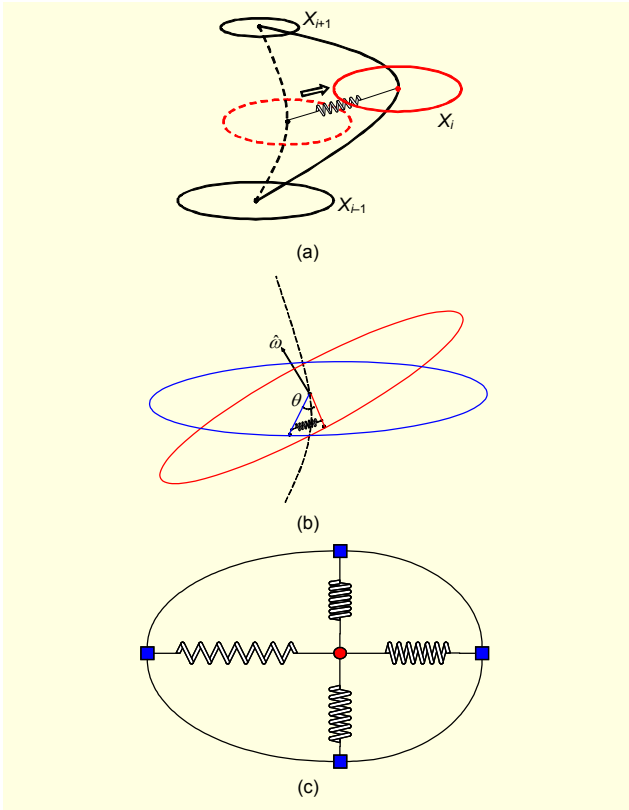


Fig. 1. Three mass-spring systems for the elastic changes of a key cross section.

make elastic changes to the orientation of a key cross section. All rotations in three-dimensional space can be represented by a rotational axis  $\hat{\omega}$  and a rotational angle  $\theta$  as shown in Fig. 1(b). We apply a mass-spring system to the rotational angle  $\theta$  to change it elastically while the rotational axis remains fixed. This rotational mass-spring system can generate deformation effects such as elastic twisting and bending of a sweep surface in a three-dimensional space.

Finally, we develop a radial mass-spring system to make elastic changes to the radii of a key cross section. Each key cross section has  $n$  radial handles which are positioned around each key cross section at uniform angular increments of  $2\pi/n$ . Figure 1(c) shows an example of a key cross section where four radial handles are connected to the origin of the key cross section using radial mass-spring systems. The boundary shape of a sweep surface which is represented by the scalar radius function  $r(\theta, t)$  is generated by interpolating the results of simulating these radial handles.

Although these mass-spring systems are quite simple and efficient, they have some limitations. Since each individual elastic change is limited to a single key cross section and a single angle or distance, such change is inadequate to generate a plausible shape deformation in a three-dimensional space. This limitation can easily be overcome by combining the

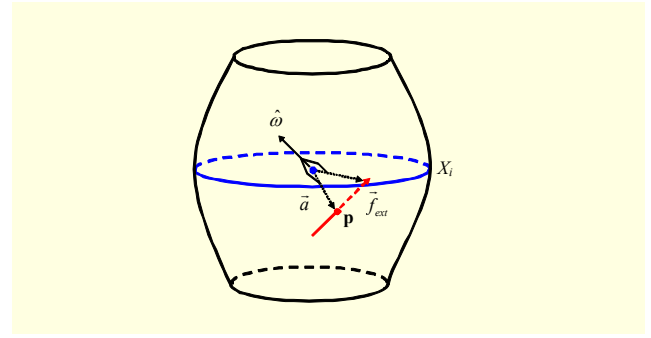


Fig. 2. Rotational axis.

simulation results from a sequence of key cross sections and interpolating them simultaneously, so as to generate more complicated and, hence, more plausible deformations of an elastic sweep surface in three dimensions.

### III. Sweep-Based Elastic Deformation

Using the elastic sweep surface introduced in section II, we further extend the sweep-based freeform deformation [3] to the elastic deformation of three-dimensional objects. Preprocessing steps for constructing control sweep surfaces and binding the vertices of deformable parts to the sweep surfaces are detailed in [3]. The main novelty is that we enhance a control sweep surface to become an elastic sweep surface and allow the user to apply external forces to these surfaces.

**External force.** An external force can be applied at a point on the sweep surface to initiate an elastic deformation. We decompose this external force into a rotational force and a radial force at each key cross section and then apply these component forces to the appropriate mass-spring systems at that key cross section. Using this approach, we can generate compound and plausible deformation effects of a sweep surface. Instead of deriving a translational force in a similar way, we use a radial force because combining rotational and radial forces gives more natural deformations.

Figure 2 shows an example of a force vector  $\vec{f}_{ext}$ , which is applied to a point  $\mathbf{p}$  on a sweep surface. First, we derive the rotational axis  $\hat{\omega}$  by computing and normalizing the cross product of  $\vec{a}$  and  $\vec{f}_{ext}$ . The rotational force is then computed as

$$f_{rot} = C_{rot} \cdot \|\vec{f}_{ext}\|,$$

where  $C_{rot}$  is a scale factor, which is necessary because the metric of a rotational force is different from that of a radial force. We apply the force around the rotational axis at the key cross section  $X_i$ . In general, the point  $\mathbf{p}$  will follow  $X_i$  to a large extent, which makes the resulting deformation behave as we would intuitively expect. Moreover,  $X_i$  can either be the closest

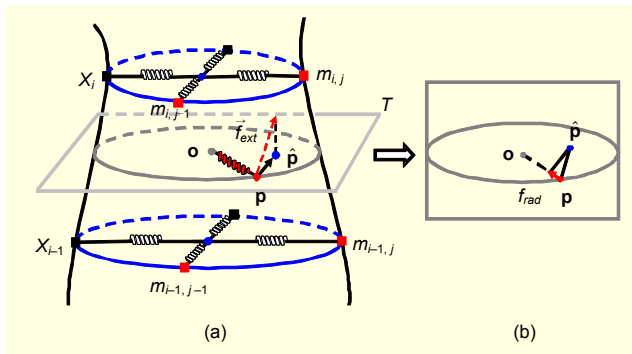


Fig. 3. Computing the radial component of an external force: (a) a longitudinal view of the sweep surface with the virtual mass-spring system shown in red and (b) the cross section containing the point  $\mathbf{p}$ .

key cross section to  $\mathbf{p}$ , or it can be chosen by the user.

Now, we explain how to compute the radial force from the external force  $\vec{f}_{ext}$ . Figure 3(a) shows the force vector  $\vec{f}_{ext}$  applied to a point  $\mathbf{p}$  on a sweep surface. A point  $\hat{\mathbf{p}}$  is obtained by projecting  $\vec{f}_{ext}$  onto the cross-sectional plane  $T$ , which contains  $\mathbf{p}$  (see Fig. 3(b)). The radial force  $f_{rad}$  can then be computed as

$$f_{rad} = C_{rad} \cdot \frac{\langle \vec{o\hat{p}}, \vec{p\hat{p}} \rangle}{\|\vec{o\hat{p}}\|},$$

where  $C_{rad}$  is a scale factor for controlling the radial force and  $\langle \cdot, \cdot \rangle$  denotes the inner product.

**Distribution of a radial force.** As shown in Fig. 3(a), the radial force  $f_{rad}$  should be applied to the virtual mass-spring system which connects the point  $\mathbf{p}$  to the origin  $\mathbf{o}$  of the cross-sectional plane  $T$ . However,  $T$  is not one of our key cross sections. Therefore, we distribute the radial force  $f_{rad}$  to the four nearest handles,  $m_{i-1,j-1}$ ,  $m_{i-1,j}$ ,  $m_{i,j-1}$  and  $m_{i,j}$  of the two adjacent key cross sections,  $X_{i-1}$  and  $X_i$ . Let  $(\theta_p, t_p)$  and  $(\theta_*, t_*)$  be the corresponding parameters of the point  $\mathbf{p}$  and the four radial handles  $m_{**}$  in  $(\theta, t)$  space. The radial force  $f_{rad}$  is distributed to four radial handles in inverse proportion to areas formed by  $(\theta_p, t_p)$  and the four corners  $(\theta_*, t_*)$  of the parameter domain. The distributed forces are then applied to these handles and allow the sweep surface to imitate the effect of the virtual radial mass-spring system at the cross section  $T$ .

**Experimental results.** Our technique was implemented using C++ on a P4 3.2 GHz PC with 1 GB main memory and an NVIDIA GeForce FX5700 128 MB graphics card. It showed real-time performance (20 frame/s) for the elastic deformation of a dinosaur model (28,000 vertices) shown in Fig. 4. An external force was decomposed into various components, and the bending and twisting effects were achieved by a rotational force, and the elastic change to cross

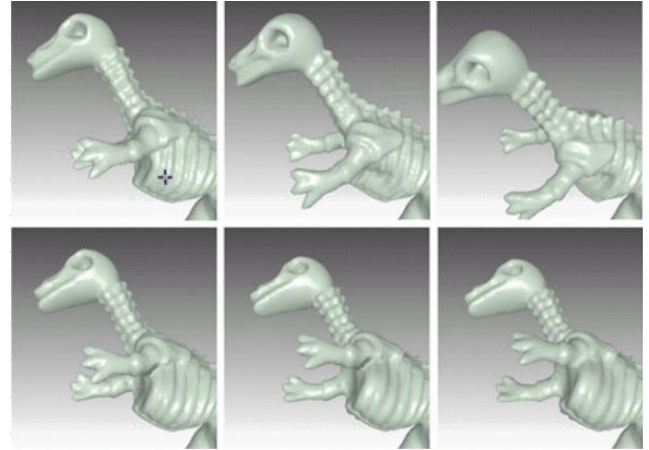


Fig. 4. A sequence of deformations of a dinosaur model: bending, twisting, and cross-sectional change are simulated simultaneously.

sections was achieved by a radial force. Our technique also provided deformation effects by decomposing the external force and applying its components selectively.

## VI. Conclusion

We have extended sweep-based modeling to include elastic deformation. Deformations are achieved by combining simple changes to key cross sections. Thus, our technique is easy to control, and can achieve real-time performance. Our technique offers various deformation effects by decomposing an external force into rotational and radial components and applying each of them selectively. We believe that our technique will enhance the plausibility of various real-time applications such as 3D games.

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